The Counting Losses in Geiger-Müller Counter Circuits and Recorders

HAROLD LIFSCHUTZ AND O. S. DUFFENDACK
University of Michigan, Ann Arbor, Michigan
(Received August 24, 1938)

The counting losses due to the finite recovery time of Geiger-Müller counter circuits and recorders are investigated. A description and critique of several experimental methods is given. The parallel method reported here gives the most accurate results. By this method the counting losses in a system consisting of a Geiger-Müller counter coupled by means of a Neher-Harper circuit to a scale-of-one recorder were determined. The method compares the counts registered by the scale-of-one against those registered by a vacuum tube scale-of-128 in parallel with it. The resolution times of both the scale-of-one and the G-M, Neher-Harper combination are thus found. The theoretical equations of Ruark and Brammer and Alaoglu and Smith are verified. The limitations and applicability of the Schiff-Volz formulation are also determined. Departures from the Ruark-Brammer formula for the Neher-Harper circuit are found at high counting rates. A corrected formula is derived. The speed of the Neher-Harper type of extinguishing circuit, determined in these experiments, is compared against the maximum counting speed possible with G-M counters. It is found that such circuits already approach the speed of the G-M tube itself. Existing vacuum tube scaling circuits and frequency meters are shown to be already faster than the G-M tube itself. The methods for correcting recorders and counter circuits for counting losses are given. These losses amount to about 20 percent for a Cenco recorder and one percent for most G-M counters at input rates of 1000 counts per minute. The losses increase rapidly with the input counting rate.

INTRODUCTION

THE statistical fluctuations in the number of disintegrations per minute of a radioactive substance give rise to the problem of low efficiency in the recording with simple apparatus of the individual disintegrations. In the case of a Geiger-Müller counter furnishing pulses to a scale-of-one recorder, it is well known that a large fraction of the counts are not registered by the recorder because of its finite resolution time even at rather low counting rates. Because of the statistical fluctuations in the time intervals between disintegrations, many of the counts come so close together as to fail to be resolved by the recorder. Since the short time intervals are the most probable, this effect is very large and increases rapidly with the counting rate. The recovery time of the Geiger-Müller counter and its coupling and quenching circuit (such as the Neher-Harper circuit) also causes counting losses due to this effect. Since the counting losses depend on the counting rate, the shape of observed radioactive decay curves will be distorted.

It is the purpose of the present paper to investigate the counting losses in a scale-of-one recorder and in the Geiger-Müller counter and Neher-Harper[1] coupling circuit, and to test the statistical theories of these losses which have been proposed.

METHODS

The efficiency of a recorder is defined as the ratio of the output counts to input counts. To test the efficiency of a recorder in the most direct way a known number of counts having a *random* distribution must be fed into it and the number registered observed. This procedure would have to be repeated for various input counting rates to obtain the efficiency as a function of the counting rate.

However, it is not possible directly to obtain such a known and variable random source of counts so that more indirect methods must be employed. Several methods may be used which may be designated as follows: (1) the film method;[2] (2) the multiple addition test method;[3] (3) the inverse square method; (4) the variable area method and (5) the parallel method.[4] We have in the past extensively investigated methods 1, 2, and 5 while methods 3 and 4 have sometimes

[1] H. V. Neher and W. W. Harper, Phys. Rev. **49**, 940 (1936).

[2] H. Lifschutz, O. S. Duffendack and M. M. Slawsky, Phys. Rev. **51**, 1027 (1937).

[3] O. S. Duffendack, H. Lifschutz and M. M. Slawsky, Phys. Rev. **52**, 1231 (1937).

[4] H. Lifschutz and O. S. Duffendack, Phys. Rev. **53**, 941 (1938). The present report modifies the conclusions given in this note.

been used by others in a qualitative way, mostly to test the speed of certain recording circuits.

A brief description and critique of the above methods may be given. In the film method, the electrical output of a G-M counter and Neher-Harper circuit actuated by a constant radioactive source was recorded by a commercial Western Electric sound system on standard 32 mm film. This sound system had an over-all frequency response flat to 10,000 cycles per second. This was about 150 times the speed of the single scale recorder to be calibrated. The recording of the G-M pulses was made at such a slow counting rate that the counting losses in the recording were very small and a quite accurate Poisson distribution of pulses was obtained on the film. This distribution was quantitatively tested by the method of Marsden and Barratt[5] in which the number of intervals of various lengths between pulses on the film was determined. A plot of the probability, $P(x)$, of an interval of length, x, *versus* the interval, x, showed good agreement with the theoretically predicted curve with a Poisson distribution. Of course small deviations would probably be masked by statistical fluctuations and the limited sensitivity of the method. The number of pulses on the film was counted visually. The film was then used as a known source of random counts by playing it back with the use of a standard sound head and photo-cell amplifier. The pulses from the film were fed to a single scale recording circuit operating a Cenco counter. The input to this recorder was determined from the measured running speed of the film and the total number of counts. The input rate was varied by varying the speed of the film. Thus a calibration curve for the single scale recorder could be determined.

The multiple addition test method is an approximation method. It makes use of a number of weak natural radioactive sources, the number of counts from which have been measured individually by the circuit being calibrated. The sources are so weak that the counting losses from one source are assumed to be negligible. These calibrated sources are then used in combination and the sum of the individually determined values is taken as the true input. A simple source holder with substitution dummy sources keeps the radiation scattered into the G-M counter constant. When only two sources are used this method reduces to the well-known simple addition test used for testing the linearity, with source strength, of counter circuits. However, the simple addition test cannot be used for calibrating a counter circuit. At most it can be used to determine when the departure from linearity becomes large, i.e., at what counting rates counting losses become appreciable.

The inverse square method makes use of the inverse square law of the variation of input counts of the source as a function of the distance from the counter. The variable area method employs a diaphragm which screens off the source from the counter. By varying the area of an opening in the diaphragm, it is assumed that the input counting rate to the G-M tube may be varied proportionately.

Before describing the parallel method the limitations of the previous methods may be pointed out. The film method determines the losses in the recording circuit of the G-M system only and not in the G-M tube and quenching circuit itself. The other three methods include the effect of losses in the G-M circuit. Statistical fluctuations are very small in the film method, extremely large in the multiple addition method, and fairly large in the inverse square and variable area method. In the film method the number of pulses on the film is fixed; fluctuations arise because of the possibility of changing distributions. The fluctuations are of the order $(\Delta N)^{\frac{1}{2}}$, where ΔN is the number of counts lost. In the multiple addition method the fluctuations in the individual calibrations go as $N^{\frac{1}{2}}$, where N is the total number of counts recorded. When the sources are added the total error in the strength of the combined source due to the errors in the individual sources caused by fluctuations may be very great. This makes it necessary to record an enormous number of counts. This consumes too much time at the low counting rates which must necessarily be used. The fluctuations in methods 3 and 4 are proportional to $N^{\frac{1}{2}}$ and are greater than that in the parallel method where again the fluctuations are of order $(\Delta N)^{\frac{1}{2}}$. The width of the pulses in the film method changes as the speed of the film is changed. This restricts the input counting rate to

[5] Marsden and Barratt, Proc. Phys. Soc. **23**, 367 (1911).

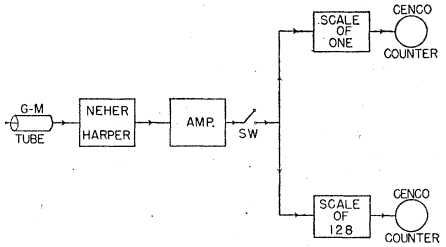

FIG. 1. Schematic diagram of set-up for parallel method.

certain limits such that the width of the pulses remains less than the resolution time of the recording circuit. Difficulties arise in methods 3 and 4 because of scattering, the lack of true point sources, and the non-uniformity of the flux of radiation over the diaphragm area. Also geometrical uncertainties in applying the inverse square law to the G-M counter cause difficulties. Many other limitations in the above methods relating to accuracy of results, flexibility, ease of application, requirements of special apparatus and sources, speed of the method and limits of counting range over which they are practicable, could easily be pointed out. In short, all the above difficulties may be best overcome by employing the parallel method. The precision and range of the results of this method are such that it only will be discussed further. The data to be presented also refer only to this method. Results were long ago obtained by the parallel method with thyratron scaling circuits. It was not until the recent development of high speed vacuum tube scaling circuits, however, that complete results were obtainable over wide enough input counting rates to allow a real test of the theory.

Experimental Arrangement and Procedure

The experimental arrangement for the parallel method is shown schematically in Fig. 1. The G-M counter is actuated by a constant radioactive source and is coupled by means of the Neher-Harper circuit to a scale-of-one and a scale-of-128 recorder in parallel. The circuit diagram of the Neher-Harper circuit and scale-of-one[6] is shown in Fig. 2. The scale-of-128 consisted of the high speed vacuum-tube type recently described[7] and was obtained by using a scale-of-16 and a scale-of-eight (both of the vacuum-tube type) in series. The resolution time of the vacuum tube scaling circuits was shown to be 6.5×10^{-6} second for input pulses of width 3.25×10^{-6} second.[7] The scaling ratio of 128 employed was so large that counting losses in the Cenco counter fed by the scaling circuit could be taken as negligible and actually will later be shown to have been zero. The G-M tube used was constructed of a solid nickel cylinder (about 2 cm long and 1.2 cm in diameter) and 3 mil tungsten anode wire. It was filled with hydrogen to a pressure of 136 mm of Hg. The method of preparation was as given previously[3] with the addition of thorough outgassing of the nickel cathode by means of an induction furnace to ensure a more permanent freedom from foreign gases. The counter had a long plateau of over 600 volts, which was quite flat over most of this range. The counter voltage used was 1200 volts; the counter threshold was at 1000 volts. A stabilized power supply was used to furnish this voltage to the counter. Inspection of the pulses generated by this counter, by use of a cathode-ray oscillograph showed the counts to be sharp, single pulses of uniform amplitude without the presence of trains of spurious pulses of varying amplitude as often seen in poor counters. It was concluded that, except for losses due to its finite resolution time, this counter furnished a true Poisson distribution of pulses when actuated by a radioactive source and was thus suitable for testing the statistical counting loss theories which were fundamentally based on such a distribution.

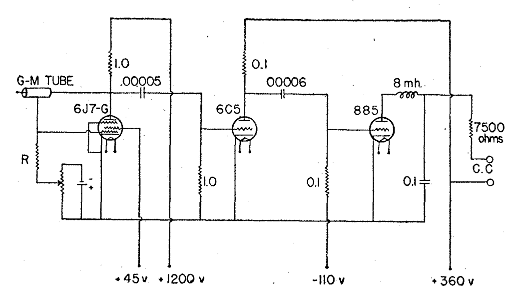

FIG. 2. The Neher-Harper circuit and scale-of-one recorder. Resistance values are in megohms and capacitance in microfarads. The amplifier indicated in Fig. 1 is not shown in this diagram.

The parallel method compares the number of counts recorded by the scale-of-one against that

[6] J. R. Dunning, Rev. Sci. Inst. 5, 387 (1934).
[7] H. Lifschutz and J. L. Lawson, Rev. Sci. Inst. 9, 83 (1937).

recorded by the scale-of-128. If the counting losses in the G-M circuit are negligible the scale-of-128 reading may be taken as the true number of counts, n_0, because the scaling circuit records all the counts coming from the N-H circuit. In this way the true input to the scale-of-one is determined and the output is simultaneously observed. This allows a direct calibration of the scale-of-one and also a direct test of any counting loss theory by giving the output as a function of the input. Since the input to both recorders is the same because the readings are simultaneous, it is clear that the statistical fluctuations in this method are very small. If the losses in the G-M circuit are not negligible for the counting rates used, the scale-of-128 does not record the true input, n_0, but does record all the pulses coming from the Neher-Harper circuit. Since the reading of the scale-of-one and also that of the scale-of-128 is a function of n_0, the simultaneous readings of the two recorders are described by two equations from which the unknown, n_0, may be eliminated. The way in which quantitative results are obtained may be found by examining the theory.

Theory

Ruark and Brammer,[8] and Alaoglu and Smith,[9] have given statistical theories of the counting losses in recorder circuits and have included the effect of losses in the G-M circuit. They have derived the following formulae for the observed readings of the recorders as a function of the true random input, n_0.

$$n_1 = \frac{n_0}{1+n_0\sigma} \exp\left(-(\tau_1-\sigma)n_0\right) \quad \text{if} \quad \tau_1 \geqslant \sigma, \tag{1a}$$

$$n_n = \frac{n_0}{n} \cdot \frac{1}{1+n_0\sigma} \quad \text{if} \quad \tau_n \leqslant n\sigma. \tag{1b}$$

Here n_1 is the observed reading of the scale-of-one, n_n that of the scaling circuit, n is the scaling ratio used, τ_1 is the resolution time of the scale-of-one and σ that of the Neher-Harper circuit. We distinguish τ_1 and τ_n as τ_n refers to the Cenco counter stage fed by the scaling circuit. These τ's are of the same order of magnitude. Here, $n=128$ and for convenience we will write hereafter τ for τ_1, and n_{128} for $n_n \cdot n$ which gives

$$n_{128} = \frac{n_0}{1+n_0\sigma}. \tag{1c}$$

The recovery time is defined as the time interval after a count which must be allowed before the circuit is ready to record another count. Clearly the statistics of a recording circuit will be different according to whether or not the recovery time of the circuit is affected by the occurrence of another count during the period of recovery. This circumstance gives rise to two formulae describing the two opposite extremes of recorder behavior. For recorders such as the Cenco counter, which can be re-excited during the period of recovery, the efficiency formula for the case of a true random input is

$$n_1 = n_0 \exp(-n_0\tau). \tag{2a}$$

This type of circuit is designated as Type I. A Type I recorder requires that a count, to be recorded, must be preceded by an interval τ during which no count arrives at the recorder. If the recovery time is completely unaffected by additional counts coming in during the recovery period we obtain a Type II recorder for which the efficiency formula is

$$n_1 = \frac{n_0}{1+n_0\tau}. \tag{2b}$$

In order to test the theory, the circuits used must satisfy the assumptions made in deriving the formulae. In referring to Eq. (1a) it is clear that with $\tau > \sigma$ and $\sigma \rightarrow 0$ Eq. (1a) becomes

$$n_1 = n_0 e^{-n_0\tau}. \tag{3}$$

In other words the recorder circuit used to test Eq. (1a) must be a Type I recorder. The scale-of-one shown in Fig. 2 is such a recorder even though the Cenco counter is driven by a thyratron since the thyratron circuit is so much faster than the mechanical counter itself. This scale-of-one was also chosen because it is a widely used circuit which is very simple, reliable and faster than most other scales-of-one developed to date.[10]

[8] A. E. Ruark and F. E. Brammer, Phys. Rev. **52**, 322 (1937).

[9] L. Alaoglu and N. M. Smith, Jr., Phys. Rev. **53**, 832 (1938). See also W. B. Lewis, Proc. Camb. Phil. Soc. **33**, 549 (1937).

[10] See, e.g. W. H. Pickering, Rev. Sci. Inst. **9**, 180 (1938).

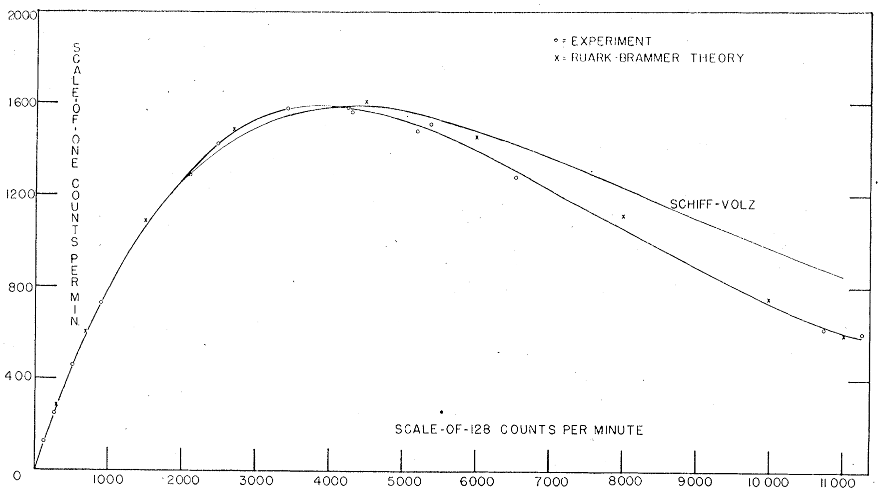

FIG. 3. The response curve for $R=20$ megohms.

Other conditions of the theory that must be met are that the recovery time of the Kth stage of scale-of-two in the scaling circuit does not exceed $2^{K-1}\sigma$, or, in other words, if we consider the first stage, its recovery time must not exceed that of the Neher-Harper circuit. From the measured resolution time of the scaling circuit given previously and preliminary measurements of σ showing it to be of the order $10^{-3}-10^{-4}$ second, it is clear that we have satisfied this condition. Likewise, it is clear that the condition $\tau_1 \geqslant \sigma$ is met and also the condition $\tau_{128} \leqslant 128 \cdot \sigma$, since the scaling ratio of 128 is so great. Moreover, rough preliminary measures of τ and σ may easily be made which also show that these latter conditions are met and later these conditions may be checked again with the final values of τ and σ.

We conclude that the circuit arrangement chosen satisfies all the conditions for providing a test of the theory.

The Eqs. (1a) and (1c) are now used to eliminate the unknown input n_0 and give

$$n_1 = n_{128} \exp\left[-n_{128}(\tau-\sigma)/(1-n_{128}\sigma)\right]. \quad (4)$$

If one takes $dn_1/dn_{128}=0$, a maximum of n_1 is found when

$$\sigma = \frac{1-[n_{128}(\tau-\sigma)]^{\frac{1}{2}}}{n_{128}}. \quad (5)$$

Defining the percent loss P as

$$P = 100 \times (n_{128}-n_1)/n_{128},$$

taking dP/dn_{128} and letting $n_{128} \rightarrow 0$, we find the slope, S_0, of the percent loss curve at the origin is given by

$$dP/dn_{128} = S_0 = (\tau-\sigma) \times 100, \quad n_{128} \rightarrow 0. \quad (6)$$

Eqs. (5) and (6) allow both τ and σ to be found from experimental observations of S_0 and the value of n_{128} when n_1 is a maximum.

The experimental procedure is now clear. By using a constant radioactive source and varying its distance from the G-M counter, the experimental curves $n_1 = f_1(n_{128})$ and $P = f_2(n_{128})$ may be obtained. The values of τ and σ are found from these curves as described above. The values are put back into the equations and the curves above computed theoretically. This gives a check of the theory. Moreover, τ and σ may also be found by independent methods and compared against the values of τ and σ found from the experimental curves. These values of τ and σ will be denoted by τ_{direct} and σ_{direct}. The value of τ_{direct} was found by applying *periodic* saw-tooth pulses from a sweep-circuit oscillator to the scale-of-one recorder and increasing the frequency till the maximum counting speed of the recorder was reached. This was determined from the reading of the recorder itself over a time interval measured with a stop watch. From the definition of recovery time it is clear that

$$\tau_{\text{direct}} = 1/n_{\max} \text{ (periodic)}. \quad (7)$$

Similarly, σ_{direct} was found by observing the

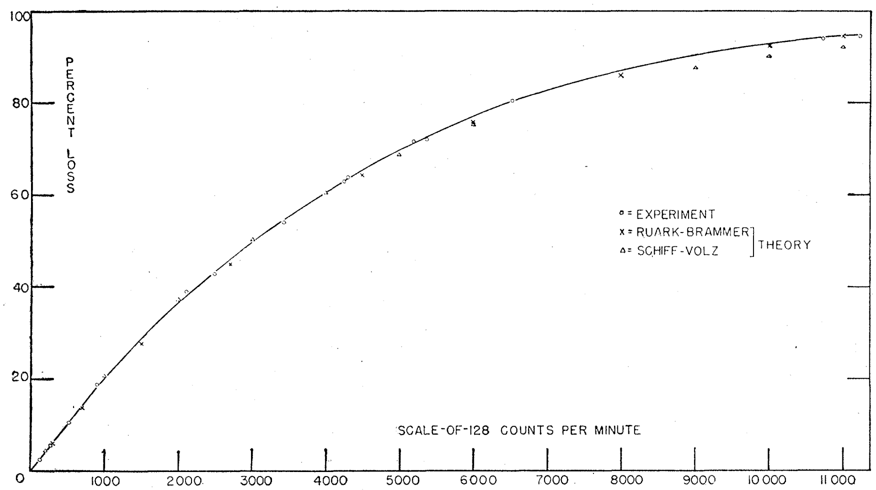

FIG. 4. The percent loss curve for $R=20$ megohms.

maximum *random* counts from the Neher-Harper circuit by means of the scale-of-128. This was accomplished simply by bringing up a strong source. Eq. (1c) shows that

$$\sigma_{\text{direct}}=1/n_{128\ \max}\ (\text{random}). \qquad (8)$$

Experimental Results

Figures 3 and 4 show the experimental curves for n_1 and P as functions of n_{128}. The predicted maximum in n_1 is observed. It will be noted that this maximum is very broad which makes an accurate determination of n_{128} at the maximum very difficult. On the other hand, the slope at the origin in Fig. 4 may be accurately determined and so also the value of $\tau-\sigma$. The value of σ is very sensitive to the value taken for n_{128} at the maximum. Since in the case here considered σ is quite small relative to τ it was decided to use for σ the value of σ_{direct} and for $\tau-\sigma$ the value from the slope of the percent curve. These values were put into the theoretical equations to calculate the points marked "Ruark-Brammer Theory." It is seen that the agreement with the experimental curves is very good. These curves were taken by using a grid resistor, R, in the Neher-Harper circuit of 20 megohms, a correct value for the G-M tube used.

In order to allow a more complete test of the theory, σ was purposely increased by increasing the Neher-Harper grid resistor to 100 megohms and the experiments repeated. The results are shown in Figs. 5 and 6. The decrease in the slope of the percent curve is found as predicted. The value of σ relative to τ was now large enough to determine τ and σ from the observed slope and maximum in spite of the broadness of the maximum. By taking as observed values $n_{128}=3850\pm125$ counts per minute, and $\tau-\sigma=1.066\times10^{-2}$ sec. the theory gives $\sigma=0.27\times10^{-2}$ sec. and $\tau=1.34\times10^{-2}$ sec. Over the limits given for n_{128}, σ varies from 0.24×10^{-2} sec. to 0.30×10^{-2} sec. These limits include the value of $\sigma_{\text{direct}}=0.286\times10^{-2}$ sec. The values of τ and σ determined from the curves as above were put into the theoretical equations and the curves calculated again as shown in Figs. 5 and 6. The agreement between theory and experiment is again very good. Although the value of σ is very sensitive to the value of n_{128} taken, the shape of the curve is not especially sensitive to σ so that the agreement with experiment is substantially the same over the limits of σ given as for the curves shown with $\sigma=0.27\times10^{-2}$ sec.

The comparison of the values of τ and σ with τ_{direct} and σ_{direct} is shown in Table I where, as previously stated, for $R=20$ megohms, σ was taken equal to σ_{direct}. The values of σ_{direct} were also checked with a vacuum tube frequency meter.[11] It will be noted that the theoretical value of τ remains practically constant in the two determinations while only σ varies. This is, of course, exactly as it should be.

[11] H. Lifschutz, Phys. Rev. **53**, 950 (1938).

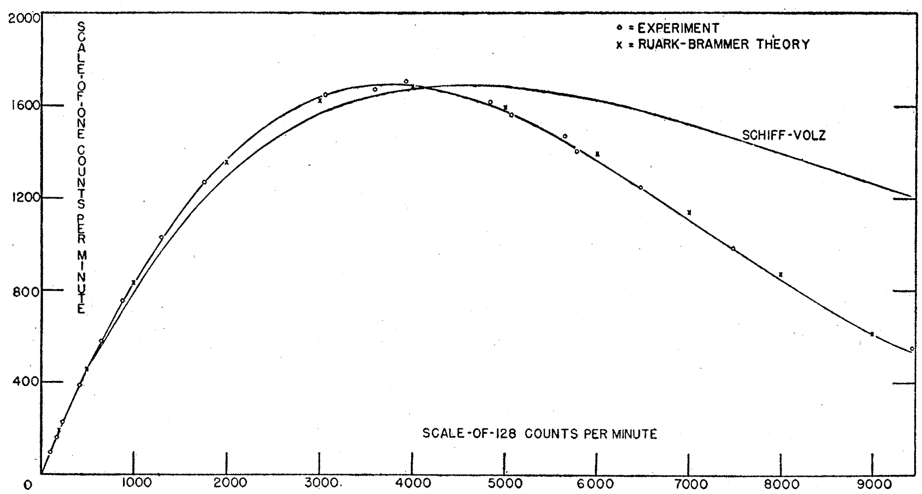

FIG. 5. The response curve for $R=100$ megohms.

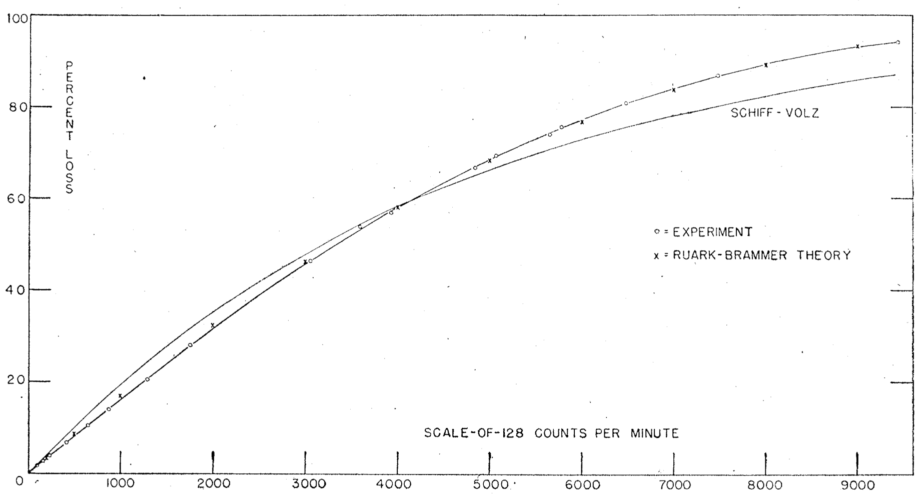

FIG. 6. The percent loss curve for $R=100$ megohms.

The original counting loss theories of Schiff[12] and Volz[13] attempted to account for counting losses in a G-M system by introducing a single recovery time, τ, to include both the effects of the counter circuit and the recording circuit. They derived the equation

$$n_1 = n_0 e^{-n_0\tau}. \tag{9}$$

Clearly the Ruark-Brammer and Alaoglu-Smith formulae reduce to this when $\sigma=0$. If the losses in the Neher-Harper circuit could be neglected, the theoretical Schiff-Volz curves shown in Figs. 3 to 6 should agree with the experimental curves for then $n_{128}=n_0$. Clearly they do not, showing in a striking way the effect of the resolution time σ and also showing at what counting rates the losses in the counter circuit become appreciable. As expected, the agreement of the Schiff-Volz theory is much better for the case in which $R=20$ megohms than in the case $R=100$ megohms since in the former case σ is smaller.

It is easily shown from Eq. (9) that

$$\tau = 1/n_{1\,\mathrm{max}}, \tag{10}$$

where $n_{1\,\mathrm{max}}$ is the maximum random counting

[12] L. I. Schiff, Phys. Rev. **50**, 88 (1936).
[13] H. Volz, Zeits. f. Physik **93**, 539 (1935).

rate. The theoretical Schiff-Volz curves shown in Figs. 3 to 6 were calculated by determining τ from the maximum of n_1 shown in Figs. 3 and 5. The τ's determined in this way were respectively 1.39×10^{-2} sec. and 1.31×10^{-2} sec. If there were no losses in the G-M circuit, the Schiff-Volz theory would give the slope of the percent loss curve at the origin as $100\times\tau$, since then $n_{128}=n_0$. Actually values of τ are found by this method to be 1.25×10^{-2} sec. and 1.066×10^{-2} sec., which are very low as expected.

As shown above, τ, determined from the maximum of n_1, is less in the case $R=100$ megohms than in the case $R=20$ megohms. The observed maximum of n_1 increases as σ increases, since, due to σ, the short intervals are cut out by the G-M circuit. Thus the recorder is able to approach more nearly its maximum *periodic* counting rate. Accordingly, it is to be expected that τ determined experimentally from $n_{1\,\max}$ would automatically give the correct kind of average value including the effects of both the counter and the recording circuits. For this reason the use of a single resolution time to give the *over-all efficiency* of the entire counting system turns out, in practice, to give a good approximation to the truth. This is shown by Fig. 7, where n_1 is plotted against n_0 (not n_{128}) from the data for the case $R=100$ megohms. The experimental curve is computed from the Ruark-Brammer theory since this has been shown to agree with experiment. The graph is plotted up to such values of n_0 as correspond to the maximum of n_{128} in Fig. 5. One must conclude that the Schiff-Volz method is adequate to correct a G-M Type I scale-of-one system for counting losses up to its maximum counting rate with a high degree of accuracy. This is fortunate since τ is so easily determined experimentally from Eq. (10).

Experimental Precautions

The statistical fluctuations in the parallel method are quite small as shown by the smoothness of the experimental curves. The number of counts per point usually was about 10,000 to 15,000. The slope of the percent loss curve at the origin may, under these conditions, be determined with an accuracy of one or two percent. In order to obtain reproducibility all conditions of the experiments must remain quite constant. The circuit constants must not vary too much, nor the voltages. Different G-M tubes have different σ even though of similar construction and used with the same N-H circuit. This is easily shown by the very different values of the maximum random counts obtainable. The capacity of the G-M tube and its leads also influences σ quite sharply. The leads should be kept short and separated. If the counter is not electrostatically shielded, the body capacity of an observer influences the counting rate quite markedly, especially at high counting rates. In short it is essential to keep the capacity of the counter with respect to its surroundings constant. If an electrostatic shield is used to keep the charges on the glass walls of the G-M tube from changing,[14] a very large increase in σ is observed as measured by the maximum counting rate. The counter used in these experiments was used without a shield. Any effects due to charges on the walls (which were quite small for this counter) were avoided by allowing the charge to come to equilibrium after changing the source strength. Effects due to photosensitivity were also avoided by light shielding.

TABLE I. *Values of τ and σ in seconds.*

	$R=2\times10^7$ ohms Theory	Direct	$R=10^8$ ohms Theory	Direct
τ	1.37×10^{-2}	1.43×10^{-2}	1.34×10^{-2}	1.43×10^{-2}
σ	0.114×10^{-2}	0.114×10^{-2}	0.27×10^{-2}	0.286×10^{-2}

The Neher-Harper Circuit

The experimental method considered above, as already stated, is not especially sensitive to the effect of σ for values of σ relative to τ as given above. Since the losses in the Neher-Harper circuit are independent of the use of fast scaling circuits or frequency meters, the only way to take account of these losses is by correcting theoretically for them. Of course, faster counters and quenching circuits would also get around this difficulty up to certain counting rates. However, it seems that such faster circuits have not yet been developed.[15] The highest counting rates

[14] J. L. Lawson and A. W. Tyler, Phys. Rev. **53**, 605 (1938).
[15] See e.g. I. A. Getting, Phys. Rev. **53**, 103 (1938); A. E. Ruark, Phys. Rev. **53**, 316 (1938); H. V. Neher and W. H. Pickering, Phys. Rev. **53**, 316 (1938); E. Y. Yetter, Phys. Rev. **53**, 612 (1938).

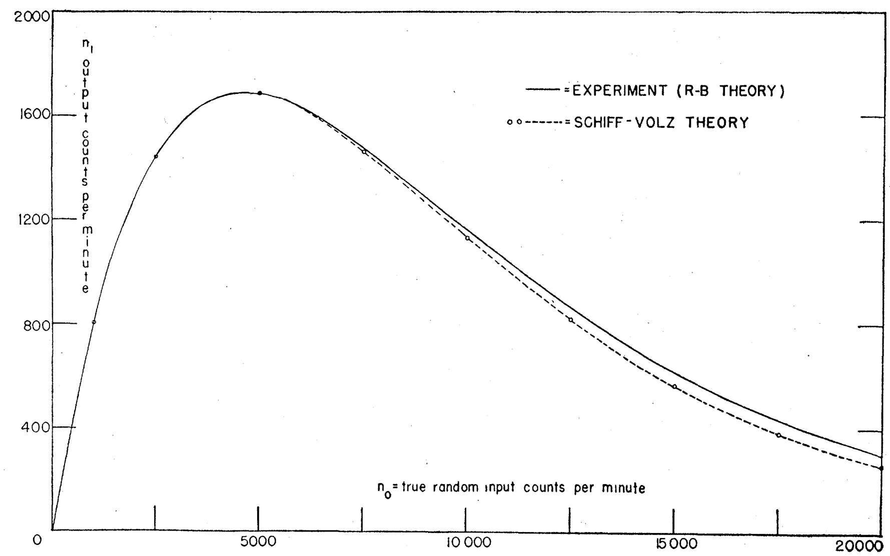

FIG. 7. Showing the approximate validity of the use of a single resolution time.

so far reported have been obtained by using the Neher-Harper circuit. Getting[15] reports values for the multivibrator type of quenching circuit of 1.2×10^5 counts per minute and probably higher values if part of the counting range of the G-M tube is sacrificed.

The counting losses due to σ are shown in Fig. 8 according to the Ruark-Brammer formula. The value of σ used corresponds to the average of most counters we have tested. In view of the magnitude of these losses it is important to know whether or not the Neher-Harper circuit does follow closely the Ruark-Brammer equation

$$n=\frac{n_0}{1+n_0\sigma} \tag{11}$$

or, perhaps, the equation

$$n=n_0\exp\left(-n_0\sigma\right). \tag{12}$$

The parallel method gives rather good support to Eq. (11), as we have seen within the limits of its sensitivity to σ. However, all counters we have tested depart from Eq. (11) at high counting rates in that they reach a maximum counting rate which is followed by a very marked decrease as the source strength is still further increased. This is in disagreement with Eq. (11), which, as noted by Ruark and Brammer, indicates a maximum *constant* value for n. Counters which showed this decrease had maximum counting rates varying from 50,000 to 175,000 random counts per minute. The effect was found both with the scale-of-128 and the frequency meter. This decrease is what would be expected if Eq. (12) were followed or might be due to a slight departure from Eq. (11). To test this point further, the following method was devised.

The grid resistor of the N-H circuit was connected by switches so that it could quickly be changed from 10^7 to 10^8 ohms. The counting rate due to a constant source was determined with the scale-of-128 for each of these grid resistor values. The grid bias was also changed with the resistor to the proper value. This procedure was repeated for all counting rates up to the maximum. Let the observed counts with the two resistors be n_7 and n_8. Then

$$n_7=\frac{n_0}{1+n_0\sigma_7}, \tag{13}$$

$$n_8=\frac{n_0}{1+n_0\sigma_8}. \tag{14}$$

Thus n_0 may be eliminated giving

$$n_8=\frac{n_7}{1+n_7(\sigma_8-\sigma_7)}, \tag{15}$$

$$P=\frac{n_7-n_8}{n_7}\times100=100\times\frac{n_7(\sigma_8-\sigma_7)}{1+n_7(\sigma_8-\sigma_7)}. \tag{16}$$

The slope of the percent curve at the origin

$$dP/dn_7 = 100(\sigma_8 - \sigma_7), \quad n_7 \to 0 \tag{17}$$

and

$$\sigma_7 = \frac{1}{n_{7\ \max}}, \quad \sigma_8 = \frac{1}{n_{8\ \max}}. \tag{18}$$

The experimentally observed percent loss curve could be compared with that calculated from Eqs. (16) and (18). This could also be done by the use of a formula for the N-H circuit similar to Eq. (12). Rough preliminary results show a decided disagreement with Eq. (12) and fair agreement with Eq. (11) up to $n_{7\ \max}$. A variant of this method may possibly be that of keeping the grid resistor constant and varying the N-H grid bias alone, as the observed counting rate is a function of this bias.

In view of the tendency to follow the equation

$$n = n_0/(1 + n_0\sigma)$$

up to counting rates near the observed maximum and also the observed departure from this equation as shown by the failure to maintain a constant maximum counting rate, it is of interest to try to look more deeply into the mechanism of the G-M counter and N-H combination. The G-M counter has a deionization time, τ_d, which is given by the time it takes for the electric field in the counter to sweep out the positively charged ions after the discharge is extinguished. Thus, τ_d may be taken as constant. It also has a discharge time, σ, which has a lower limit equal to τ_d and which has a value which depends not only on the G-M tube but also on the extinguishing circuit. Since the counter tube cannot be re-excited by an incoming particle during the discharge period, σ, the counter itself may be classed as a Type II recorder. The extinguishing circuit introduces still another resolution time, ρ. The resolution time, ρ, of the N-H circuit would be approximately equal to σ if the succeeding circuits had no thresholds. To see this, consider a discharge in the G-M counter. In the extinguishing process, as is well known, the potential on the counter is reduced to the threshold value or very slightly below. The voltage then starts to recover at a rate depending mainly on the RC value of the N-H circuit. If another discharge occurs in the G-M counter when the potential has recovered to a value only slightly above the threshold value, the electrical impulse developed will be very weak since these impulses are equal to the overvoltage. The pulse thus fails to be recorded because the succeeding circuits require a certain minimum signal for response. Such circuits are the discriminator circuit, the first stage of scale-of-two in the scaling circuit or the mechanical counter stage if no scaling circuit is used. In this way the time constant of the N-H circuit does introduce an effective recovery time, $\rho > \sigma$. It is clear that a count coming within a time greater than σ and less than ρ of a preceding count will not only fail to be recorded but will re-excite the Neher-Harper circuit. In other words we may reasonably take the efficiency formula as that of a Type I recorder. If one assumes σ to be reasonably constant, the usual derivation gives for the over-all efficiency

$$n = (n_0/(1 + n_0\sigma)) \exp(-n_0(\rho - \sigma)) \text{ if } \rho > \sigma \tag{19}$$

where n is the number of counts from the N-H circuit. This, then, is the corrected Neher-Harper equation. If ρ and σ are approximately equal, this equation reduces essentially to

$$n = n_0/(1 + n_0\sigma)$$

at counting rates considerably below the maximum. This explains why the parallel method showed agreement with this equation, since the counting rates were well below the maximum. The exponential "guillotine factor" in Eq. (19) is then responsible for the observed decrease after the maximum counting rate has been reached. This requires $\rho > \sigma$. Since the Neher-Harper cir-

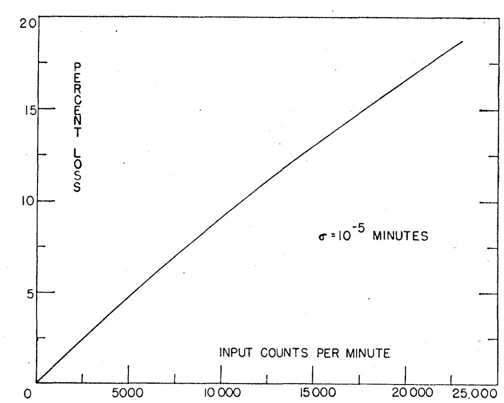

FIG. 8. The counting losses in the G-M circuit.

cuit cannot recover the voltage till the counter has stopped discharging, it is clear that $\rho > \sigma$ always. However, ρ and σ are not independent. The variation of σ by changing the N-H grid resistor shows a dependence of σ on ρ. Ruark and Brammer describe the situation by saying that the counter and circuit "work together" as a single unit so that the dependence is described by the relation $\rho \cong \sigma$. This approximate relation, $\rho \cong \sigma$, plus the relation $\rho > \sigma$ and Eq. (19) thus seem capable of explaining all the experimental facts. Qualitative tests of Eq. (19) with the inverse square method show a more or less exponential decay after reaching the maximum counting rate. However, it is clear that a more detailed justification of Eq. (19) would be desirable, especially in order to throw more light on the exact relationship between ρ and σ. This might lead to circuit designs which would allow σ to be decreased. It seems in the present case that enough amplification was already used to reduce ρ to a value not much greater than σ. No important increase in counting speed thus seems possible by increasing the amplification between the N-H circuit and the recorder.

The decrease in amplitude of the pulse generated by the G-M tube as described above may easily be shown in the following simple experiment. The output of the Neher-Harper circuit is applied directly to the plates of a cathode-ray oscillograph. A strong source (say one mg of radium) is moved continuously toward the G-M counter. As the input counting rate increases the amplitude of pulse in the oscillograph will be seen to drop until it almost vanishes. This effect cannot be ascribed to the regulation of the high voltage supply since the drop in applied voltage due to regulation is observed to be very small compared to the overvoltage applied to the counter.

Lyshede and Madsen[16] have measured the deionization time of G-M counters by forced quenching of the counter. The quenching was accomplished by applying periodic pulses from a thyratron oscillator to the counter in a special circuit. When the frequency was increased to such an extent that the counter discharge was not extinguished by successive pulses a direct measure of the deionization time was obtained. Values of 10^{-4} second were found for air filled counters. Hydrogen counters were three or four times faster. These values agreed with calculations of the time taken to sweep out the positive ions using the mobility of the ions, size of the counter, and electric field distribution. This agrees with similar calculations we had made (unpublished) about two years ago.

The deionization time fixes the maximum counting rate possible with G-M counters roughly as 10^6 counts per minute. Taking account of the counting losses, one could thus measure input rates up to about 10^7 per minute. The fastest counter we have measured in combination with the N-H circuit gave a maximum counting rate of the order 2×10^5 counts per minute. This is the same value as reported by Neher and Harper[1] for one of their counters. Thus about a fivefold increase in speed may be possible with faster quenching circuits. However, one must conclude that existing quenching circuits are already capable of allowing a close approach to the maximum counting speed possible. It is also clear that existing vacuum tube scaling circuits and frequency meters are already much faster than necessary for G-M work as the resolution time of these circuits is less than the deionization time of the counters themselves. For practical purposes, one is interested in attaining a close approach to the maximum speed in order that the necessity for correcting for counting losses may be avoided at ordinary counting rates, as these losses set in at relatively low counting rates. It should be stressed that whether or not counting losses are great depends not only on the counting rate at which one is working, but also on the σ of the particular counter being used, which may be quite large.

Conclusion

Alaoglu and Smith[9] have derived the statistical formulae giving the counting losses for a scaling circuit and including the effect of σ. They find

$$n_n = \frac{n_0}{n(1+n_0\sigma)}[1 - I[n_0(\tau - n\sigma), n]], \quad \text{if} \quad \tau > n\sigma \qquad (19)$$

and $$n_n = \frac{n_0}{n}\frac{1}{1+n_0\sigma} \quad \text{if} \quad \tau \leqslant n\sigma. \qquad (20)$$

[16] J. M. Lyshede and J. C. Madsen, Zeits. f. Physik **108**, 777 (1938).

Here n is the scaling ratio and the function $I(n_0(\tau-n\sigma), n)$ is the ratio of the incomplete Γ-function $\Gamma(n_0(\tau-n\sigma), n)$ to the complete Γ-function, $\Gamma(n)$. The upper limit of the incomplete Γ-function is $n_0(\tau-n\sigma)$. The details may be found in the paper of Alaoglu and Smith. The interesting feature of these equations is that the counting losses in the mechanical recorder stage fed by the scaling circuit drop out when $\tau\leqslant n\sigma$ as shown by Eq. (20). This fixes the maximum scaling ratio it is necessary to use. Of course, it is assumed in the derivation that the resolving time of the scaling circuit itself is less than σ. In the parallel method reported here, the relation was satisfied so that the losses in the Cenco counter stage were zero. It is also desired to point out here, that Eqs. (19) and (20) may also be tested by using the parallel method, as was first pointed out to the authors by Smith. The simplest case would be that of comparing a scale-of-two following Eq. (19) against a circuit following Eq. (20) such as the scale-of-128 used in the present experiments. A series method similar to the parallel method, which could also be used, would involve observing the number of counts in scales-of-one connected to the output of successive stages of scales-of-two in series.

The question of the best way to correct for the counting losses due to σ in actual experimental work may be briefly discussed. The parallel method in which the scaling circuit used is compared against a scale-of-one is not sensitive enough to accurately measure small values of σ. The determination of σ from measurements of the maximum random counting rate depends on fulfilling the relation $\tau\leqslant n\sigma$. This might require a larger ratio scaling circuit than is available. To overcome this σ may purposely be increased. A frequency meter affords a convenient way of measuring σ from the maximum. In this connection the vacuum tube frequency meter already reported[11] is of interest. This frequency meter employs the same circuit as the vacuum tube scaling circuit[7] discussed above. A scaling circuit of this type may thus be built in which one of the stages of scale-of-two also serves as a frequency meter. This frequency meter could thus be used for measuring σ even though $n<\tau/\sigma$. The addition of a tank circuit would not be necessary.

For common values of τ and σ which actually occur, namely $\tau=1.5\times10^{-2}$ sec. and $\sigma=5\times10^{-4}$ sec. we obtain $n=30$. Thus a scale-of-32 would in most cases suffice for the determination of σ from the maximum as well as to prevent losses in the mechanical counter stage. We have found that a vacuum tube scale-of-eight follows a vacuum tube scale-of-16 in parallel with it up to about 30,000 counts a minute showing that the losses in the Cenco counter stage were negligible up to such counting rates. For this case σ was about 10^{-3} sec. As Alaoglu and Smith show, σ cuts out the short intervals and allows the lower ratio scaling circuit to follow the higher ratio one up to higher counting rates than would be the case if a true random input came from the Neher-Harper circuit. We may conclude then that a vacuum tube scale-of-eight is sufficient for most counting rates if means are provided for measuring and correcting for the losses due to σ, which losses would be the same no matter what the scale of the circuit.

It is a pleasure to acknowledge the very helpful discussions of the theory and its applications to the parallel method which we have had with Mr. N. M. Smith, Jr. The help of Dr. M. M. Slawsky with the earlier work, and the discussion with and encouragement of Professor S. A. Goudsmit is also deeply appreciated. Finally, support from the Horace H. Rackham Fund is gratefully acknowledged.