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"Transverse" Acoustic Waves in Rigid Tubes

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Recently considerable work has been done, both theoretical and experimental on the transmission of electromagnetic waves through dielectric wave guides and hollow metal tubes. An analogous phenomenon is the propagation of transverse acoustic waves in gaseous media confined by rigid cylindrical tubes. Apparatus and methods are described for the production of some of the lower ordered acoustic waves and measurements are made with a tube of circular section filled with air. Measured values of wave-length, threshold frequency, and pressure distribution for some of the possible harmonic wave types are found to be in excellent agreement with theoretical values. Absorption measurements by the standing wave method in which transverse waves are used are suggested.

THE problem of transverse aerial vibrations within a rigid cylindrical tube received attention many years before the analogous problem of electromagnetic oscillations in hollow metal tubes and dielectric columns. The latter phenomenon which has recently been the subject of great interest was investigated mathematically by Lord Rayleigh[1] in 1897. The reality of the predicted waves was experimentally verified by Zahn[2] in 1916. Improvements in the means for producing ultra-high frequency radio waves and very likely the possibility of practical use for the tubular radio wave guides has renewed interest in the phenomenon. For a bibliography of the previous principal work in the field an article by Schelkunoff[3] may be consulted.

It is only natural that the revived interest in the electromagnetic problem should call attention to the older and similar acoustic problem. Indeed it was a consideration of the remarkable properties of the dielectric wave guides which led the writers to investigate what seemed like an analogous phenomenon manifesting itself as a nuisance in an experimental tube intended for the measurement of acoustic impedance by the standing plane wave method. It was observed that, above a certain impressed frequency, difficulty in obtaining uniformly spaced nodes and antinodes in the tube was experienced. This was recognized by its behavior as an effect chargeable to transverse particle velocity. This same difficulty had also been reported by Davis and Evans,[4] who using a pipe having a diameter of 15 cm found undesirable "radial vibrations" to set in above 1290 cycles per second. So far as the writers are aware, however, no systematic experimental investigation of the transmission of transverse acoustic waves had been made. Recent workers in the field of electromagnetic tubular guides have invoked acoustic analogies to aid in making familiar some of the properties of the radio waves, but apparently went no further in their thinking than plane acoustic waves propagated through tubes or radiated from horns. An exception, however, must be noted in the remarks by Leon Brillouin[5] at the close of a very recent paper entitled "The Theoretical Study of Dielectric Cables" in which a more far reaching comparison is drawn.

Lord Rayleigh,[6] probably before 1875, made a systematic theoretical study of the normal modes of aerial vibration within a cylindrical tube, and credits Duhamel[7] with a still earlier investigation limited, however, to the symmetrical vibrations within a cylindrical boundary. Accordingly the theoretical beginnings of the phenomenon before us date back nearly 90 years.

Experimental methods, however, lagged far to the rear. Thus Rayleigh is constrained to quote the results[8] of very limited observations, but careful and excellent ones, on the period of waves produced on water in cylindrical vessels in sup-

[1] Lord Rayleigh, Phil. Mag. **43**, 125–132 (1897).
[2] H. Zahn, Ann. d. Physik **49**, 907–933 (1916).
[3] Schelkunoff, Proc. I. R. E. **25**, 1491 (1937).
[4] A. H. Davis and E. J. Evans, Proc. Roy. Soc. **127**, 89 (1930).
[5] Electrical Communication, April 1938, No. 4, Vol. 16.
[6] Lord Rayleigh, *Theory of Sound*, Vol. II, p. 297.
[7] Duhamel, Liouville J. Math. **14**, 69 (1849).
[8] F. Guthrie, Phil. Mag. **50**, 290–302 (1875).

port of the general correctness of his computation of normal vibration periods. Such has been the advance, however, in acoustic measurement and the production of sustained and controlled waves that what was not possible in 1875 may now be done readily.

It is perhaps not necessary to emphasize that a knowledge of the behavior of transverse acoustic waves is of advantage in the illumination of ideas concerning the analogous electromagnetic phenomenon, particularly in questions relating to the stability of certain types of waves when the assumed theoretical conditions are departed from, as for example the actual ellipticity of a tube supposed circular, the effect of bends, etc. In the acoustic case means are available for measuring continuously the relative phase conditions in various parts of the wave and consequently wave-type identifications can be made more readily than in the case of ultra-high frequency radio waves where as yet this cannot be done. Moreover, the acoustic experimental technique is relatively simple and lends itself to intuitive guidance and to the experimental solution of problems not readily obtainable by mathematical methods.

Transverse Acoustic Waves, Theoretical

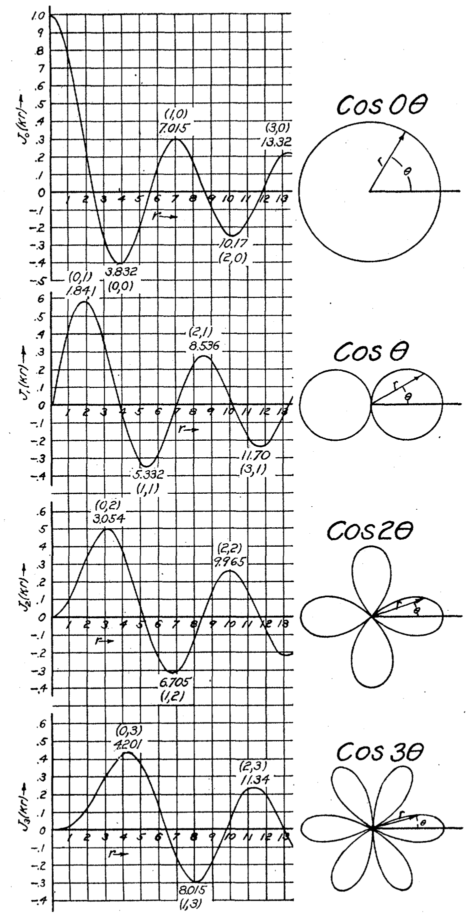
Suppose a smooth tube has an inner radius R and extends indefinitely in the z direction. It will be assumed that there is no energy dissipation in the tube. Then in cylindrical coordinates the wave equation for excess pressure is

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial P}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 P}{\partial \theta^2}+\frac{\partial^2 P}{\partial z^2}=\frac{1}{c^2}\frac{\partial^2 P}{\partial t^2}. \quad (1)$$

Let us first determine the frequencies of the internal normal transverse oscillations, which are defined as those which do not depend on z. With this restriction the solution of Eq. (1) is

TABLE I. *Lower roots of $J_n'(kr)=0$; m is the number of internal circular nodes.*

	$n=0$	$n=1$	$n=2$	$n=3$
$m=0$	3.832	1.841	3.054	4.201
$m=1$	7.015	5.332	6.705	8.015
$m=2$	10.174	8.536	9.965	11.344
$m=3$	13.324	11.706		



(a) (b)

FIG. 1. Rectangular graphs of Bessell's J_n functions each associated with the polar graph of the appropriate circular function, from which the pressure variation at a tube section may be visualized.

the sum of terms of the type

$$J_n(kr)[A \cos n\theta+B \sin n\theta] \sin kct,$$

where $J_n(kr)$ is the Bessell's function of the first kind and n is zero or a positive integer. Since the particle velocity at $r=R$ is zero, $\partial P/\partial r=0$ and accordingly[9] $J_n'(kr)=0$. The lower roots of this equation are shown in Table I, taken from the *Theory of Sound*,[6] p. 298, and the positions of the roots are indicated on the graphs of the appropriate functions in Fig. 1(a). In Fig. 1(b) are shown, associated with the proper $J_n(kr)$ function, the polar graphs of cos $n\theta$.

[9] This is the same boundary condition that holds in the case of electromagnetic waves of the so-called H type.

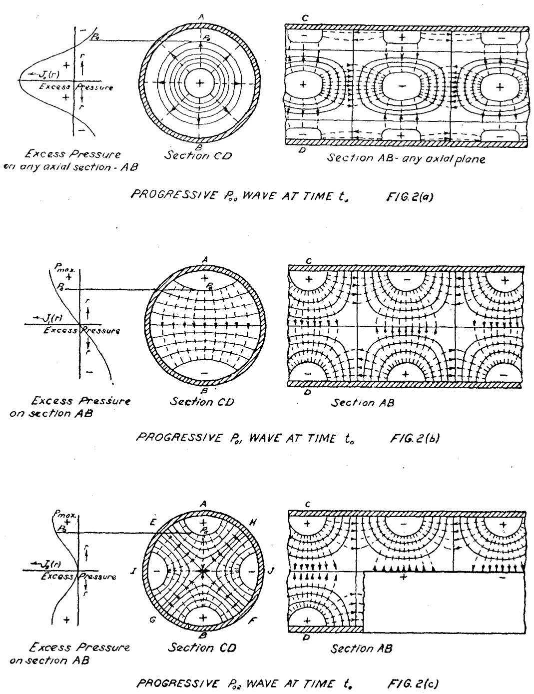


FIG. 2. Approximate configuration of stream lines and surfaces of equal excess pressure. Dotted lines represent stream lines with arrows to indicate the direction of instantaneous particle velocity. Solid lines represent sectional traces of equal pressure surfaces.

The tabular numbers will be denoted by $(kR)_{mn}$. If $(kR)_{mn}/R$ is denoted by k_{mn} and f_{mn} is defined as the frequency of normal oscillation of the type mn, and c is the velocity of plane sound transmission for the medium within the tube, then $k_{mn}=2\pi f_{mn}/c$. For any particular tube radius and sound velocity,

$$f_{mn}=(kR)_{mn}c/2\pi R. \quad (2)$$

If one turns to Eq. (1) and removes the restriction that the excess pressure is independent of z, the general solution may be expressed as the real part of a series of terms of the type

$$J_n(k_{mn}r)[A\cos n\theta+B\sin n\theta] \times\exp[i(\omega t\pm k\psi_{mn}z)], \quad (3)$$

where A and B are arbitrary constants having in general different values for each value of m and n, $\psi_{mn}=[1-(f_{mn}/f)^2]^{\frac{1}{2}}$, $\omega=2\pi f$, and $k=\omega/c=2\pi/\lambda$. Now suppose that by means of external harmonic forces a pressure distribution is maintained in the steady state on the tube section at $z=0$ corresponding to that of the mn normal mode of transverse oscillations, then for a proper choice of signs it is seen that (1) when the frequency of the impressed force is constant and less than f_{mn}, the phase angle of the propagated acoustic wave is independent of z and its amplitude tends toward zero as $z\rightarrow\infty$, but (2) if the impressed frequency is greater than f_{mn}, then the amplitude of the wave is propagated with undiminished magnitude as $z\rightarrow\infty$ while the phase angle decreases linearly. Consequently only in the latter case are progressive waves possible.

An appropriate particular solution of Eq. (1) for the investigation of progressive waves propagated in the positive z direction is,

$$P_{mn}=J_n(k_{mn}r)[A\cos n\theta\sin(\omega t-2\pi z\psi_{mn}/\lambda) +B\sin n\theta\cos(\omega t-2\pi z\psi_{mn}/\lambda)]. \quad (4)$$

The wave-length of the mn wave relative to that of the plane wave is

$$\lambda_{mn}/\lambda=\psi_{mn}, \quad f>f_{mn}. \quad (5)$$

In Eq. (4) when $n=0$ a particularly simple wave type results in which circular symmetry exists and the particle velocity is in axial planes.

$$P_{m0}=J_0(k_{m0}r)A\sin(\omega t-2\pi z\psi_{m0}/\lambda). \quad (6)$$

The resulting pressure distribution and stream lines are shown in Fig. 2(a) for the case $m=0$.

It will be observed Fig. 1(a) that $J_0(kr)$ satisfies the boundary condition $J_0'(kr)=0$ for one additional value not shown in Table I, namely $kr=0$, which however does not imply transverse oscillations but rather represents the ordinary plane wave. If we denote the corresponding critical frequency by $f_{-1,0}$ it will be seen that Eq. (2) predicts the correct result namely that plane waves of all frequencies may be transmitted through tubes.

For every admissible value of n different from zero two independent components of excess pressure are predicted by Eq. (4). If for the sake of concreteness $m=0$, $n=1$,

$$P_{01}=J_1(k_{01}r)[A\cos\theta\sin(\omega t-2\pi z\psi_{01}/\lambda) +B\sin\theta\cos(\omega t-2\pi z\psi_{01}/\lambda)], \quad (7)$$

where A and B are arbitrary. The pressure distribution and stream lines for the case $B=0$, and appropriate choices of z and t, are shown in Fig. 2(b). The wave may be said to have a kind of linear polarization since each particle moves in a line. It possesses a diametral nodal plane section AB on which the normal particle velocity is zero, and an antinodal plane at right angles on which the excess pressure is zero. These nodal planes are not present in the general (0,1) wave, for the second component is in space and time phase quadrature and the resultant wave is in general elliptically polarized and all particles except those on the tube axis and the circumference vibrate in elliptical paths. The wave can be line polarized by introducing a rigid diametral plane in the tube coinciding in position with the nodal plane of one of the components which accordingly is unaffected, the other component suffering reflection.

Obviously similar relations hold for the higher n-ordered transverse waves, one term of Eq. (4) producing a wave linearly polarized, let us say, by sectors, the second component being space shifted $\pi/2n$ radians and in time quadrature combines to give a resultant elliptical polarization. At this point it may be noted that in the experimental investigation of transverse waves it is of great advantage to be able to measure phase relationships. For example if $A=B$ in Eq. (4), then the pressure amplitude and particle velocity amplitude remain constant as θ is varied. Consequently there is nothing to distinguish a wave of one n-order from another except phase and radial variations.

In Fig. 2(c) are shown the lines of equal pressure and of flow for one component of the (0, 2) wave. Obviously the complete delineation of one of the higher ordered (mn) waves is a matter of considerable labor and difficulty.

Apparatus

The experimental tube, Fig. 3, was constructed of thin galvanized iron, the circularity of the cross section being maintained by closely fitted external wooden rings. The tube was inclosed in a compartment filled with sand to prevent the elastic response of the tube walls to acoustic excitation. The tube was mounted on a pair of heavy wooden rails which extended so as to form a guide for a sliding carriage, the latter carrying a heavy, shielded brass chamber within which was mounted in felt a telephone watchcase receiver which served as a sound pressure microphone. Through the end of the brass chamber extended a sound probe consisting of a nest of brass tubes, the outer one $\frac{1}{2}$ inch in

TABLE II. *Frequencies of (m, n) modes of vibration for a rigid tube of circular section filled with air, $c=35{,}000$ cm/sec, diameter 14.92 cm.*

	$n=0$	$n=1$	$n=2$	$n=3$
$m=0$	2,860	1,374	2,279	3,135
$m=1$	5,235	3,979	5,004	5,981
$m=2$	7,575	6,370	7.437	8,466
$m=3$	9,943	8,736		

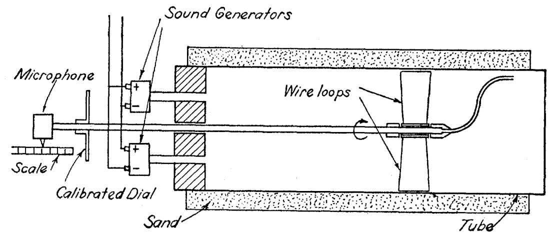


FIG. 3. Sectional diagram of experimental acoustic tube arranged to generate one component of the (0,1) wave.

diameter, the inner one, which served as the actual sound probe, had an opening $\frac{3}{16}$ inch in diameter. Spiral servings of wool yarn were applied between the tubes so as to reduce the lateral sound transmission. The outer tube was fitted loosely so that it could be turned with respect to the inner tubes and was provided with a circular dial marked in degrees at one end, and with a nozzle into which off-set probe tubes of different off-set radii could be inserted. The nozzle end was supported by a collar fitted with slideable wire loops adjusted to the inside of the main tube. It was possible by these means to explore the excess pressure continuously along a cylinder having the radius of the particular off-set probe tube selected and coaxial with the galvanized main tube. To measure pressures continuously along a diameter a special horizontal support was cemented to the inside of the galvanized tube, and a straight probe drawn across the top of the support to definite positions by a music wire cable.

It was of no consequence that the sound probe and microphone varied in sensitivity with frequency as comparisons were made only with pressures at the same frequency. A check of the apparatus made by stopping the end of the sound probe verified that the voltages developed by the

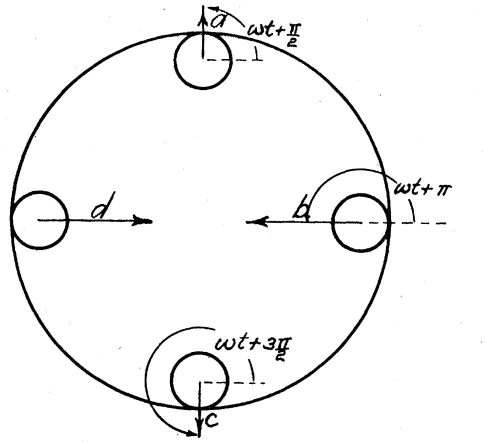


FIG. 4. Section of tube showing location and time phase relation of loudspeakers for producing the (0,1) wave.

microphone were due solely to sound entering at the probe. The output of the microphone was amplified by a vacuum-tube amplifier having 90 decibels gain and then filtered with an adjustable inductance-capacity circuit to eliminate extraneous harmonics. A 3-inch cathode-ray oscilloscope was used for measuring the relative pressure amplitudes, it having been previously established that vertical oscilloscope deflection varied linearly with excess pressure at the end of the sound probe. The horizontal deflecting plates of the oscilloscope were connected through a phase shifting device to the same vacuum-tube oscillator which supplied power to the loudspeakers producing the sound pressures in the main tube. Thus relative phase and amplitude measurements could be made simultaneously. With one exception to be noted later, Western Electric 555 type loudspeaker units were employed as sound generators and the power source was a battery operated 6010B Western Electric oscillator having an excellent sinusoidal wave form, careful frequency calibration, and a very low frequency drift.

Procedure and Experimental Results

In the investigation of transverse waves in a given tube filled with a particular medium, it is convenient to have a table of the lower frequencies below which a given type of vibration is not expected. Table II presents this information computed by Eq. (2) for the experimental tube with a diameter of 14.92 cm. Air is the medium and the velocity of sound propagation is taken to be 35,000 cm per second. It will be observed that when $m=0$, $n=1$ the lowest critical frequency for transverse waves is predicted. Consequently the (0,1) wave type is particularly easy to elicit without at the same time producing disturbing transverse waves of other types. It is necessary only to avoid the production of plane waves in the tube in the approximate frequency interval 1370–2270 in order to have substantially a pure wave form of the (0,1) type. While, strictly speaking, a pressure distribution on the plane $z=0$ as given by Eq. (7) is theoretically indicated, a sufficiently good approximation is obtained by providing relatively concentrated harmonically varying forces as shown in Fig. 4. This may be done by means of loudspeakers arranged as shown in Fig. 3. Any effect due to a departure from the required pressure distribution is rapidly damped out as is the case for plane waves.

The pressures indicated with arbitrary magnitudes and suitable time phase by the arrows in Fig. 4 obviously are such as to inhibit the generation of plane waves. For experimental simplicity a single pair of balanced pressures (*bd*) was employed. The balancing was performed at a frequency just under the critical (0,1) frequency by adjusting the input voltage of one loudspeaker and it was verified that sound pressures in the tube at points distant, say 150 centimeters, from the loudspeakers could be reduced to negligible amplitudes. Now as the frequency was increased and passed $f=1360$,

TABLE III. *Calculated and measured values of wave-length in cm for the (0,1) wave.*

Frequency	Wave-length, (0,1) type Calculated	Measured	Difference	Percent Error
1400	130.0	130.8	0.8	0.6
1420	97.51	95.5	−2.0	−2.1
1440	81.11	77.0	−4.1	−5.0
1460	70.91	69.3	−1.7	−2.4
1500	58.13	58.00	−0.13	−0.2
1600	42.68	42.64	−0.04	−0.1
1700	34.96	35.07	0.11	0.3
1800	30.10	29.95	−0.15	−0.5
1900	26.67	26.76	0.09	0.3
2000	24.08	23.93	−0.15	−0.6
2200	20.37	20.33	−0.04	−0.2
2400	17.79	17.82	0.03	0.2
3000	13.12	13.14	0.02	0.15

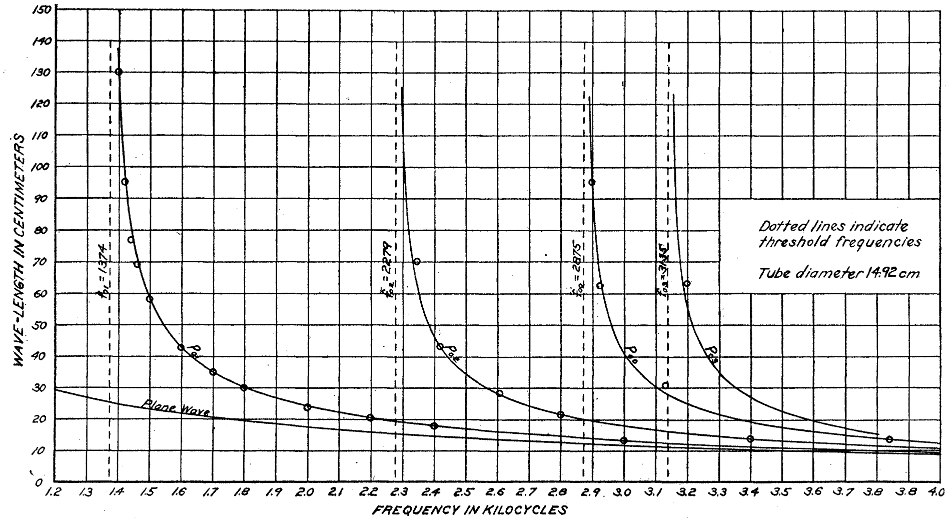


FIG. 5. Variation of wave-length as a function of frequency, for the plane wave and for the (0,0), (0,1), (0,2) and (0,3) type of waves. Curves are theoretical, points are measured values.

the measured excess pressures increased with extreme rapidity and reached such large amplitudes that any residual plane wave component was entirely negligible. Indeed even with a single speaker the (0,1) wave predominated. In spite of the sharpness of the onset of the transverse wave, however, the critical frequency $f_{0,1}$ could be located more accurately through the measurement of the wave-lengths of standing waves. These were produced through reflection by stopping one end of the pipe with a plane-faced cast-iron plug. The longitudinal location of successive minima could then be made for the higher frequencies with considerable accuracy as shown by the degree of equality between successive half-wave-length steps. Data obtained in this way are shown in Table III. It is clear that although the measurements above 2200 cycles were subject to interference from waves of the (0,0) and (0,2) type, the mode of excitation employed was sufficiently unfavorable to substantially suppress the latter. The computed wave-lengths in Table III were obtained by the use of Eqs. (2) and (5) with $R=7.46$ cm and $c=35{,}000$ the latter determined from plane wave measurements in the tube made under the same temperature conditions. The value of $f_{0,1}$ thus obtained was 1374 cycles per second. These wave-length data are also shown in Fig. 5 as the centers of the small circles. The solid lines are the graph of Eq. (5), for the wave type under consideration and others, in which the appropriate computed values of f_{mn} are inserted.

Measurements were also made of the variation of excess pressure in a progressive wave consisting of one (0,1) component, measured at a tube cross section and along the circumference of a circle with center at the tube axis. At one end of the tube a disk of $1\frac{1}{4}$ inch Celotex covered with loose felt provided an almost perfect absorber of the (0,1) wave at a frequency of 1500 cycles per second. The data obtained by use of a probe having a radius of 6.5 cm are shown by the small circles, Fig. 6. The two solid line circles shown within the tube section are the corresponding theoretical curves. It was observed that the pressures recorded as circles in Fig. 6 were in identical time phase, and that points (not shown) corresponding to the lower circle were in phase opposition. It may be remarked, however, that when an attempt was made to produce one component only of the (0,1) wave the quadrature component was also sometimes faintly excited. On investigation it was found that a very slight ellipticity of the tube, the axis being oblique to the axis drawn through the two loudspeaker inlets, Fig. 3, was sufficient to produce this result. By means of simultaneous observations of phase and amplitude the quadrature component could be definitely identified. Measurement of

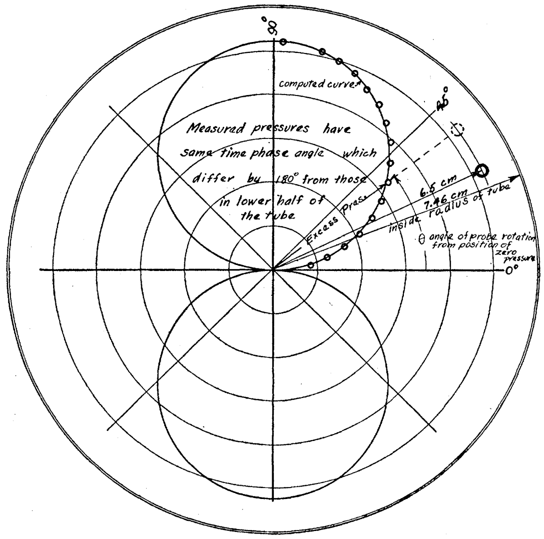


FIG. 6. Excess pressure variation for the (0,1) type of progressive wave measured on the circumference of a 6.5 cm circle coaxial with the tube axis. The curve is the polar graph of sin θ, the points are measured values of relative excess pressure. Frequency 1500 cycles per second.

the maximum amplitudes of the two components and substitution in Eq. (7) yielded values for the resultant which were in excellent agreement with measured resultant values for corresponding values of θ.

The pressure measurements made on the (0,1) wave component were completed by a set in which a straight probe was carried along a tube diameter by a mechanism previously described. The diameter selected was one which passed through the points of maximum peripheral pressure. The frequency was 1500 cycles and the wave progressive. The experimental data obtained are shown by the circles in Fig. 7, the solid line showing the theoretical variation as a function of r, Eq. (7), in which the numerical average of the extreme measured pressure amplitudes was taken as the maximum amplitude of the $J_1(k_{01}r)$ function. In view of the difficulty in directing the motion of the probe accurately along a diameter and through the tube axis with the inflexible means used, the results obtained are quite satisfactory. The departure from the theoretical curve of the actual pressures near the extremities of the diameter may perhaps be attributed to the influence of the probe on the pressure at these points.

To produce other types of transverse waves, special excitation arrangements were, of course, necessary and interference from transverse waves having a lower f_{mn} frequency was expected. In setting up longitudinal standing waves, however, advantage may be taken of the possibility of increasing the amplitude of the desired wave type through tuning by adjusting the length of the air column. By this means it was found possible to excite the (0,0), (0,2) and (0,3) modes with sufficient freedom from interference to obtain the wave-length data shown in Fig. 5, the relative abundance of the points shown near each theoretical curve being a rough measure of the ease of obtaining them. For the (0,2) wave, two speakers with inlets at a and b, Fig. 4, and driven in time phase opposition proved to be sufficiently close to the theoretical pressure distribution to produce a strong (0,2) wave, not however without an admixture of the (0,1) wave type. The latter was satisfactorily minimized by longitudinal tuning. The wave-length data obtained are shown in Table IV as well as graphically in Fig. 5.

On investigating excess pressure in circles coaxial with the tube, it was found that the second (0,2) quadrature component was also present, and as a result the pressure varied with θ approximately as cos 2θ as shown in Fig. 1(b)

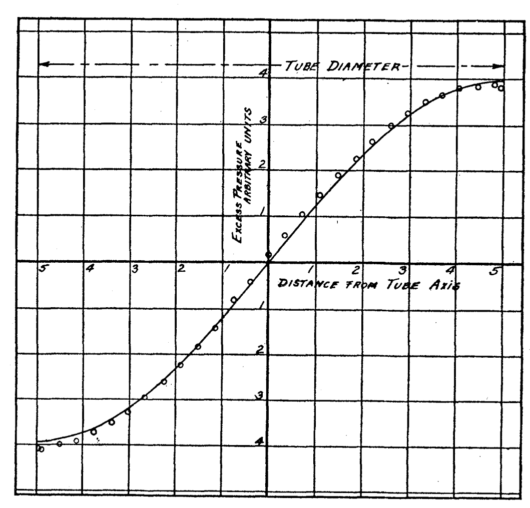


FIG. 7. Excess pressure variation along a diameter of maximum pressure for the (0,1) type of progressive wave, frequency 1500 cycles per second. The curve represents the graph of the Bessell function $J_1(x)$ from $x=0$ to $x=1.84$. Points are measured values of excess pressure.

except that the pressure amplitude at the minimum points was different from zero, leading and lagging respectively in time phase the pressures at points just ahead and behind by one quadrant.

To elicit the (0,3) wave type it would seem from the cos 3θ graph, Fig. 1(b), that at least six loudspeakers equally spaced around the periphery of the tube and alternately driven in time phase opposition would be required. However it was found that a single pair arranged for the generation of the (0,1) wave produced a strong (0,3) wave when the frequency was above the computed threshold value namely 3135 cycles per second. Furthermore, by selecting a frequency and a tube length most favorable for the (0,3) wave and least favorable for the (0,1) wave type, a relatively pure wave form of the (0,3) type was produced. The wave-length at a single frequency was measured and was in satisfactory agreement with the theoretical curve, Fig. 5. It was also verified that the (0,3) wave was still present at a frequency of 3150 cycles, and was definitely absent at 3100 cycles, thus roughly defining the threshold frequency.

When the end of the tube, remote from the generating end, Fig. 3, was left open, radiation to the outside atmosphere naturally occurred, and the general nature of the wave, whether it had one or more diametral antinodes, could be determined directly by listening with one ear covered. The relative amount of the radiation was large and on investigating the standing wave pattern within the tube it was concluded that under some conditions the open end provided a fairly good approximation to an ideal absorber. As in the case of electromagnetic transverse wave guides the analogous conception of an average acoustic impedance of the tube to transverse waves may be introduced.

TABLE IV. *Calculated and measured values of wave-length in cm for the (0,2) wave.*

FREQUENCY	WAVE-LENGTH, (0,2) TYPE CALCULATED	MEASURED	DIFFERENCE	PERCENT ERROR
2345	62.5	70.2	7.7	12.0
2420	43.2	43.3	0.1	0.3
2607	27.7	28.3	0.4	1.4
2800	21.5	21.2	−0.3	−1.4
3000	17.95	17.9	−0.05	−0.3
3400	13.87	13.88	0.01	0.1

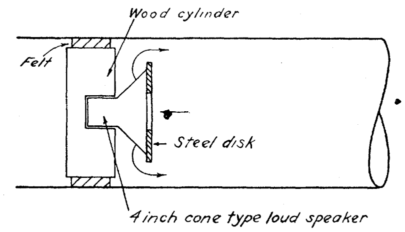


FIG. 8. Sectional diagram showing an arrangement for generating the (0,0) wave.

Discussion of the (0,0) wave type has been deferred until now because this wave is unique and requires a special driving apparatus, (Fig. 8), to produce it. Only waves of the (m,0) type, Fig. 1(b), possess circular symmetry and consequently they alone consist of one component only. Moreover, only waves of this type have excess pressures at the tube axis different from zero. The fact that the pressure for the (0,0) wave is maximum on the tube axis while the contributions of all other waves (except the plane wave) are zero is a circumstance of considerable convenience in making wave-length measurements. Such measurements were made by using the sound generator described and by plugging the loudspeaker holes in the cast iron cylinder, Fig. 3, which thus served as a reflector. It should be observed that in this case the sound probe was included in the portion of the tube in which the measured standing waves were formed, one effect of which is to reduce the effective tube diameter. When this was allowed for, as well as a change in the velocity of the plane sound wave transmission due to temperature conditions prevailing, the normal frequency of the (0,0) transverse mode was computed to be 2875 cycles per second, a small departure from the value in Table II. The experimental wave-length data obtained are shown in Fig. 5 and are in satisfactory agreement with the corresponding theoretical values. The principal irregularity was due to the presence of plane waves which were minimized by longitudinal resonance tuning. On investigating the wave at other points than on the tube axis, however, it was found that a strong (0,1) wave was also present. Indeed the latter component was very difficult to eliminate, such devices as longitudinal diametral barriers being relatively ineffective, probably due to slight residual dissymmetries in the generator, barrier and tube.

Attenuation

The effects of energy dissipation in the tube were observed only in the standing wave-length measurements. Successive minima caused by the interference of the incident and the reflected wave steadily became larger with the distance from the reflecting surface. Measurements made on the (0,1) wave type indicated that the effect of attenuation can be satisfactorily allowed for by means of an exponential attenuation factor. While, of course, attenuation is a fact of major importance in the practical consideration of the long distance transmission of electromagnetic waves through hollow tubes, the principal features of the theoretical dissipationless transverse wave phenomena in short tubes appear to be very similar to those in which small energy losses by dissipation are present.

Discussion

Obviously other than circular cylindrical sections may be used for determining and transmitting transverse acoustic waves. Indeed various cross sections have already been studied[5] for the analogous electromagnetic waves. The normal modes of oscillation for rectangular sections were examined theoretically by Lord Rayleigh[6] many years ago. Also it may be remarked that many features of aerial vibrations within cylinders are similar to the vibration characteristic of plates, which have been extensively investigated experimentally as well as theoretically.

One practical use for acoustic transverse wave transmission has already been suggested by Brillouin,[5] who pointed out that with suitable terminal devices for exciting and detecting a single transverse wave type, a high-pass filter could be constructed. Such a filter could be made to have an extremely sharp cut-off frequency and the amount of attenuation provided in the stop-band could be increased to any desired amount simply by increasing the length of the tube. Acoustic filters of the Stewart[10] type, while they may be designed as high-pass filters are in reality intrinsically band-pass filters so that transmission is not uniform in the so-called pass band.

In determining acoustic absorption coefficients of various commercial materials, the reverberation chamber method is largely used although the results obtained thereby by different laboratories are discordant. The standing plane wave method, on the other hand, gives reproducible coefficients but they are in general lower than those obtained by the reverberation chamber method. It is objected that wave incidence is entirely normal and consequently the conditions of the measurement do not agree with the actual conditions of use. It is to be expected, and some evidence is at hand, that if standing transverse waves are used, higher absorption coefficients will be obtained than for the corresponding plane wave measurement. This matter is now being examined.

In conclusion a caution may be expressed against the use in experimental apparatus of large ducts and pipes intended for plane wave measurements, but which in reality are excellently adapted to the transmission of transverse waves, thereby largely vitiating the results.

[10] W. P. Mason, Bell Sys. Tech. J. **6**, 258 (1927).