The multiplicity $M'(S, T; \lambda)$ associated with the irreducible manifold $[\lambda]$ is related to $M(S, T; \mu)$ by the equation

$$M(S, T; \mu) = \sum_{\lambda} (\lambda/\mu) M'(S, T; \lambda). \quad (46)$$

Eq. (46) can be used as a recurrence formula for the computation of $M'(S, T; \lambda)$; since $(\lambda/\lambda) = 1$,

$$M'(S, T; \mu) = M(S, T; \mu) - \sum_{\lambda \neq \mu} (\lambda/\mu) M'(S, T; \lambda). \quad (47)$$

One needs also the starting values

$$M'(S, T; 4\cdots 4) = 1, \quad S = T = 0,$$
$$= 0, \quad \text{for all other } S, T.$$

$$M'(S, T; 4\cdots 41) = M'(S, T; 4\cdots 43) = M(S, T; 1, 0). \quad (48)$$

$$M'(S, T; 4\cdots 42) = M(S, T; 0, 1).$$

Finally

$$M'(S, T_z; \lambda) = \sum_{T \geqq |T_z|} M'(S, T; \lambda) \quad (49)$$

and

$$M(S, T_z; \mu) = \sum_{T \geqq |T_z|} M(S, T; \mu). \quad (50)$$

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The Focusing of Charged Particles by a Spherical Condenser

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The paths of charged particles traversing a portion of an ideal spherical condenser are worked out. The section of the condenser considered is bounded by two rays, enclosing an angle Φ, from the common center of curvature, O, of the equipotential surfaces. It is shown that a group of particles, homogeneous in energy, leaving a point P on a normal to one of these boundaries and entering the condenser along this normal as a diverging bundle, will be brought to a focus at a point Q lying on the line PO extended, if the proper potential is applied to the condenser. This permits the whole condenser gap to be used as a focusing energy analyzer, or monochromator, of very large useful aperture. The velocity dispersion and reduced velocity dispersion are calculated for the most general case, and are found to take the same simple form as do the corresponding expressions for the limited homogeneous magnetic field spectrograph. The expressions for the reduced velocity dispersion are identical in the two cases. Compensation for edge effect is discussed. The relativistic modification of the theory required for high speed particles is discussed and results are presented which indicate that the simple theory of the electrostatic spectrograph may be inadequate even for fairly low values of v/c. It is suggested that this difficulty may be avoided by the choice of suitable instrument parameters.

An analyzer is described which has a useful aperture of 0.210 steradians, a theoretical reduced dispersion of 1010, and which requires a total focusing potential of 0.315 E, where E is the particle energy in equivalent volts. The operation of the analyzer in focusing electrons accelerated by a field designed to furnish an equivalent point source is described.

INTRODUCTION

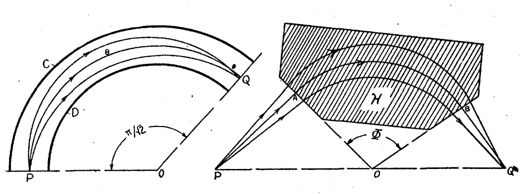
THE possibility of deflecting and focusing a slightly diverging beam of charged particles by means of a cylindrical condenser was first demonstrated by Hughes and Rojansky.[1] In Fig. 1(a), a beam of particles of the same charge and initial energy, diverging from P and traveling between the plates C and D of a cylindrical condenser, will be approximately focused at Q, if the circular arc PBQ, subtending an angle of $\pi/\sqrt{2}$ or 127° 17′, is the trajectory of those particles which leave P in a direction perpendicular to OP. This device is essentially an *energy*-analyzer, for the trajectory of a (nonrelativistic) particle in any given electrostatic field depends only on its initial position and direction and the ratio of its charge to its initial kinetic energy. Such analyzers have been incorporated in successful mass spectrographs.

[1] A. L. Hughes and V. Rojansky, Phys. Rev. **34**, 284 (1929).

A similar focusing effect is obtained in a uniform magnetic field, after traversal of 180°, for particles with the appropriate ratio of charge to momentum, a result so well known and widely applied as to make further description superfluous. Certain properties of the more general case, in which the extent of the magnetic field is limited, may be recalled, however. In Fig. 1(b), the homogeneous magnetic field (normal to the figure) in the shaded area is assumed to be cut off sharply at the boundaries OA and OB. Particles of the proper charge-to-momentum ratio leaving P and traveling near the normal PA, are approximately focused at Q, the intersection of the normal to the OB boundary and PO extended. The fact that the conjugate points lie on a line through O was pointed out by Barber.[2] In the corresponding general case of the cylindrical condenser, which has been investigated by Herzog,[3] no such simple rule holds, of course.

It will be noted that the focusing analyzers mentioned above are all two-dimensional, that is, they are analogous to optical systems composed of prisms and cylindrical lenses.

In the work to be described here, the focusing properties of the spherical condenser were investigated. The possibility of using a portion of a spherical condenser as an analyzer was suggested by Aston[4] in 1919. He remarked that particles of the proper energy entering the condenser in the proper direction would follow great circles and be united on the axis of the figure. The fact that there would also be a focusing action in a plane through the axis was not brought out, and no proof or further details were given. Our analysis will show that such a focusing effect does exist, and that it is described by formulas remarkably similar to those obtained



(a) (b)

FIG. 1.

[2] N. F. Barber, Proc. Leeds Phil. Soc. **2**, 427 (1933).
[3] R. Herzog, Zeits. f. Physik **89**, 447 (1934).
[4] F. W. Aston, Phil. Mag. **38**, 710 (1919).

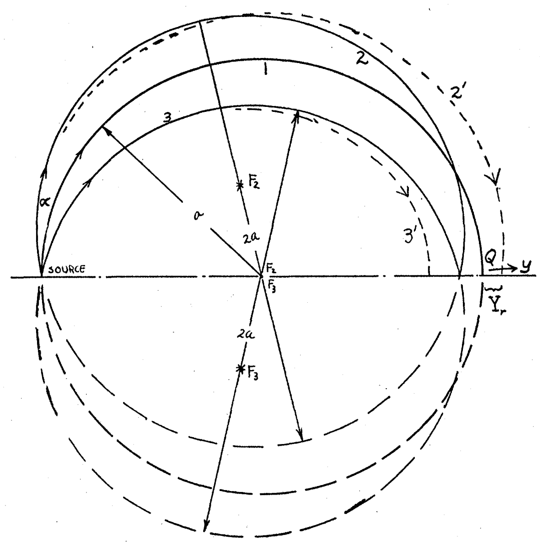


FIG. 2. The partial re-focusing of orbits of equal energy in an inverse square field.

for the general case of the homogeneous magnetic field spectrograph. Moreover it will be shown that the conjugate points are determined by a rule exactly corresponding to the relation found by Barber for the magnetic spectrograph; this result is particularly important here, for it permits the construction of a three-dimensional analyzer of very large useful aperture.

THEORY

If one recalls that Kepler orbits of the same total energy in a given field have the same major axis, it appears from the construction in Fig. 2 that approximate re-focusing of orbits passing through a given point, the "source," and grouped about the circular orbit through that point, is obtained after a revolution of 180°. Clearly, from the symmetry of the figure, the intercept, y, measured from Q, is an even function of α, the angular separation of a trajectory at the source from the circular trajectory, and hence vanishes to the order of αa in the neighborhood of $\alpha=0$. A slightly diverging bundle of trajectories through P will therefore be nearly focused at the "receiving slit," Q. The similarity to the homogeneous magnetic field spectrograph, in which all trajectories are circular, is here first evident. It is this point which encourages one to attempt the analysis of the more general case,

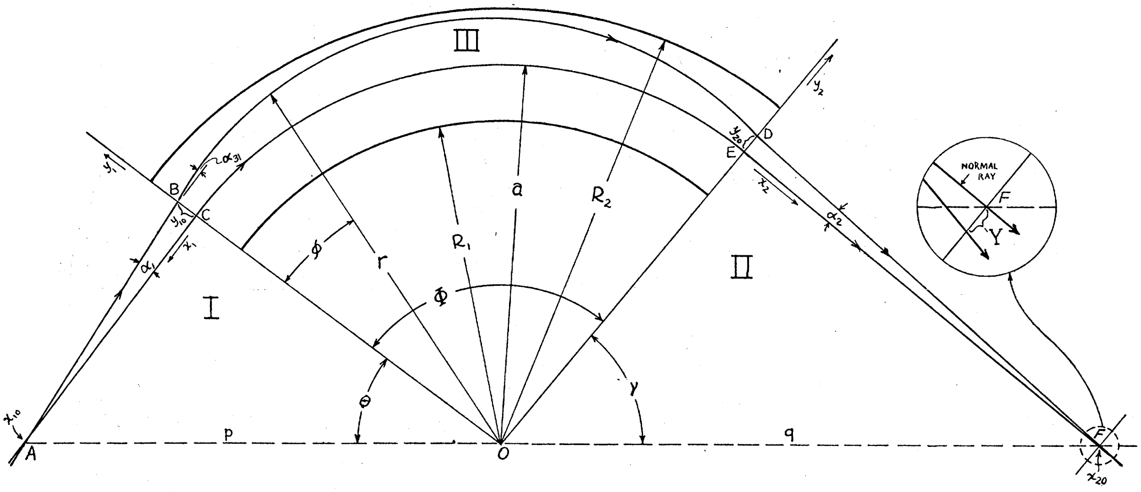


FIG. 3.

in which a section of the inverse-square field is considered; the "source" or "receiving slit" or both are permitted to lie in a field-free region outside, as in Fig. 3.

The problem will be treated as one of two dimensions. Only trajectories lying in a plane through the axis of symmetry of the condenser will be considered. Practically, this means that in the spectrograph finally suggested the source is supposed to be confined to this axis. Space charge will, of course, be neglected, as will, for the present, any modification required by relativity. The fringing field at the condenser edge will be ignored; the effect of this rather drastic simplification will be discussed later. The procedure, which is standard, is to calculate the trajectories approximately, by assuming α, the angle at the source between the trajectory in question and the "normal" trajectory, $ACEF$, to be small. Most of the results in which we shall be interested are obtained from a simple calculation carried only to the first power in α.

A particle of specific charge e/m travels along the trajectory $ABDF$ in Fig. 3. The first part of the trajectory, in the field-free region I, is defined by x_{10} and y_{10}, the intercepts on the x_1, y_1 axes. At the boundary $x_1=0$, the particle enters the electrostatic field between the spherical surfaces R_1 and R_2, which differ in potential by V_f. The original velocity, v_1, of the particle in region I, is: $v_1=v_0(1+\beta)$ if the circle of radius a is the proper orbit for a particle velocity v_0 entering along the x_1 axis. β is small, and represents a possible spread in velocity of the rays from the source. It is not necessary to consider a spread in mass as well, for the mass need not appear explicitly in the calculation. The particle leaves the field (again abruptly) at the boundary $x_2=0$ between the region III and the field-free region II. The problem is to find the path DF of the particle in II.

The radial field is $\mathcal{E}(r)=V_fR_1R_2/(R_2-R_1)r^2$, and the condition on the circular orbit requires that $\mathcal{E}(r)=amv_0^2/er^2$. It follows that, if $mv_0^2=2eE$, that is, if E is the energy in equivalent volts of the normal particle in I, and if $a=(R_1+R_2)/2$, the focusing potential must be given by:

$$V_f=E(R_2/R_1-R_1/R_2). \qquad (1)$$

From the equations of motion, $\ddot{r}-r\dot{\varphi}^2=-\mathcal{E}e/m=-k^2/r^2$, and $r^2\dot{\varphi}=A$. where $k^2=av_0^2$, we obtain, by making the substitution $u=1/r$, $d^2u/d\varphi^2+u=k^2/A^2$, the integral of which is:

$$u=P\cos\varphi+Q\sin\varphi+k^2/A^2. \qquad (2)$$

The boundary conditions are now applied and the powers of α_1 and β higher than the first are neglected. The equation of the trajectory in I is, $y_1=\alpha_1(x_{10}-x_1)$. Upon entering the field at $(0, y_{10})$ the particle experiences a change in kinetic energy which is: $ey_{10}\mathcal{E}(a)$, to the first order, or $v_{31}=v_0(1+\beta-y_{10}/a)$; v_{31} here means the velocity of the particle just after entering region III. The angular deflection of the path at the boundary is clearly of the second order. Now $A = r^2\dot{\varphi} = av_{31}(1+y_{10}/a) = av_0(1+\beta)$. If we define z by $u = 1/a(1+z)$, we can write the equation of the trajectory in *III*, to the present approximation, as

$$z = 2\beta - aP\cos\varphi - aQ\sin\varphi. \quad (3)$$

Determining P and Q by the position and direction of the trajectory at the *I–III* boundary we find that,

$$z = \alpha_1 \sin\varphi + (\alpha_1 x_{10}/a - 2\beta)\cos\varphi + 2\beta. \quad (4)$$

The particle enters the region *II* at y_{20} given by:

$$y_{20} = az(\Phi) = a[\alpha_1 \sin\Phi + (\alpha_1 x_{10}/a - 2\beta)\cos\Phi + 2\beta].$$

The angle $\alpha_2 = -dz/d\varphi$ (at $\varphi = \Phi$) $= -\alpha_1\cos\Phi + (\alpha_1 x_{10}/a - 2\beta)\sin\Phi$. Then in the region *II*,

$$\begin{aligned} y_2 &= y_{20} - \alpha_2 x_2 \\ &= a[\alpha_1\sin\Phi + (\alpha_1 x_{10}/a - 2\beta)\cos\Phi + 2\beta] \\ &\quad + x_2[\alpha_1\cos\Phi - (\alpha_1 x_{10}/a - 2\beta)\sin\Phi]. \end{aligned} \quad (5)$$

A bundle of "rays" from a source at $(x_{10}, 0)$ will be brought to convergence in *II* if y_2 can be made independent of α_1 for some value of x_2, say x_{20}. From (5), this requires that:

$$a(x_{10}+x_{20}) = \tan\Phi(x_{10}x_{20} - a^2). \quad (6)$$

It will now be shown that the conjugate points, which we may conveniently call the "source" and the "slit," lie on a line through O. In Fig. 3, $\tan\gamma = x_{20}/a$; $\tan\theta = x_{10}/a$; $\tan(\theta+\gamma) = a(x_{10}+x_{20})/(a^2 - x_{10}x_{20}) = -\tan\Phi$, by (6). This is the property found by Barber for the case of the homogeneous magnetic field. It is fortunate that it holds here, for it permits one to make a three-dimensional spectrograph, using the whole gap between the spherical shells for focusing. Our actual analyzer, then, would be formed like the surfaces generated by rotating Fig. 3 about AOF, and all particles of the proper energy leaving A in a hollow cone would be focused at F. If the above result did not hold, the source at A would project as a *ring* in region *II*.[5]

The velocity dispersion may now be obtained from (5). At x_{20} determined by (6) we have:

$$Y_2 = 2\beta a(1 - \cos\Phi + (x_{20}/a)\sin\Phi). \quad (7)$$

Y_2 is the distance, measured from the normal ray, EF, by which a particle with velocity $v_0(1+\beta)$ misses F, the focal point for particles of velocity v_0. If we call AO, the "source distance," p, and OF, the "slit distance," q, (7) reduces to: $Y_2 = 2\beta a(1+q/p)$. The velocity dispersion proper, D_v, is defined as Y_2/β. We then have:

$$D_v = 2a(1+q/p). \quad (8)$$

This is just twice the dispersion found for the case of the limited homogeneous magnetic field spectrograph.[6] It is perhaps more appropriate to speak of the *energy* dispersion of an electrostatic spectrograph. Since the energy dispersion D_e is just one-half the velocity dispersion, it will be given by exactly the same expression as is the velocity dispersion of the magnetic spectrograph.

A better figure of merit for an analyzer than the dispersion alone is represented by the *reduced dispersion*. This takes into account the broadening of the image of the source due to the fact that the focusing is only approximate, and that rays at an inclination α_1 to the normal ray will miss the focus by a small amount proportional to $(\alpha_1^2 + \cdots)$. If B is the "trace width" so caused, then the reduced dispersion, Δ, is defined by $\Delta = |D/B|$. This is equivalent to the *resolving power* (for complete separation of lines) for an analyzer with an infinitely small receiving slit. It will be the energy resolution if D_e is used, the velocity resolution if D_v is used.

In order to obtain B we must carry the previous calculation to the order of α_1^2. Let $\delta = y_{10}/a$; that is, $\delta = \alpha_1 x_{10}/a$. For convenience, the subscript will be dropped from α_1. Starting from (2), we wish to apply the boundary conditions with an accuracy of α^2. We are here considering only particles with $v_1 = v_0$, or $\beta = 0$.

[5] One might wonder whether a similar spectrograph of large aperture could not be made by a modification of the magnetic analyzer of Fig. 1 (*b*), in which the field would be "bent around in a ring." This was suggested by Stephens, Phys. Rev. **45**, 513 (1934), who was one of the first to study in detail the focusing by a sector of a magnetic field. This is not possible however, without modifying the shape of the field boundaries, for the requirement curl $H = 0$ in the space through which the particles pass introduces an inhomogeneity which is necessarily large enough to invalidate Barber's rule. It is essentially an additional inhomogeneity of this order which makes our results here so different from those for the cylindrical condenser.

[6] Brüche and Scherzer, *Geometrische Elektronenoptik* (J. Springer, 1934). See especially p. 142. Their z is the same as p/q here.

At the boundary I–III we must take into account not only the deflection of the path, but also the variation of the field with r. The change in kinetic energy at the boundary is $-mv_0^2\delta/(1+\delta)$. Then $v_{31}=v_0[1-2\delta/(1+\delta)]^{\frac{1}{2}}$. Now $v_{0r}=v_{31r}$. That is, the radial component of the velocity is not changed for the particle merely crosses the boundary suddenly into a region of different potential. Thus we can find $v_{31\varphi}$ from:

$$v_{31}^2=v_0^2[1-2\delta/(1+\delta)]=v_{0r}^2+v_{31\varphi}^2,$$

which yields:

$$v_{31\varphi}=v_0[1-2\delta/(1+\delta)-\alpha^2]^{\frac{1}{2}}.$$

Since

$$A=rv_{31\varphi}=av_0(1+\delta)[1-\alpha^2-2\delta/(1+\delta)]^{\frac{1}{2}}$$

the equation of the trajectory in III is:

$$1/(1+z) =aP\cos\varphi+aQ\sin\varphi+1/(1-\delta^2-\alpha^2). \quad (9)$$

Determining P and Q at the I–III boundary, we find: $aP=-(\delta+\alpha^2)$. $aQ=-(dz/d\varphi)/(1+z)^2=-\tan\alpha_{31}/(1+\delta)$, at $\varphi=0$. But $\tan\alpha_{31}=v_{31r}/v_{31\varphi}=\alpha[1-\alpha^2-2\delta/(1+\delta)]^{-\frac{1}{2}}$, which yields:

$$Q=-\alpha/a.$$

Writing δ' for y_{20}/a, or $z(\Phi)$, we obtain, after inserting the above values of P and Q in (9),

$$\delta'=(\alpha\sin\Phi+\delta\cos\Phi)+(\alpha\sin\Phi+\delta\cos\Phi)^2 +\alpha^2(\cos\Phi-1)-\delta^2.$$

But, as one can easily show, $(\alpha\sin\Phi+\delta\cos\Phi)=\alpha x_{20}p/aq=\alpha\sin\gamma/\cos\theta$, so that we have finally for δ':

$$\delta'=\alpha\sin\gamma/\cos\theta+\alpha^2\sin^2\gamma/\cos^2\theta +\alpha^2(\sin\theta\sin\gamma-\cos\theta\cos\gamma-\sec^2\theta). \quad (10)$$

The angle α_{32} must now be determined from $\tan\alpha_{32}=(dr/d\varphi)/r$ at $\varphi=\Phi$ and from this α_2 must be found, taking into account the velocity change at the boundary, which will, of course, involve δ'. After some calculation, in which the appropriate approximations are made, one obtains: $\tan\alpha_2=\alpha\cos\gamma/\cos\theta+\alpha^2\sin\Phi$, to the order of α^2. Now Y_2, the amount by which the trajectory misses F, will be given by: $Y_2/a=\delta'-\tan\gamma\tan\alpha_2$. Using the results above for δ' and $\tan\alpha_2$, and the identities arising from $\theta+\gamma+\Phi=\pi$, one finally arrives at:

$$Y_2/a=-\alpha^2(\cos^2\gamma/\cos^2\theta+\cos\theta/\cos\gamma) =-\alpha^2(p^2/q^2+q/p). \quad (11)$$

The trace width, B, is then $-a\alpha^2(p^2/q^2+q/p)$. The minus sign indicates that the rays on the outside of the bundle are bent too much and pass underneath F in Fig. 3. Now the expression above is just twice the corresponding expression for the homogeneous magnetic field case, and hence, from the result expressed in (8), the reduced velocity dispersions of the two types of spectrograph are identical. For each:

$$\Delta=\left|\frac{D}{B}\right|=\frac{2(1+q/p)}{\alpha^2(p^2/q^2+q/p)}=\frac{2/\alpha^2}{1-p/q+p^2/q^2}. \quad (12)$$

The maximum value of Δ is attained for $q=2p$ and is $8/3\alpha^2$. For the symmetrical case, $p=q$, $\Delta=2/\alpha^2$. For comparison we note[6] that the reduced dispersion for the cylindrical condenser is $3/2\alpha^2$.

The effect of finite source-width on the trace is also of practical interest. The path of a ray emerging from a point near the source, and traveling in a plane through the axis of the system, can be constructed if one makes use of the general expression (11) for Y_2, the expression (10) for δ', and the fact that the conjugate points are on a line through O. (The intersection of the ray in question with the normal ray from the source is taken as a new source.) It is hardly profitable to carry this analysis out unless one has settled on definite values for Φ and p/q. However the following statement can be made: If the source width is not much greater than α^2a, where α is the maximum divergence of rays from the normal ray at the source, the spreading of the trace due to source-width will be of the same order as the spreading due to the aberration expressed in (11). One can justify this statement most readily by recalling that any given trajectory is *reversible*. A spreading of this order will also be caused by those rays which do not travel in a plane through the axis of the condenser, when the source has finite extent.

The detailed calculation of these second-order effects is of doubtful value because of the uncertainty introduced by our neglect of the fringing field of the condenser. If the gap is small compared to the total path in the condenser, the first-order results should still be valid, and one can go further and use a grounded guard-diaphragm of the proper proportions, as suggested by Herzog,[7] to compensate for the edge effect as far as possible. This course was followed in the design of the present analyzer. Herzog's calculations were made for a plane condenser, and one cannot rely on the compensation to the order of α^2 because of the variation of the field strength across the spherical (or even the cylindrical) condenser. Thus strict validity cannot be claimed for the expression for the reduced dispersion, in a practical case; this is true, of course, for any kind of particle spectrograph except the 180° focusing magnetic spectrograph.

Correction for Relativity

In certain possible applications, involving high speed electrons, the modification of the above theory required by relativity may be important.[8] The extent to which the theory of the electrostatic spectrograph is affected by relativity does not seem generally to be appreciated. One might at first suppose that, since the trajectories with which one is concerned are nearly circular, and only small changes in the energy of the particles as they traverse the condenser are involved, the effect of the relativistic change in mass would first be felt in the second-order calculation, at least for reasonably small values of v/c. It turns out, however, that there will be, at the nonrelativistic focus, a spreading of the relativistic beam of the order αa, even when v/c is only a few tenths.

It is fortunately fairly easy to understand and to calculate what happens in the spherical condenser spectrograph, for we have essentially the problem of the relativistic Kepler orbits which arises in the calculation of the relativistic fine structure in the quantum theory. It is well known that the effect of the relativistic correction is to introduce a precession of the orbits,[9] essentially replacing the argument φ in (2) by $\gamma\varphi$, where, for orbits near the circular orbit, $\gamma \approx 1/(1+E)$, E being the particle kinetic energy, now measured in mc^2 units. In Fig. 2 the dotted trajectories, 2′ and 3′, have been roughly sketched in to show the modification of 2 and 3 caused by such a precession. It will be noted that although the orbits 2 and 3 are *very nearly* circular, the precession nevertheless shifts the intercept by a rather large amount, for the precession is about the appropriate focus, and not about the center of the orbit.

If we let Y_r represent the distance by which a relativistic trajectory misses the nonrelativistic focus, it is of interest to calculate k_1 in $Y_r/a = k_1\alpha + k_2\alpha^2 + \cdots$. In Fig. 2, k_1 is seen to be positive for the 180° case. The calculation is too long to be reproduced here. It has been carried out to the first order in α, but without any restriction on $\beta = v/c$. The result can be written in the following form:

$$k_1 = \frac{1+\beta^2}{1-\beta^2}(x_{10}/a)\cos\gamma\Phi + \frac{\sin\gamma\Phi}{\gamma} - \frac{2\beta^2(x_{10}/a)}{1-\beta^2} + (x_{20}/a)[\cos\gamma\Phi - (x_{10}\gamma/a) \times \sin\gamma\Phi(1+\beta^2)/(1-\beta^2)], \quad (13)$$

where $\gamma^2 = 1-\beta^2-2\beta^4\alpha$, $\beta = v/c$, and the other quantities have their former meaning.

The general symmetrical case, $x_{10}/a = x_{20}/a = \cot(\Phi/2)$ was investigated in some detail; k_1 was calculated numerically for $E = 0.2$ (100 kev electrons) for several values of Φ, including 90° and 180°. The interesting result of this calculation is that k_1 changes sign between these limits of Φ. For $\Phi = 90°$, k_1 is -0.48, and for $\Phi = 180°$, k_1 is 0.60, representing in each case a very serious departure from focusing. By trial, the zero of k_1 was found to lie very near $\Phi = 131°$. For 131°, $k_1 = 0.003$, for a 100 kev electron. k_1 is even less for lower energies, and is small clear up to 500 kev, the value for this energy is 0.064, and does not yet represent a serious spreading of the beam.

There is still one parameter available for adjustment, namely the ratio of x_{10} to x_{20}, or, what amounts to the same thing, p/q. It may be that a more favorable case could be found in which the coefficient of relativistic spreading is small by exploring the dependence of k_1 on this parameter as well. One would expect the same

[7] R. Herzog, Zeits. f. Physik **97**, 596 (1935).

[8] F. T. Rogers, Rev. Sci. Inst. **8**, 22 (1937) has calculated the effect of relativity on the *dispersion*, but not on the *focusing itself*.

[9] See, for example, Sommerfeld, *Atombau und Spektrallinien*, fifth edition, p. 272ff.

difficulty to arise in the cylindrical condenser spectrograph, but there, unfortunately the analysis, especially that of the general case, is more difficult.

The Construction and Operation of a Spherical-Condenser Spectrograph

The results of the preceding analysis seemed to warrant the construction of an experimental analyzer. The case $\Phi = 90°$, $p = q = \sqrt{2}a$, was chosen for its simplicity. The construction is easier because of the symmetry, and the reduced dispersion is still high ($\Delta = 2/\alpha^2$). The radius of the inner spherical surface was 8.69 cm, of the outer surface, 10.16 cm, making the total source-to-slit distance 21.65 cm. These dimensions yield a value of 0.315 for the theoretical ratio, V_f/E, of focusing potential to particle energy. The width of the annular aperture of the guard-diaphragm, i.e., the width of the beam at the entrance to the condenser, is limited by the requirement of clearance for the whole beam inside the condenser. Ideally, the maximum width of the beam inside is $\sqrt{2}$ times its width at the entrance to the condenser. After the aperture width (8 mm) has been chosen, the separation of the guard diaphragm from the condenser edge was adjusted according to Herzog's[7] curves.

Figure 4 is a cross section of the analyzer. It will be noted that the focusing electrodes, *1* and *2*, which are of spun copper, are supported between two heavier, pie-pan shaped plates (the grounded guard diaphragms) in which the entrance and exit apertures, *5*, are cut. The lavite blocks, *6*, insulate and space the parts. The analyzer, including the accelerating electrode structure beneath, is supported as a mechanical unit by the lavite feet, *11*, resting on the wall of the glass bulb. The entire instrument has threefold rotational symmetry.

It was desired to test the operation of the analyzer with electrons, and for this purpose a source was needed which, while acting effectively as a point source, would yield a hollow cone (semi-angle 45°) of electrons of the same energy. The problem is somewhat similar to the one which arose in the e/m measurement of Busch.[10] It was here solved by the use of accelerating electrodes arranged as shown in Fig. 5, which also shows the internally heated cathode in cross section. The latter is a nickel tube carrying an oxide-lined cup at one end. From the mouth of this cup electrons are accelerated in the desired directions if the potential V_b is near that of the cathode V_c, V_a is the plate potential. The central pin is maintained at the potential V_b by a wire which the electrons never "see." The alignment of the cathode is assured by two pins, *13*, in Fig. 4. The lens action of the accelerating electrodes was not a serious difficulty, and the source was entirely satisfactory.

At the other end of the spectrograph a Faraday cage collects the beam current through a 1.4-mm diameter slit. The Faraday cage assembly, which can be moved about from the outside, carries a small fluorescent screen, and during part of the work this whole assembly was replaced by a larger screen. The position of the slit can quickly be determined to 0.1 mm by the settings of the three tilting screws. The collector and the filament, and also, when desired, the whole accelerating electrode assembly, from *9* on down in Fig. 4, can be removed and replaced through the respective ends of the tube.

The glass envelope of the analyzer was an 11-liter flask, cut along the equator and sealed together again on the glass lathe. All leads are flexible and it will be noted that the lead to the inner sphere lies in one of the three narrow segments of the condenser which are closed to the beam; its presence did not disturb noticeably the adjacent parts of the beam. The analyzer was mounted with its axis along the earth's field and care was taken to avoid other magnetic disturbances. A weak field along the axis has a small and symmetrical effect on the focusing which can easily be estimated.

All potentials applied to the analyzer were tapped off a line of stable bleeder resistances connected across a 3000-volt rectifier. This is a convenient and satisfactory supply, for the operation of an electrostatic analyzer depends only on the ratios of the various potential differences. Fluctuations in the over-all voltage do not matter. Tapped resistors in the bleeder line allowed one to vary the electron energy by small, known steps, with the focusing potentials on the spheres held constant.

[10] H. Busch, Physik. Zeits. **23**, 438 (1922).

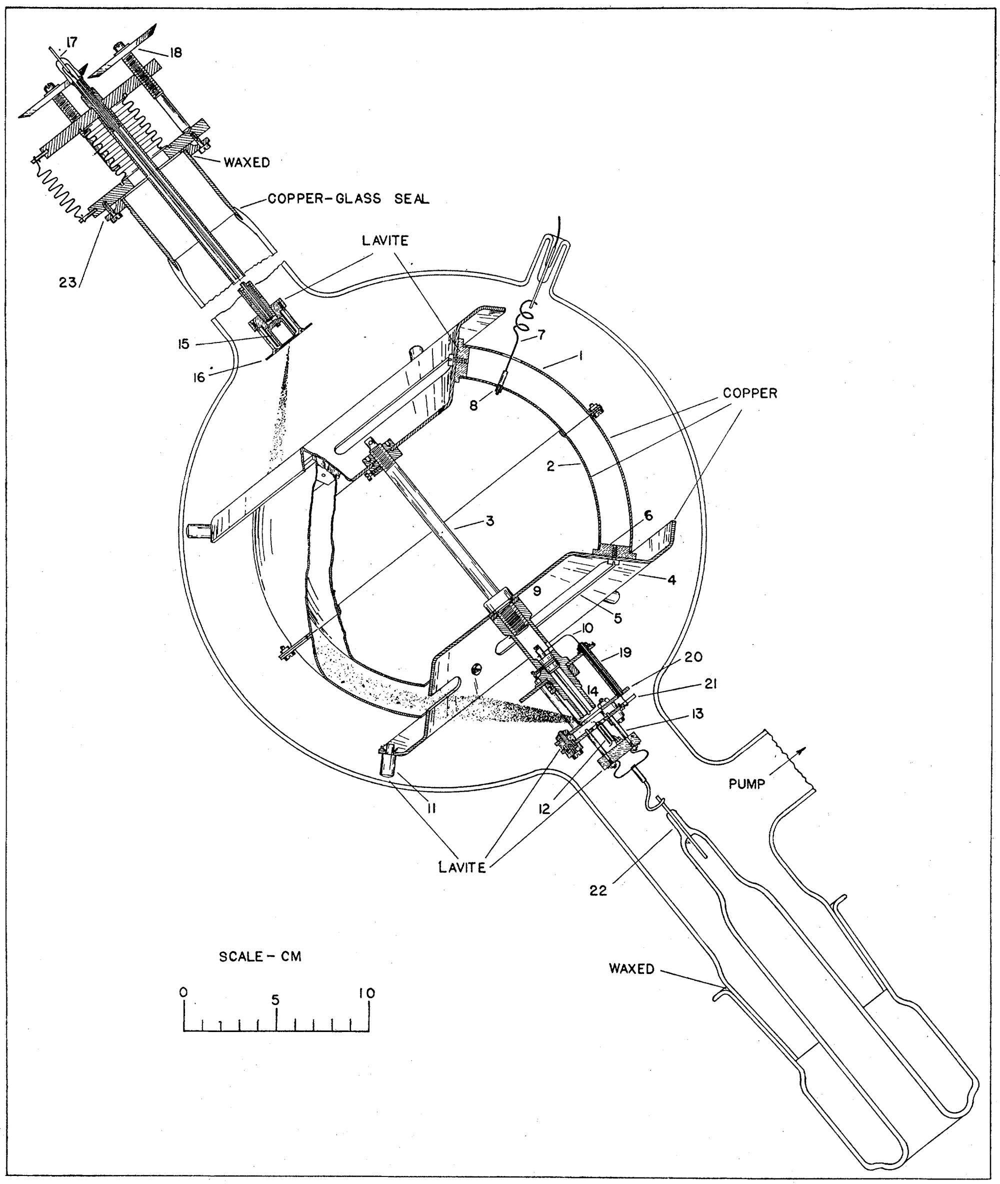


FIG. 4.

The analyzer was operated with the cathode about 2000 volts below ground, the inner and outer spheres being then about 300 volts above and below ground, respectively. Under these conditions a pattern was observed on the screen, but it did not have the simple form expected. For a given focusing potential one would expect to observe at approximately the correct plate potential, a ring, or rather three segments of a ring, which should shrink down to a spot as the electron energy is adjusted to exactly the right value. Instead, the different parts of the ring appeared to shrink into focus at somewhat different accelerating potentials. The resulting asymmetry of the pattern was rather simple, suggesting, at first, a source off the axis. It was finally shown, however, after several experiments, to have its origin in an asymmetry of the condenser itself; it became clear that the two spheres were not concentric. It is believed that an accidental dislocation of the parts during their rather severe ordeal in the glass lathe caused the trouble. A departure of about 1 mm from concentricity is indicated. One cannot get at the condenser to remedy this, and this lack of the flexibility which was sacrificed in favor of clean vacuum conditions in the present apparatus, was keenly felt at this stage. It was possible, however, to check the theory.

It should be remembered that an analyzer of this type is really "many spectrographs in parallel." If the spheres are not quite concentric it is, to the first order, as if the various spectrographs were not adjusted to focus at the same V_f/E value. By moving the receiving slit along a chosen perpendicular to the axis of the system at the focal point we can confine our attention, as we did in developing the theory, to one of the spectrographs, i.e., to a plane through the axis.

The current through the slit at successive steps of the accelerating potential was measured, for various positions of the slit off the axis. From the shifting of the current peaks one can find the dispersion, and from the narrowing of the peaks as the slit nears the theoretical focal point the focusing action can be observed. The results thus obtained agreed satisfactorily with the indications of the theory. The width of the trace at the focus was not appreciably greater than the width of the source itself, confirming the conclusions drawn in the discussion of second-order effects. Both from visual and electrical studies of the pattern it seemed likely that, had the condenser not been distorted, the entire beam leaving the exit aperture of the condenser would have entered the 1.4-mm diameter receiving slit when the focusing and accelerating potentials were in the indicated ratio of 0.315.

Conclusion

In the design of a charged particle spectrograph the particular compromise between resolution and useful aperture which is finally reached will depend on the use to which the instrument is to be put. The particular merit of the spherical condenser spectrograph is that, being in effect a three-dimensional instrument, it offers, for any desired resolving power, a very large aperture. This advantage is well illustrated by the constants of the instrument described above. The theoretical reduced dispersion is 1010; the actual useful aperture, measured in solid angle at the source, is 0.210, or 1/60 of the whole sphere. The results of the experimental work indicate that a considerably larger aperture could be used without serious trouble from edge effect and other second-order disturbances. The allowable aperture is governed to some extent also by the practical limitations on V_f/E.

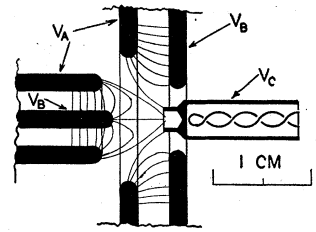


FIG. 5. Arrangement of cathode and accelerating electrodes, showing lines of force.

A disadvantage of the complete three-dimensional spectrograph is its inherent difficulty of construction. However, it is believed that in the light of experience a considerably better design than the present one can be evolved. This problem is being attacked at present with a view to some possible applications of the instrument. The simplicity and generality of the results of the theoretical analysis permit one to choose fairly easily the most practical form of the spectrograph for a particular application.

To Professor Bainbridge, who suggested this problem, the author is deeply indebted for advice and encouragement. The difficult task of assembling the glass envelope of the analyzer was accomplished by Mr. H. W. Leighton.