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The Angular Spread of Hard Cosmic-Ray Showers

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In order to account for the experimental results of Schmeiser and Bothe, the "dynaton" theory has to be modified by the introduction of a "form-factor" which reduces the probability of showers with large angular divergence. A particular invariant "cutting off" rule is discussed.

SCHMEISER AND BOTHE[1] have shown that in the hard cosmic-ray showers, the primaries and secondaries of which are both of the penetrating type, the directions of the shower particles are strongly correlated; the angular divergence of two particles is less than 10° on the average.

From the comparatively large number of these hard showers[2] it may be concluded that the majority of all penetrating cosmic-ray particles are involved in their production. For if only a few percent of the incoming particles, for instance only those of highest energies, were responsible for the hard showers, the cross section of those particles would have to be assumed so large that they could not pass for "penetrating" particles at all; at any rate the observations of Schmeiser and Bothe concerning the connection of the hard showers with the second maximum of the Rossi curve would be difficult to explain in this case. Thus there can hardly be any doubt that the primary radiation which produces the hard showers comprises the bulk of the penetrating cosmic radiation, which, according to present day knowledge, consists of heavy electrons having an average energy between 10^9 and 10^{10} ev. Obviously the secondary particles must be heavy electrons too, since they are distinctly more penetrating than ordinary electrons. It may be admitted, therefore, that the processes in question are nuclear processes in which one heavy electron, with an energy of the order 10^9 or 10^{10} ev in most cases, is absorbed or scattered and several heavy electrons of somewhat lower energy are created ("multiple emission process").

This is, indeed, also in general agreement with theoretical expectation if the heavy electrons are identified with the "dynatons" which, according to Yukawa's theory, give rise to the nuclear forces. The cross sections of the processes in question, as estimated by Heitler[3] on the basis of Yukawa's theory, are so large at energies of the order 10^9 ev that dynatons of about this energy may well be assumed to be the primaries of the hard showers.

In this case, however, as Heitler[3] remarked already, there arises a difficulty as to the explanation of the angular correlation of the hard shower particles. For, according to the unmodified quantum theory, a pronounced intensity maximum in

[1] Schmeiser and Bothe, Ann. d. Physik **32**, 161 (1938).

[2] Professor Bothe kindly informed the writer that the number of hard showers produced in 17 cm of lead (second maximum of the Rossi curve) is rather like, and certainly not much inferior to, the number of the ordinary soft (cascade) showers produced in 1.7 cm of lead (first maximum).

[3] Heitler, Proc. Roy. Soc. **166**, 529 (1938).

the forward direction is to be expected, just as in the case of Compton scattering, only if the momentum p of the incident particle is large in comparison with Mc (M = mass of the scattering proton-neutron), and the spread angle will then be of the order $(Mc/p)^{\frac{1}{2}}$. If this angle were to be as small as 5 or 10° as found by Schmeiser and Bothe, the primary energy would have to be assumed of the order 100 Mc^2 or 10^{11} ev, in contradiction with the foregoing estimation; at any rate it is hard to believe that the energy spectrum of the penetrating radiation has sufficient intensity above 10^{11} ev[4] to account for the many hard showers observed. (For this reason, Heitler[3] considers the possibility that the hard showers originate in impacts with electrons rather than with nuclei; but the assumptoin of so large an interaction between heavy and ordinary electrons can hardly be reconciled with the long lifetime of the β-radiators and of the heavy electron itself.)

Yet the observed angular correlation cannot be said to contradict the dynaton interpretation of the hard showers absolutely, since it is well known from other applications that Yukawa's theory, like quantum electrodynamics, must be assumed to break down in problems involving very large energies. This is usually expressed by saying that the high energy states have to be "cut off." As to the question where the cut should be made, the only information available up to now appears to be that resulting from theoretical speculations on the masses and the magnetic moments of the proton and the neutron, which are rather doubtful; *a priori*, of course, such a limitation remains arbitrary to some extent. Now the observations of Schmeiser and Bothe may be considered to furnish direct experimental evidence in this respect: The nonoccurrence of hard showers with larger angular divergence may have the simple meaning that these showers are forbidden by certain "cutting off" rules.

This interpretation may be supported by the following reasoning. As is well known, the high energy states are canceled automatically if the elementary particles are assumed to have a nonvanishing spatial extension, and the canceling rules are closely connected with the assumptions about the structure and the dimensions of the particles. In the mathematical formalism they manifest themselves in "form factors" which are, e.g., in the case of scattering, functions of $(r/\lambda)\sin\vartheta/2$, where r denotes the linear dimension of the scattering system, $\lambda=\hbar/p$ the wavelength and ϑ the scattering angle. The form factor equals 1 if $(r/\lambda)\sin\vartheta/2\ll1$; this defines the field of validity of the ordinary theory ($r=0$). On the other hand the processes with $(r/\lambda)\sin\vartheta/2\gg1$ are almost excluded. At high energies ($\lambda\ll r$) the domain of the allowed scattering angles is $\lesssim\hbar/rp$. If one identifies r with the classical electron radius (e^2/mc^2) or with the Compton wavelength of the proton ($\hbar/Mc$), the average scattering angle will be about 5° if the energy is of the order 10^9 or 10^{10} ev, respectively.

Of course the question remains open whether or how far there is a physical meaning in speaking of the spatial extension of elementary particles. The tentative introduction of form factors F may therefore only have the significance that the equation $F=1$ defines the domain in which the unmodified quantum theory holds true.

In the present case particularly, it is readily seen that a too straightforward introduction of form factors into the Hamiltonian does not help to remove the above-mentioned difficulty. Let the matrix element of the multiple emission process be expressed in the usual manner by products of the matrix elements of the corresponding single absorption and emission processes. If, then, the form factors representing the spatial structure of the proton and the neutron are introduced into the matrix elements of the *single* processes, the only success will be that the energy of each absorbed or emitted particle will be limited, but there will be no restriction as to the angles between the directions of the various particles.

However, the desired angular correlation may be obtained by multiplying the total matrix element of the compound process, irrespectively of the virtual intermediate states, with a "form factor" saying that the amount of the total change of momentum of the heavy particle (proton-neutron) shall be limited:

$$|\Delta\mathbf{P}|\lesssim\hbar/r, \tag{1}$$

i.e., that the large changes of momentum

$$|\Delta\mathbf{P}|\gg\hbar/r$$

[4] Compare Euler, Naturwiss. **26**, 382 (1938).

are forbidden. For, if one applies this to high energy collisions, the total momentum of the emitted particles will equal the momentum of the incident particle within an accuracy of the order $\hbar/r$, and consequently the expectation value of the angle between two emitted particles will be of the order $\hbar/rp$, in accordance with observation.

In order to be Lorentz invariant $|\Delta\mathbf{P}|$ should be measured in an invariant reference system, such as the system in which the total momentum vanishes, or the system in which the energy of the proton-neutron remains constant ($\Delta E=0$). These two systems do not, in general, coincide, but provided that $\hbar/r \lesssim Mc$, the limits defined by (1) will be practically the same whether $|\Delta\mathbf{P}|$ is measured in the one or in the other reference system or also, say, in the initial rest system of the proton-neutron. If (1) holds in the system where $\Delta E=0$, this condition reads

$$|\Delta\mathbf{P}|^2-(\Delta E/c)^2 \lesssim (\hbar/r)^2 \qquad (2)$$

in an arbitrary reference system.

A rule which establishes a discrimination between the various particles involved may appear strange at first sight. But it must be remembered that, according to the theory in question, the proton-neutron actually plays a privileged role, in that it remains present during the whole process, through all intermediate virtual phases. Thereupon one should also expect that the restrictive rule must be essentially the same when the particles involved are replaced by particles of similar nature (e.g., the proton-neutron by an electron, or a heavy electron by a photon). Thus, for instance, a shower of heavy electrons produced by an incident *photon* of corresponding energy should have the same angular spread, assuming that the "cutting-off" radius r is independent of the nature of the particles to be absorbed or emitted. Actually Schmeiser and Bothe[1] find that the hard showers appearing at the *first* maximum of the Rossi curve, most of which are probably created by the soft (photon) component of the cosmic radiation, have about the same angular distribution as those connected with the second maximum.

Another point to be discussed is the bearing of the form factor on the total cross section of the heavy electron. As has been observed by several authors, the probability of the multiple emission processes in question must be expected to be relatively large, compared with a multiple photon emission, for two reasons: Firstly[5] the parameter α, the powers of which define the successive approximations in the perturbation method, is probably rather large ($\alpha\sim 1/10$), at any rate considerably larger than the corresponding parameter in quantum-electrodynamics ($e^2/\hbar c \cong 1/137$); this can be deduced from the absolute magnitude of the nuclear forces. Secondly,[6] the matrix element of the interaction of the dynaton with the proton-neutron, provided that it is chosen such that the proton-neutron force gets the right spin character (essentially Majorana force), contains terms with an energy dependence which is by one degree higher than in the interaction of the photon with the electron in quantum-electrodynamics. Because of this the theoretical cross sections of high multiple emission processes increase very rapidly with increasing energy as long as no form factors are introduced, rather like the cross sections of multiple β-emission processes as discussed by Heisenberg on the basis of Fermi's β-theory.

For the following computation the heavy electron may be assumed to have the spin 1, for this is the most probable value according to the theories of the proton-neutron force.[6, 7] If such a particle, endowed with an energy $E \gg \mu c^2$ (μ = mass of the heavy electron), collides with a proton or neutron, the theoretical cross section of the process in which n heavy electrons are emitted into the solid angles $d\Omega_1, d\Omega_2 \cdots d\Omega_n$ $(1 \leqq n \ll E/\mu c^2)$ has the order of magnitude

$$\alpha^{n+1}\left(\frac{\hbar}{\mu c}\right)^2\left(\frac{E}{\mu c^2}\right)^{2n} d\Omega_1 d\Omega_2 \cdots d\Omega_n, \qquad (3)$$

at least for small scattering angles ϑ. Without form factors, this holds for any energy as far as $\vartheta \sim (Mc^2/E)^{\frac{1}{2}}$; the integral cross section would therefore be of the order

$$\alpha\left(\frac{\hbar}{\mu c}\right)^2\left(\alpha\frac{M}{\mu}\frac{E}{\mu c^2}\right)^n,$$

[5] Wentzel, Naturwiss. **26**, 273 (1938).

[6] Kemmer, Proc. Roy. Soc. **166**, 127 (1938); Bhabha, Proc. Roy. Soc. **166**, 501 (1938).

[7] Yukawa, Sakata, Taketani, Proc. Phys.-Math. Soc. Japan **20**, April, 1938; Fröhlich, Heitler, Kemmer, Proc. Roy Soc. **166**, 154 (1938); Stueckelberg, Helv. Phys. Acta **11**, 245 (1938).

and heavy electrons with energies above 10^9 ev would no longer be "penetrating," which, of course, can hardly be true. If, however, a form factor is added which allows only scattering angles $\vartheta \lesssim \hbar c/rE$, the integral cross section turns out to be of the order

$$\alpha\left(\frac{\hbar}{\mu c}\right)^2\left(\alpha^{\frac{1}{2}}\frac{\hbar}{\mu c}\frac{1}{r}\right)^{2n}. \qquad (4)$$

In this case the cross section will not only become constant with increasing energy but also the dependence on n will be comparatively weak, so that showers with many particles $(1 \ll n < E/\mu c^2)$ can be expected to occur with an accordingly reduced probability. The order of magnitude of the total cross section of all processes in question will not be very much larger than the cross section for simple scattering, which is obtained by putting $n=1$ in (4); also this value seems to be in substantial agreement with the experimental data.

Thus, the "canceling rule" as suggested above can be said to be well adapted to the observed facts, including the angular correlation as well as the absolute frequency of the hard showers.

Recently Heisenberg has expressed the opinion that the quantum-theoretical formulae for the transition probabilities should be applicable only to such processes in which the invariant changes of momentum $||\Delta\mathbf{P}|^2-(\Delta E/c)^2|^{\frac{1}{2}}$ remain below the finite limit $\hbar/r$ for *each* particle involved in the process.[8] If $\hbar/r < \mu c$, the absorption and emission of heavy electrons would then be altogether beyond the confines of the present theory. According to the above results, however, it seems rather, that the range of validity of the theory is *larger* than Heisenberg assumes, in that the restrictive condition applies *only* to the change of momentum of the *proton-neutron*, or, speaking more generally, to that particle which remains present during the whole compound process.

If several such particles cooperate in a process, the restricting rule must naturally be applied to each of them.[9] So, for instance, in the case of the proton-neutron collision, large changes of momentum of each particle will be forbidden, and this means, obviously, that the potential of the proton-neutron force remains finite at zero distance, the wave-lengths $\lambda \ll r$ being canceled in the Fourier integral. This warrants the finiteness of all quantities related to the proton-neutron interaction, such as collision cross sections or the binding energy of the deuteron, even in higher approximations of the perturbation method.

On the other hand, self-energies and similar quantities (e.g., the magnetic moments which can be calculated from the self-energy of the proton or neutron in an external magnetic field) are, obviously, not affected by the form factor proposed, and thus remain infinite. (But in this respect it would be of no use either to extend the restrictive condition (2) to the *single* virtual processes, since for instance the left-hand expression in (2) vanishes identically in the case of an absorbed or emitted light quantum and therefore the electromagnetic self-energy would remain unaltered.) Presumably infinities of this type can only be eliminated by means of a subtraction formalism such as has proved indispensable in the "hole theory" of the positron. It is generally admitted that this problem is closely connected with other fundamental problems the solution of which will require an improved knowledge of the phenomena occurring at high energies. The hope may be justified that such knowledge will be supplied by further cosmic-ray experiments and their theoretical examination.

In conclusion, the writer wishes to express his admiration and his gratefulness to Professor Arnold Sommerfeld on the occasion of his seventieth birthday.

[8] Similar ideas seem to dominate in a new theory suggested by Wataghin (compare the preliminary notes in Nature **142**, 393 (1938) and Comptes rendus **207**, 358 (1938)). A consistent explanation of all cosmic-ray phenomena will scarcely be possible on these lines.

[9] If the electron-positron-pair creation is interpreted according to the "hole theory," the question arises whether the particle which is lifted from a negative to a positive energy state is perhaps also subject to a rule like (2) (with the reversed sign of the left-hand expression).