TABLE I.

SUBSTANCE	CONTAMINATED WITH	EXPANSION RATIO	SIGN PREFERENCE
C_6H_6	Pure, B.P. 80.0°C.	1.8	No sign preference
C_6H_6	With 0.007% H_2O by weight	1.7	Negative
C_6H_6	With 0.057% H_2O by weight (Saturated)	1.5	Negative
CCl_4	Pure, B.P. 76.4°C	1.4	No sign preference

C_6H_6 saturated with water added to 90 percent pure C_6H_6 a definite sign preference was shown for the negative ion. The work on C_6H_6 has been repeated and measurements have been carried out to determine the minimum percentage of H_2O which could be detected. Measurements were made on carefully purified CCl_4. As further experiments in this direction had to be abandoned for the present because of the departure of one of us (J.W.B.), the results to date are here presented. The commercial C. P. grade C_6H_6 was carefully purified by standard chemical procedures and finally boiled in a reflex condenser over metallic Na for three hours. The fraction boiling at 80°C was used and all glassware used was heated to 100° to remove moisture from the gases. The commercial grade of C. P. CCl_4 was purified according to standard practice and was redistilled, the fraction boiling within 0.5°C of the accepted boiling point was used (see Table I). In the case of C_6H_6 with 0.007 percent H_2O the negative cloud was very weak; only a few drops were observed. Indications are that the condensation on the negative ions in this case was clearly due to water vapor present in small traces. The fields used to draw out ions in both cases were of the order of two volts per cm.

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[1] Loeb, Kip and Einarsson, J. Chem. Phys. 6, 264 (1938).

A New Form of the Electromagnetic Energy Equation When Free Charged Particles are Present

To deal with the energy relations in electron beam tubes a new form of the electromagnetic energy equation is required which expresses conservation of energy in a region in which both free charged particles and electromagnetic fields are present. At the outset we shall make the restriction that the electrons or other charged particles are so far distant from each other that classical rather than quantum theory applies, a condition which is fulfilled within many orders of magnitude in any vacuum device.

This restriction makes it possible to ignore the electromagnetic origin of mass, for the electric field and magnetic fields can each be divided into two parts, that arising from charges at a distance which is microscopically uniform, and that excess which is appreciable only in the immediate neighborhood of the individual charged particles. The excess electric field appears as the mass and the excess magnetic field as the kinetic energy of the particles. Accordingly, in the electromagnetic equations which will be used E refers to the electric field due to particles at a distance, or what amounts to the same thing, to the field of a smoothed-out charge distribution, and the same holds for H, the magnetic field.

The equation,

$$\frac{1}{2}\frac{\partial}{\partial t}\int_\tau (E^2+H^2)d\tau + c\int_\sigma \mathbf{E}\times\mathbf{H}\cdot d\boldsymbol{\sigma} + \Sigma\int \rho_i \mathbf{v}_i\cdot\mathbf{E}d\tau = 0, \quad (1)$$

then applies in a region evacuated except for the presence of groups of charge distribution ρ_1, ρ_2, etc. Each has the respective space distribution of velocity $\mathbf{v}_1$, $\mathbf{v}_2$, etc., and each is characterized also by the same particle mass, m_1, m_2, etc. This is a slight generalization of Eq. (155–2) of Page's *Introduction to Theoretical Physics*, second edition.

Let us confine our attention to a single charge distribution in the third term of Eq. (1). The dot product of $\mathbf{v}$ with the differential equation of motion

$$d\mathbf{v}/dt = (e/m)(\mathbf{E}+\mathbf{v}\times\mathbf{H}/c) \quad (2)$$

gives

$$\mathbf{v}\cdot\mathbf{E} = (m/2e)d(v^2)/dt. \quad (3)$$

This permits the elimination of E. As integrand we then have $\rho d(v^2)/dt$ which we convert to partial derivatives:

$$\rho d(v^2)/dt = \rho\partial(v^2)/\partial t + \rho\mathbf{v}\cdot\nabla(v^2). \quad (4)$$

If we bear in mind that ρ and v^2 are scalars and $\mathbf{v}$ a vector the last term can be rearranged by means of the relation

$$\nabla\cdot(u\mathbf{v}) = u\nabla\cdot\mathbf{v}+\mathbf{v}\cdot\nabla u$$

to give

$$\nabla\cdot(v^2\rho\mathbf{v}) - v^2\nabla\cdot(\rho\mathbf{v}).$$

We now note that

$$\nabla\cdot(\rho\mathbf{v}) = -\partial\rho/\partial t$$

and that

$$\rho d(v^2)/\partial t + v^2\partial\rho/\partial t = (\partial/\partial t)(\rho v^2)$$

so that Eq. (4) becomes

$$\rho\partial(v^2)/\partial t = \nabla\cdot(v^2\rho\mathbf{v}) + (\partial/\partial t)(\rho v^2).$$

By converting the volume integral of the second term above to a surface integral, and using $\boldsymbol{\psi}$ for the current density $\rho\mathbf{v}$, we find Eq. (1) takes the form

$$\frac{1}{2}\frac{\partial}{\partial t}\int_\tau (E^2+H^2)d\tau + c\int \mathbf{E}\times\mathbf{H}\cdot d\boldsymbol{\sigma} + \Sigma(m_i/2e)\frac{\partial}{\partial t}\int \rho_i v_i^2 d\tau + \Sigma(m_i/2e)\int_\sigma v_i^2\boldsymbol{\psi}_i\cdot d\boldsymbol{\sigma} = 0, \quad (5)$$

in which the rate of change of kinetic plus electromagnetic energy in the region is related to the energy flows of both types across the boundary.

Incidentally, we see that although the third term of Eq. (1) is the rate at which the electromagnetic field does work on the charges within τ this is not the rate of increase of kinetic energy in the region because of the outflow of kinetic energy across σ.

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