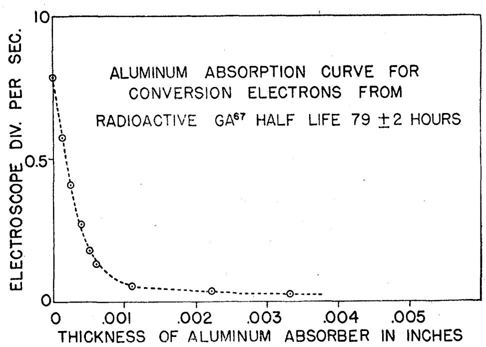


FIG. 5. Aluminum absorption curve for conversion electrons from radioactive $Ga^{67}$.

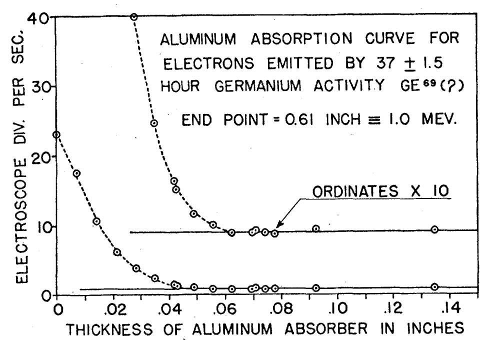


FIG. 6. Aluminum absorption curve for electrons emitted by $37 \pm 1.5$ hour germanium activity $Ge^{69}$(?).

Mev and 1.0 Mev were obtained, respectively, for the maximum energies of the electrons emitted by $Ga^{70}$ (19.8 minutes) and $Ge^{69}$ (37 hours). The disintegration products of $Ga^{67}$ (79 hours) have been exhaustively investigated by Alvarez[4] who found that $Ga^{67}$ transformed to $Zn^{67}$ by capture of a $K$ shell electron followed by emission of an x-ray characteristic of zinc. The soft electrons which are emitted, the absorption curve for which is shown in Fig. 5, have been found by Alvarez to be due to conversion of a 100 kev gamma-ray in the $K$ and $L$ shells. In the present instance, the absorption coefficients of the x-rays have also been measured and a complete identification of the results with those obtained by Alvarez has been established.

It is again a pleasure to be able to express my thanks to Professor E. O. Lawrence and to the staff of the Radiation Laboratory for all their help and cooperation. My thanks are also due to the Commonwealth Fund for the award of a fellowship which has made my stay here possible. The research has also been aided by grants to the laboratory from the Research Corporation, the Chemical Foundation and the Josiah Macy, Jr., Foundation. The experiments have been facilitated by assistance from the W.P.A.

[4] L. W. Alvarez, Phys. Rev. **53**, 606 (1938).

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# Note on $K$-Electron Capture in $Be^7$

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There is no observed positron emission in $Be^7$. This sets an upper limit of 2.09 $mc^2$ for the mass difference $Be^7$–$Li^7$ from Fermi's theory and 2.3 $mc^2$ from the Konopinski-Uhlenbeck theory. This upper limit is compared with other estimates of the same mass difference. The gamma-ray emission accompanying the disintegration probably takes place after the $K$ capture from an excited state of $Li^7$ having a spin $\frac{1}{2}$. The $K$-capture transition to $i=\frac{1}{2}$ from $i=\frac{3}{2}$ is according to experiment only $\frac{1}{10}$ as weak as that to $i=\frac{3}{2}$. This speaks for the Gamow-Teller type of selection rules. The mean lives of $He^6$, $Be^7$, $C^{11}$ are compared. It is found that the observed mean lives are relatively shorter for the lighter nuclei than Fermi's theory predicts. This is in apparent contradiction with the preference shown for the Fermi theory by the alpha-particle distribution from $Li^8$. From a theory without derivatives, the facts seem to point either to an appreciable increase in the many-body aspect of nuclei from $He^6$ to $C^{11}$ or else to a decrease in the intrinsic $\beta$-emitting powers of nuclear particles in heavier nuclei. On the Konopinski-Uhlenbeck theory some of the disagreement can be removed but the $\beta$-particle distribution from $Li^8$ (Gamow and Teller) speaks strongly against using this theory for mean lives.

THE experiments of Roberts, Heydenburg and Locher[1] show that $Be^7$ captures its $K$ electron to form $Li^7$. They observe roughly one gamma-ray quantum per every ten $Be^7$ atoms produced. The gamma-radiation is explained by them as being due to capture of a $K$ electron into an excited state. Their measurements give

[1] R. B. Roberts, N. P. Heydenburg, G. L. Locher, Phys. Rev. **53**, 1016 (1938); R. B. Roberts and N. P. Heydenburg, Abstract No. 78, 1938, Washington Meeting American Physical Society.

$425 \pm 25$ kev for the energy of the gamma-ray. This agrees within experimental error with the position of the level of $Li^7$ (450 kev) determined by the reaction[2] $Li^6+H^2=Li^7+H^1$. Richardson[3] finds that $N^{13}$ emits in addition to the annihilation radiation also another somewhat softer $\gamma$-ray. The gamma-radiation of $N^{13}$ was used as a comparison standard in the determination of the radiation from $Li^7$. The energy $(Li^7)^*-Li^7$ is, therefore, possibly lower than stated and the level responsible for gamma-radiation may be different from that observed in $Li^6+H^2=Li^7+H^1$. The exact identity of the level makes little difference for the discussion below as long as its spin is $\frac{1}{2}$. It is likely and it will be assumed below that the level in question is identical with that observed in proton emission and that its spin is $\frac{1}{2}$ because the energy measurements of gamma-rays are difficult and because it is difficult theoretically to explain the presence of two levels within 450 kev of the ground state.

TABLE I. *Approximate values of* $lg_{10}(P_+/P_K)$.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $511(\Delta w-1)$kev | 10.2 | 41 | 101 | 172 | 278 | 511 | 590 kev |
| $[\lg_{10}(P_+/P_K)]_F$ | $\bar{4}.4$ | $\bar{2}.6$ | 0.0 | 0.7 | 1.4 | 2.3 | 2.5 |
| $[\lg_{10}(P_+/P_K)]_{\text{K-U}}$ | | | | $\bar{2}.7$ | $\bar{1}.7$ | | 1.2 |

The mass difference $Be^7-Li^7$ is not known directly with great certainty. The neutron distribution from $Li^6+H^2$ obtained by Rumbaugh, Roberts and Hafstad[4] indicates that their maximum energy is at least 3.3 Mev for 800 kev incident neutrons. This gives $Li^6+H^2 \geqq Be^7+n+3.2$ Mev. From the measurements of Cockcroft and Walton, Bethe and Livingston obtain $Li^6+H^2=Li^7+H^1+5.02 \pm 0.12$ Mev and Rumbaugh, Roberts and Hafstad[4] obtain 5.0(3) Mev. This gives $Be^7 < Li^7+1.8 \text{ Mev} - 0.00080 = 0.00091 = 1.6(7)\ mc^2 = 0.91$ mmu. Using the mass of $Li^6$ as given by Livingston and Bethe[5] and Bethe's revised neutron mass[6] one obtains from the $Be^7+n$ process; $Be^7=Li^7+1.9(0)\ mc^2=Li^7+1.04$ mmu. The disagreement between these estimates makes it desirable to estimate the upper limit of the energy by making use of the apparent improbability of the positron emission from $Be^7$.

This was done by using the formulas of Yukawa and Sakata[7] for the ratio of the positron probability $P_+$ to $K$-capture probability $P_K$. It was supposed that the neutrino mass is zero. One has then in a sufficiently good approximation

$$\frac{P_+}{P_K}=\frac{\int_1^{\Delta w} F(\epsilon,\,0)d\epsilon}{2\pi(\alpha Z)^3(1+\Delta w)^2}. \qquad \text{(Fermi)} \quad (1)$$

$$\frac{P_+}{P_K}=\frac{\int_1^{\Delta w} F(\epsilon,\,0)(\Delta w-\epsilon)^2 d\epsilon}{2\pi(\alpha Z)^3(1+\Delta w)^4}. \qquad \text{(Konopinski-Uhlenbeck)} \quad (2)$$

Here $\Delta w-1$ is the energy available for positron emission in $mc^2$ units, $\alpha=1/137$ and $Z(=4)$ is the atomic number of the unstable nucleus.

$$F(\epsilon,\,0)=\epsilon(\epsilon^2-1)^{\frac{1}{2}}(\Delta w-\epsilon)^2\frac{x}{e^x-1}; \qquad x=2\pi\alpha Z\epsilon(\epsilon^2-1)^{-\frac{1}{2}}.$$

The $\alpha$-particle distribution from $Li^8$ indicates, according to Gamow and Teller,[8] that the Fermi theory accounts for mean lives as a function of available energy in $\beta$-decay better than the Konopinski-Uhlenbeck theory for the disintegrations of light nuclei. It appears safest, therefore, to use the formulas of the Fermi theory in this case also. One obtains the approximate values given in Table I of $\lg_{10}(P_+/P_K)$ on the Fermi (F) and the Konopinski-Uhlenbeck theories.

Experiment shows that the number of $\gamma$-rays is about 1/10 of the number of neutrons. An emitted positron should give two $\gamma$-rays. The number of emitted positrons is, therefore, less than 1/20 of the number of $Be^7$ atoms which is practically the same as $P_+/P_K<1/20$. According to Table I this corresponds to $Be^7-Li^7<45$ kev $+2\ mc^2=2.08(8)\ mc^2$ on the Fermi theory and $Be^7-Li^7<172$ kev$+2\ mc^2=2.34\ mc^2$ on the K-U theory. Since the K-U theory underestimates the emission at the end of the range the K-U calculation should give too low values of

[2] L. H. Rumbaugh and L. R. Hafstad, Phys. Rev. **50**, 681 (1936).

[3] J. R. Richardson, Phys. Rev. **53**, 610 (1938).

[4] L. H. Rumbaugh, R. B. Roberts and L. R. Hafstad, Phys. Rev. **54**, 657 (1938); the writers are indebted to Messrs. Rumbaugh, Roberts and Hafstad for communicating to them their results before publication.

[5] M. S. Livingston and H. A. Bethe, Rev. Mod. Phys. **9**, 245 (1937).

[6] H. A. Bethe, Phys. Rev. **53**, 313 (1938).

[7] H. Yukawa and S. Sakata, Proc. Phys. Math. Soc. of Japan **17**, 467 (1935); **18**, 128 (1936); C. Moller, Phys. Rev. **51**, 84 (1937).

[8] Considerations of Gamow and Teller quoted by L. H. Rumbaugh, R. B. Roberts and L. R. Hafstad, Phys. Rev. **51**, 1106 (1937).

$P_+/P_K$ for the correct energy and too high values of $Be^7-Li^7$ for a given $P_+/P_K$. The estimate from Fermi's theory is preferable. Positrons with 45 kev maximum energy would easily escape detection and the only direct contradiction to assuming the mass difference to be $2.09\ mc^2=1.14$ mmu lies in the neutron energy. Plotting the mass differences $He^3-H^3$, $C^{11}-B^{11}$, $N^{13}-C^{13}$, $O^{15}-N^{15}$, $F^{17}-O^{17}$ against $Z$ one estimates that 0.9 mmu $<Be^7-Li^7<$ 1.2 mmu. These estimates are seen to agree with each other. From the fact that $C^{11}-B^{11}$, $O^{15}-N^{15}$ have relatively larger values than $N^{13}-C^{13}$, $F^{17}-O^{17}$ values below 1.0 mmu appear unlikely so that the range between 1.0 mmu $=1.8\ mc^2$ and 1.14 mmu $=2.09\ mc^2$ appears to be available for $Be^7-Li^7$.

Since the normal state of $Li^7$ has a spin $\frac{3}{2}$, it is natural to suppose that the same is true for the normal state of $Be^7$. There is some probability that the level of $Li^7$ at 450 kev has a spin $\frac{1}{2}$ and that it is the ${}^2P_{1/2}$ partner of the ${}^2P_{3/2}$ normal level. If so, the $K$ capture occurs here into two nuclear states differing from each other mainly through the relative orientation of total particle spin with respect to total orbital angular momentum. On the Fermi theory one would expect the transition to ${}^2P_{1/2}$ to have negligible intensity in comparison with that to ${}^2P_{3/2}$. Experiment indicates the opposite and speaks in favor of the Gamow-Teller type of selection rules.[9] With the interaction energy given by their Eq. (2) the transition probabilities are proportional to 5 and 4 for ${}^2P_{3/2}$ and ${}^2P_{1/2}$ respectively (for $l=1$, the ratio is $2l+3:4l$ on a single particle model with $l=$ orbital angular momentum) and besides to $(1+\Delta w)^2$. Using the ratio 10 : 1 for the transition probabilities and $0.88\ mc^2$ for $(Li^7)^*-Li^7$ one obtains $\Delta w$ by solving

$$\frac{(1+\Delta w)^2}{(0.12+\Delta w)^2}=\frac{4}{5}\frac{9}{1}.$$

This gives $\Delta w=0.40$ which makes $Be^7-Li^7=1.4\ mc^2=0.8$ mmu. This value is too low. For a mass difference of $2.09\ mc^2$ the expected ratio of the transition probabilities for ${}^2P_{3/2}$ and ${}^2P_{1/2}$ is 3.7 which corresponds to 1/4.7 as many $\gamma$-rays as neutrons. The difference between 1/4.7 and 1/10 may not be significant in view of the difficulty of $\gamma$-ray intensity measurements and the uncertain nature of the theory. On the K-U theory modified with spin terms for heavy particles the ratio 10 : 1 gives in the same way $\Delta w=1.3$ which corresponds to $Be^7-Li^7=2.3\ mc^2$ and is only slightly too high. For $2.09\ mc^2$ the expected ratio of the transition probabilities is 12 which agrees better with experiment than the Fermi-like theory. This comparison is, however, only apparently favorable to the K-U form because Gamow and Teller's[9] matrix element $M_2$ cannot be used by itself. Their matrix element $M_1$ will increase the ratio considerably making the Fermi-like result approach the experimental value and the K-U value recede from it.

An appreciable intrinsic preference for a transition to ${}^2P_{3/2}$ in comparison to ${}^2P_{1/2}$ is apparently ruled out by the approximate value 10 : 1 of $Be^7:\gamma$. Thus even if there were only a factor 10 favoring ${}^2P_{3/2}$ the ratio of transitions to $i=\frac{3}{2}$ and $i=\frac{1}{2}$ would be on the Fermi theory $10(1+\Delta w)^2/(0.12+\Delta w)^2$. The smallest possible value of $\Delta w$ is 0.7 which gives 40 for this ratio and the largest $\Delta w$ is 1.1 which gives 30. On a K-U theory with an intrinsic preference 10 : 1 for $\frac{3}{2}$ one has a ratio of 180 for $\Delta w=0.7$ and 90 for $\Delta w=1.1$. All of these values are appreciably greater than 10.

The $Be^7$ nucleus differs from $Li^7$ presumably only through the conversion of a proton into a neutron. The matrix element $M$ of the $\beta$-ray theory should be, therefore, of the order of magnitude 1 for the transition $\frac{3}{2}\rightarrow\frac{3}{2}$. It is thus natural to expect that the mean life of $Be^7$ should correspond to an anomalously high $|M|^2$ with Fermi's $g$ which was obtained from more complicated and, therefore, less probable transitions. The increase of $|M|^2$ with decreasing atomic number has been pointed out by Nordheim and Yost[10] and has been attributed by them to the increasing importance of the many-body aspect for heavier nuclei. A comparison will be made here with the period of $C^{11}\rightarrow B^{11}$ which gives about the same value of $|M|^2$ as $N^{13}\rightarrow C^{13}$. Before discussing the numerical relations it is necessary to consider the dependence of $|M|^2$ on other nuclear properties. The isotopic spin notation is the most

[9] G. Gamow and E. Teller, Phys. Rev. **49**, 895 (1936).

[10] L. W. Nordheim and F. L. Yost, Phys. Rev. **51**, 943 (1937).

convenient.[11] The wave function is then antisymmetric for simultaneous interchanges of space coordinates $x$, spins $\sigma$ and isotopic spins $\tau$.

The matrix element for positron emission is on Fermi's theory

$$M=(\varphi,\ \sum_k \tfrac{1}{2}(\tau_k{}^\xi+i\tau_k{}^\eta)\psi). \tag{3}$$

Here $\psi$, $\varphi$ are the wave functions of initial and final nuclei. The operators $\tau^\xi$, $\tau^\eta$, $\tau^\zeta$ are Pauli's matrices for isotopic spin so that $(\tau^\xi)^2=1$ etc. $\cdots$. The wave-lengths of the light particles are supposed to be long compared with the nuclear radius, so that there is no difference in the values of the light particle wave functions at the different heavy particles within the nucleus. The above formula takes into account the interference of the light particle field due to emissions from different heavy particles. For a symmetric Hamiltonian the normal state of the final nucleus has

$$\begin{aligned}&\varphi=\sum \tfrac{1}{2}(\tau_k{}^\xi+i\tau_k{}^\eta)\psi,\\ \text{so that}\quad &M=(\psi,\ (T_\xi-iT_\eta)(T_\xi+iT_\eta)\psi).\end{aligned} \tag{3'}$$

Evaluating the operator in the scalar product one finds

$$\begin{aligned}M&=T(T+1)-T_\zeta(T_\zeta+1)\\ &=(T-T_\zeta)(T+T_\zeta+1),\end{aligned} \tag{4}$$

where one has set $(\psi,\psi)=1$ since the wave function must be normalized. Here $T_\zeta$ is $\frac{1}{2}(N-Z)$ $=\frac{1}{2}\Sigma\tau_k{}^\zeta$ and $T(T+1)$ is the characteristic value of $T_\xi{}^2+T_\eta{}^2+T_\zeta{}^2$ where $T_\xi$, $T_\eta$ are defined similarly to $T_\zeta$. If the number of protons is the minimum possible with a given $T$ then $T_\zeta=T$ and $M=0$. This should be the case because there can be no final nucleus with the same $T$ with a smaller number of protons. The values $T_\zeta$, $T_\zeta+1$ are seen to be the values of $T_\zeta$ in the initial and final nucleus. For the disintegrations $Be^7\rightarrow Li^7$, $C^{11}\rightarrow B^{11}$, $N^{13}\rightarrow C^{13}$, $T_\zeta=-\frac{1}{2}$ so that

$$M=(T+\tfrac{1}{2})^2.$$

There is considerable probability[11] that $T=\frac{1}{2}$ for the ground states. This gives $M=1$ for the three cases.

The validity of Eq. (3′) was essential for Eq. (4). If the Hamiltonian is symmetric the $\varphi$ of Eq. (3′) will be a solution of the wave equation and will correspond to a possible final nucleus. For a Hamiltonian that is not symmetric this is not the case. Even the inequality between unlike particle and like particle forces matters. For example, the Heisenberg interaction between unlike particles can be expressed as

$$\tfrac{1}{2}\sum_{k>j}(\tau_k{}^\xi\tau_j{}^\xi+\tau_k{}^\eta\tau_j{}^\eta)J(r_{ij}).$$

For long range forces this becomes

$$[(T_\xi{}^2+T_\eta{}^2+T_\zeta{}^2)-T_\zeta{}^2-(n/2)]J(0).$$

The term $T_\zeta{}^2$ does not commute with $T_\xi+iT_\eta$ and Eq. (3′) does not hold in general. This is due to the fact that an "unsymmetric Hamiltonian" is not invariant to rotations of the isotopic spin and does not commute with $T_\xi+iT_\eta$. The equation $H(T_\xi+iT_\eta)\psi=0$ then does not follow from $H\psi=0$. If, on account of a difference between like and unlike particle forces, the four neutrons of $Li^7$ should be somewhat farther out than the three protons, then similarly in $Be^7$ the four protons will be somewhat farther out than the three neutrons. When a $K$ capture occurs the neutralized proton finds itself in the right position to be a neutron of $Li^7$ but the three remaining protons are too far out. Such a condition would be expected to decrease the transition probability. The model just mentioned is meant only as an illustration of principle and not in a literal way. Similarly an inequality between proton-proton and proton-neutron interactions introduces terms in $T_\zeta$ and $T_\zeta{}^2$. A surplus repulsion between protons will make $Be^7$ slightly larger than $Li^7$ and $C^{11}$ slightly larger than $B^{11}$. Due to this cause, however, one will expect only very small effects because the Coulomb barrier in carbon is at the most 6 $mc^2$ and changes at the most by 1 $mc^2$ in the positron emission. The mass differences $Be^8+H^3-B^{11}=0.012$ and $Be^8+He^3-C^{11}=0.0097$ show that $H^3$ and $He^3$ must be quite thoroughly distorted in $B^{11}$ and $C^{11}$. It is probable that these nuclei do not have any loosely attached particles for which the Coulomb effect is especially important. In $Li^7$ the $H^3$

[11] The most complete account of this is given by E. Wigner, Phys. Rev. **52**, 106 (1937); first introduced by W. Heisenberg, Zeits. f. Physik **77**, 1 (1932); cf. also B. Cassen and E. U. Condon, Phys. Rev. **50**, 846 (1936). Wigner's notation is used here.

combination is bound only by 0.0028 mass units and here the $H^3$ combination may be behaving somewhat as a unit. The Coulomb field in $H^3$ is too weak, however, to cause much distortion in $H^3$. It is difficult to see, from such average field effects, a reason for a smaller $|M|^2$ in $C^{11}\rightarrow B^{11}$ than in $Be^7\rightarrow Li^7$.

Using the Fermi theory and disregarding the presence of transitions that violate the selection rules, one obtains, with $g=4.0\times10^{-50}$, $C^{11}-B^{11}-2\,mc^2=2.26\,mc^2$, $|M|^2=1$ an expected mean life of 270 minutes. The observed mean life is 20.5 min./0.694 = 29.6 min. This requires $|M|_C{}^2=9.(0)$. In $Be^7$ with the same $g$ and $|M|^2$ and with $Be^7-Li^7=2\,mc^2$, the expected mean life is 5.3 years. The observed mean life is 43 days/0.694 = 62 days. This requires $|M|_{Be}{}^2=30$. The ratio of the two $|M|^2$ is 3.3. One can try to lower the ratio by increasing the value of $Be^7-Li^7$ used in the calculation of the expected mean life. The small number of emitted positrons makes it impossible to assume more than 2.1 $mc^2$ for $Be^7-Li^7$. The transition probability due to $K$ capture is then increased by 10 percent in the total transition probability due to positron emission. The discrepancy is then still represented by a factor $\sim$2.6.

According to Bjerge and Bjerge and Brostrum[12] the half life of $He^6$ is $\sim$1 sec. and the available energy $\sim$3.7 Mev corresponding to $|M|^2\sim120$. This $|M|^2$ is even larger than that in $Be^7$ and is more surprising because $He^6$ and $Li^6$ are not homologous. The $\beta$-disintegrations of the light elements show in this way an interesting contradiction in pointing for $Li^8$ to the dependence of probability on energy predicted by Fermi's theory, as has been noticed by Gamow and Teller, and in showing besides a systematic increase in the probability with decreasing $Z$.

The K-U theory when applied to $C^{11}$ and $Be^7$ gives approximate agreement of $|M|_C{}^2$ and $|M|_{Be}{}^2$ with the use of the inspection limit for $C^{11}$ and $\Delta w=1$ for $Be^7$. It is difficult to believe that this agreement means much because the K-U theory does not account for the $\alpha$-particle distribution of $Li^8$ but it should be noted that the extreme shortness of the life of $He^6$ in comparison with that of $C^{11}$ is accounted for more easily on the K-U than on the Fermi theory. According to Gamow and Teller, however, the alpha-particle distribution from $Li^8$ speaks strongly against the K-U theory.

The approximate regularity of mass differences in $He^3-H^3$, $C^{11}-B^{11}$ and their partial agreement with expected Coulomb energies indicates the partial validity of Eqs. (3′) and (4). To be sure, Eq. (4) holds only for the Fermi interaction and not for the generalization of Gamow and Teller. Nevertheless the increase in the probability with decreasing $Z$ appears to be too strong to be explicable by the geometrical factors that may be expected to appear in Gamow and Teller's theory. It is, perhaps, worth while to consider the possibility of a decrease in the intrinsic $\beta$-emitting powers of nuclear particles with increase in atomic weight. This would amount to a change of $g$ in the calculations. Such an effect is conceivable if the interactions between nuclear particles are intimately connected with the $\beta$-ray field.

It should also be remembered that the equality of unlike and like particle interactions which is essential for (3′) has so far its experimental support only for particles with zero relative angular momentum.

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[12] The anomalous behavior of $He^6$ was noticed independently by H. A. Bethe. For measurements on $He^6$ see Bjerge, Nature **138**, 400 (1936); Bjerge and Brostrum, Nature **138**, 400 (1936).