Determination of e/m from the $H\alpha-D\alpha$ Interval

ROBLEY C. WILLIAMS*
Cornell University, Ithaca, New York
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Photographs were taken with a 3 mm étalon of the interference pattern of $H\alpha$ and $D\alpha$ from a discharge in the two gases mixed in equal proportions. The interval between the intensity peaks due to the low frequency major components of the two isotopes was measured with the microphotometer used as a comparator. Corrections were made to the observed positions of the intensity peaks to change them to the positions of the centers of the components. These corrections were based on the intensities, positions, and half-intensity breadths of the components as experimentally determined, and upon certain assumptions relative to the shape of the components. An interval between components of 4.14700 ± 0.0004 cm^{-1}, when reduced to vacuum, was obtained. By combining this with recently determined values of the atomic weights of hydrogen and deuterium as reported by Livingston and Bethe, the wave number of $H\alpha$ as determined by Houston, and the physical Faraday as computed by Birge, a value of $1.7579 \pm 0.0004 \times 10^7$ e.m.u./gram is obtained for the value of e/m.

HISTORICAL

VALUES of e/m determined by measuring the difference in wave number between two spectrum lines have been reported by several investigators within recent years. W. V. Houston[1] determined e/m spectroscopically in 1927, using the intervals between the λ4686 line of ionized helium and $H\alpha$, as well as $H\beta$. He obtained by this method a value of $e/m = 1.7606 \pm 0.001 \times 10^7$ e.m.u./gram, which Birge later changed to $1.7603 \pm 0.001 \times 10^7$ e.m.u./gram upon detecting a slight error in Houston's calculations. In 1933 Gibbs and Williams[2] determined the value of e/m provisionally by measuring the interval between similar components of the $H\alpha$ and $D\alpha$ lines, and obtained a value of $1.757 \pm 0.001 \times 10^7$ e.m.u./gram. Kinsler and Houston[3] in 1934 obtained a value of $1.7570 \pm 0.0007 \times 10^7$ e.m.u./gram for e/m from the Zeeman patterns of zinc, cadmium and helium. In 1935 Shane and Spedding[4] used the $H\alpha$–$D\alpha$ interval to obtain a value reported as $1.7579 \pm 0.0003 \times 10^7$ e.m.u./gram. Houston[5] has recently determined a provisional value of 1.760 ± 0.0015 e.m.u./gram for e/m, based upon a Fourier analysis of the fine structure patterns of $H\alpha$ and $D\alpha$.

* Now at the Observatory of the University of Michigan.
[1] Houston, Phys. Rev. **30**, 608 (1927).
[2] Gibbs and Williams, Phys. Rev. **44**, 1029 (1933).
[3] Kinsler and Houston, Phys. Rev. **45**, 104 (1934); **46**, 533 (1934).
[4] Shane and Spedding, Phys. Rev. **47**, 33 (1935).
[5] Houston, Phys. Rev. **51**, 446 (1937).

DETERMINATION OF e/m FROM THE $H\alpha$–$D\alpha$ INTERVAL

The equations used for determining the mass of the electron in atomic weight units, and e/m are derived in a previous publication[6] and are given here

$$m = \frac{H^+D^+(\nu_D - \nu_H)}{D^+\nu_H - H^+\nu_D} \tag{1}$$

and

$$e/m = \frac{F(D^+\nu_H - H^+\nu_D)}{H^+D^+\Delta\nu_{H-D}}, \tag{2}$$

where F is the Faraday, ν_H and ν_D the wave numbers of like components of hydrogen and deuterium lines, and H^+ and D^+ the atomic weights of the hydrogen and deuterium nuclei.

The advantage of using the two isotopes of hydrogen for the determination of e/m is at once apparent: $\Delta\nu_{H-D}$ is the only quantity to be measured, while the atomic constants and the wave numbers are known to a relatively high degree of precision.

EXPERIMENTAL

By means of the discharge tube, spectrograph and interferometer described in the preceding paper, interference fringes were obtained of $H\alpha$ and $D\alpha$. The gas in the tube was maintained at approximately half hydrogen and half deuterium, with a total pressure of about 3.1×10^{-1} mm of mercury, and a current density of about 20

[6] Gibbs and Williams, Phys. Rev. **44**, 1029 (1933).

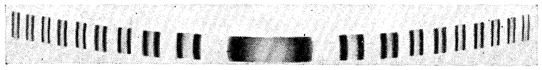

FIG. 1. Interferometer fringes of H and D obtained with a 3-mm étalon spacing.

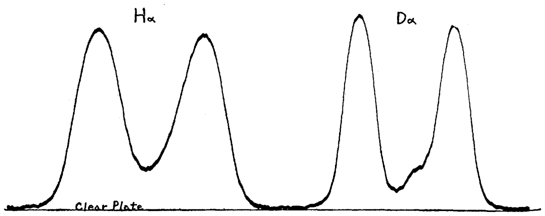

FIG. 2. Microphotometer tracing of the fringe pattern shown in Fig. 1.

ma/cm^2. Five excellent plates were taken with the aid of a 3-mm étalon spacing.

The choice of the proper optical equipment for the analysis of Hα and Dα on the same plate must be in the nature of a compromise. One possibility is to use a primary instrument of great dispersion, so that the spectrum lines of Hα and Dα are separated sufficiently to prevent the interference patterns overlapping. This was done by Shane and Spedding. The most serious objection to this type of apparatus is its lack of speed. This lack of speed, which is likely to introduce distortions because the optical apparatus and source cannot readily be maintained in the same adjustment for the necessary time, can be overcome by a much lighter coating of silver on the étalon plates, but this leads to a greatly reduced optical resolution. The second possibility is to use a lower-dispersion primary instrument, and an étalon spacing of approximately 3 mm. This spacing causes a given Dα fringe to be overlapped about two and a half orders from the Hα fringe of the same order, but does not cause the Hα and Dα patterns to be confused. Fig. 1 is a reproduction of such a fringe system. Fig. 2 shows that the intensity of the pattern drops truly to zero between each Hα and Dα fringe. The resolving power of a 3-mm étalon silvered to reflect over 90 percent is over 250,000, while the temperature broadening of even Dα makes a resolving power much greater than this relatively unimportant. This point is illustrated in the half-intensity breadths obtained for Dα with a 3-mm and a 5-mm étalon spacing. The former spacing yielded a half-intensity breadth of about 0.120 cm^{-1}, and the latter spacing gave a breadth between 0.115 cm^{-1} and 0.120 cm^{-1}.

The peak-to-peak separations of peak 1 of Hα and Dα were measured in the manner indicated in the preceding article. The fractional Δp interval between the two similar peaks was determined, and to this number the integer 2 was added, since it was known that the fringes overlap approximately two and a half orders of interference.

The $\Delta\nu$ interval was found by dividing $2+\Delta p$ by the customary denominator $2d\cos\theta$. By use of a standard neon spectrum, the étalon spacing was determined with an accuracy of about 1 part in 1,000,000. Neon fringes were taken with each plate of Hα and Dα.

The $\cos\theta$ factor was carefully accounted for. At the center of the interference pattern, the $\Delta\nu$ between orders is strictly equal to $1/2d$, but as the distance from the center increases, the $\Delta\nu$ decreases as $1/\cos\theta$. Hence in computing the $\Delta\nu$ between the peaks of a pattern, the measured Δp must be divided by $2d$ times the cosine of the angle made by the peaks in question. This angle is the angle whose tangent is the radius of the fringe divided by the focal length of the camera lens of the spectrograph.

RESULTS

The observed values of Δp between peak 1 of Hα and peak 1 of Dα are listed in Table I, by plate number and by fringe number. (To each

TABLE I. *Observed values of Δp between peak 1 of* Hα *and peak 1 of* Dα.

PLATE NUMBER	FRINGE 2	3	4	2d (cm)
212*a*		0.48926 0.48854	0.48800	0.600163
212*b*	0.48748	0.48710 0.48743	0.48804	0.600163
212*c*	0.48807	0.48842 0.48870	0.48827	0.600163
212*d*		0.48871 0.48845	0.48829	0.600163
212*e*	0.48677	0.48977 0.48899	0.48926	0.600163
	$\cos\theta=$ 0.99987	$\cos\theta=$ 0.99976	$\cos\theta=$ 0.99962	

TABLE II. *Values of $\Delta\nu$, the frequency difference between peak 1 of Hα and peak 1 of Dα.*

Plate Number	Fringe 2	Fringe 3	Fringe 4
212*a*		4.14863 cm^{-1} 4.14743	4.14711 cm^{-1}
212*b*	4.14521 cm^{-1}	4.14503 4.14558	4.14718
212*c*	4.14619	4.14723 4.14770	4.14756
212*d*		4.14772 4.14728	4.14756
212*e*	4.14403	4.14948 4.14818	4.14981

fractional Δp the integer 2 must be added since the fringes are overlapped two and a half orders of interference.)

At the right-hand side of the table are listed the $2d$ values for each plate, and at the bottom is listed the $\cos\theta$ value for each fringe. Dividing each $2+\Delta p$ by the proper $2d\cos\theta$, gives the values of $\Delta\nu$ in cm^{-1} shown in Table II.

The average peak-to-peak interval is:

$$\Delta\nu \text{ (air)} = 4.14716 \pm 0.0004 \text{ cm}^{-1}$$

and $$\Delta\nu \text{ (vac.)} = 4.14610 \pm 0.0004 \text{ cm}^{-1}.$$

To this interval a correction must be made because of the differential effects in the case of Hα and Dα of components 3 and 4 on the position of the center of component 1. The intensity ratio of component 1 and the disturbing components, and the magnitude of these corrections are listed in Table III. These corrections are based on the following assumptions and hypotheses: (1) The intensity distribution in each component is that given by Doppler broadening, and is the same for all components. (2) The position of component 4 is that determined by theory. (3) The position of component 3 in Hα is the same as that found by analysis for Dα. The differential correction is in such a direction as to increase the measured peak-to-peak interval of Hα and Dα.

Adding this amount to the average $\Delta\nu$ (vac.) just obtained, there results the completely corrected $\Delta\nu$ (vac.) value of the separation of the $2p\ ^2P_{3/2}-3d\ ^2D_{5/2}$ component of Hα from the corresponding component of Dα. This is:

$$\Delta\nu \text{ (vac.)} = 4.14700 \pm 0.0004 \text{ cm}^{-1}.$$

The values of and authorities for the atomic constants used in computing the atomic weight of the electron, m, from Eq. (1), and e/m from Eq. (2) are these:

$\nu_{H\alpha}$(vac.) = 15233.094 cm^{-1}	Houston[7]
$H^+ = 1.00813 - 0.00055$	Livingston and Bethe[8]
$D^+ = 2.01473 - 0.00055$	Livingston and Bethe[8]
$F = 9651.3 \pm 8$ abs. coulombs	

(F has been adjusted to the atomic mass scale, and is based upon an O^{16}/O^{18} ratio of 537.)

Substituting these values in Eq. (1) we obtain the value of the mass of the electron in atomic weight units:

$$m = 5.4902 \pm 0.0005 \times 10^{-4}.$$

The e/m ratio is determined by dividing the value of the Faraday by the atomic weight of the electron:

$$e/m = 9651.3/5.4902 = 1.7579 \pm 0.0004 \times 10^7 \text{ e.m.u./gram.}$$

The probable error of the determination of the Hα–Dα interval is ± 1 part in 10,000, and that of the Faraday is ± 1 part in 12,000. The effect of the probable errors of the atomic masses and the absolute wave number of Hα is negligible. Upon combining the two significant probable errors, a value of ± 0.0003 e.m.u./gram is obtained for e/m. This has been increased to ± 0.0004 e.m.u./gram to allow for any constant error. The uncertainties in the *corrections* of the component positions are not included in the adopted probable error.

After making slight changes[9] in the value of $R_{He}-R_H$ as used by Houston, adjusting the value of F to the physical mass scale as used in the present paper, and changing to the most recently reported[8] atomic mass values, Houston's earlier value of e/m becomes 1.7600 $\pm 0.001 \times 10^7$ e.m.u./gram.

TABLE III. *Magnitude of the corrections to the frequencies of* Hα *and* Dα.

	Hα	Dα
Half-intensity width	0.165 cm^{-1}	0.120 cm^{-1}
I_1/I_4	7	6–7
I_1/I_3	3.5	3.5
Correction due to component 4	−0.0041 cm^{-1}	+0.0038 cm^{-1}
Correction due to component 3	+0.0012 cm^{-1}	0.0000 cm^{-1}
Net correction	−0.0029 cm^{-1}	+0.0038 cm^{-1}
Differential correction (+ increases interval)	+0.0009 cm^{-1}	

[7] Houston, Phys. Rev. **30**, 608 (1927).
[8] Livingston and Bethe, Rev. Mod. Phys. **9**, 373 (1937).
[9] Williams and Gibbs, Phys. Rev. **45**, 491 (1933).

TABLE IV. *Corrections to component 1 due to component 3 for* Hα.

I_1/I_3	$X_h=0.150$ cm^{-1}			I_1/I_3	$X_h=0.160$ cm^{-1}			I_1/I_3	$X_h=0.170$ cm^{-1}		
3	0.0006	0.00075	0.0011	3	0.0009	0.0012	0.0017	3	0.0013	0.0018	0.0025
4	0.0004	0.0006	0.00085	4	0.0007	0.0009	0.0014	4	0.0011	0.0014	0.0020
5	0.0003	0.00045	0.00065	5	0.00055	0.0007	0.0011	5	0.0010	0.0013	0.0016
Separation of I_1 and I_3 in cm^{-1}	0.200	0.190	0.180		0.200	0.190	0.180		0.200	0.190	0.180

Upon correcting to *vacuum* the measured $\nu_{H\alpha}-\nu_{D\alpha}$ interval as reported by Shane and Spedding and with the above-mentioned atomic mass values, their value for e/m becomes $1.7581\pm0.0003\times10^7$ e.m.u./gram.

In his latest report on this subject Dunnington[10] has announced a highly definitive value of $1.7597\pm0.0004\times10^7$ e.m.u./gram for e/m_0 which he obtained from magnetic deflection methods. The precision of his measurements and the extreme care taken by him to correct for numerous sources of error lend considerable weight to his determination.*

Critical Examination of Possible Errors

In the evaluation of e/m spectroscopically by use of hydrogen and deuterium, the errors in the constants used are probably relatively small compared to the error in the measured $\Delta\nu$ between the similar components of Hα and Dα.

The most accurate value is that of $\nu_{H\alpha}$, obtained by Houston in 1927. Even granting that this value is in error[9] by 0.0027 cm^{-1} the resultant percent error is negligibly small.

It now appears that the proton and deuteron mass values have become well established and are known to an accuracy considerably beyond that of the measured interval.

The observations on the fringe diameters of the interference patterns seem to be free from any chance of systematic errors. Errors in the screw of the microphotometer would introduce only differential effects, since absolute wave numbers are not being measured, but only the difference of two nearly equal wave numbers. That the actual determination of the points of maximum intensity of the fringes can be done with great accuracy is attested by the very small probable error of ±0.0004 cm^{-1} in the peak-to-peak measurements, and the maximum spread of the observations of about ±0.006 cm^{-1}.

The recent correction of the Faraday to fit the physical scale of masses is an important contribution to the accuracy of the calculations of e/m. The probable error of the Faraday value remains the same as before, about ±8 parts in 100,000.

All of the above-mentioned sources of error are probably smaller than that introduced by the correction of the positions of peak 1 in Hα and Dα. In the preceding paper there is a resumé of the conditions and assumptions underlying the type of correction. By reference to Table III in that paper, which shows the corrections of peak 1 due to the influence of component 4, it is seen that by adopting any reasonable value for I_1/I_4 and for the separation of component 4 from component 1, the *differential* correction between Hα and Dα ranges between 0.0002 cm^{-1} and 0.0004 cm^{-1}.

From Table IV it is seen that the range of corrections probable for peak 1 due to component 3 for Hα is from 0.0005 cm^{-1} to 0.0018 cm^{-1}. The correction due to component 3 is entirely negligible for Dα. The net differential correction, between Hα and Dα, to the position of peak 1 is estimated at $+0.0009$ cm^{-1} for the most probable value, but could conceivably be as high as $+0.0015$ cm^{-1} or as low as $+0.0002$ cm^{-1}. In the former case, e/m would have the value of 1.7576×10^7 e.m.u./gram, and in the latter case it would be 1.7582×10^7 e.m.u./gram.

At the beginning of this research it was thought that the value of e/m might possibly appear to vary with changing discharge conditions, even when the interval between the $2p\ {}^2P_{3/2}-3d\ {}^2D_{5/2}$ components (components 1) of Hα and Dα were measured. Five plates, called series H-D213, were taken with the discharge tube operating under the extreme conditions of 2.5 mm of Hg pressure, and a current density of 60 ma/cm^2. The average peak-to-peak separation of peak 1

[10] Dunnington, Phys. Rev. **52**, 475 (1937).

* *Note added in proof:* In a recent paper, Phys. Rev. **54**, 193 (1938) A. E. Shaw reports upon the use of crossed electric and magnetic fields, from which he obtains $e/m_0=1.7571\pm0.0013\times10^7$ e.m.u./gram.

was found to be 2.48666 orders of interference for these plates, compared with an average interval of 2.48832 orders of interference for the normal discharge conditions. This indicates that the peaks were about 0.003 cm^{-1} closer together for the 213 series than for the 212 series. However, the net corrections to the positions of peak 1 in the 213 series are $+0.0016$ cm^{-1} for Hα and $+0.0029$ cm^{-1} for Dα. This means that the differential correction is 0.0045 cm^{-1} in size, and in such direction to increase the Hα–Dα interval. The net result of this is that the Hα–Dα interval for the high pressure, high current density plates was found to be about 0.0015 cm^{-1} greater than for the plates taken under normal conditions. However, the corrections for the 213 series are much more uncertain than those for the 212 series because of the much larger half-intensity breadths of the fringes in the former case.

Component 2, resulting from a $2p\,^2P_{1/2}-3d\,^2D_{3/2}$ and a $2s\,^2S_{1/2}-3p\,^2P_{3/2}$ transition, was not used in this research, but instead the $2p\,^2P_{3/2}-3d\,^2D_{5/2}$ component was used. This was done for these reasons: The last-named component arises from a single transition and consequently is not split by a weak-field Stark effect, nor has it a large hyperfine structure splitting. Bechert and Meixner[11] show that the fine structure and hyperfine structure patterns of Hα and Dα should be identical; accordingly we should not expect any relative shift of component 1 in the fine structure patterns of the two isotopes.

[11] Bechert and Meixner, Ann. d. Physik **22**, 525 (1935).

Summary

Calculations of the value of e/m, based upon the interval between the $2p\,^2P_{3/2}-3d\,^2D_{5/2}$ components of Hα and Dα, result in a value of $1.7579\pm0.0004\times10^7$ e.m.u./gram. The new value of the Faraday, based on the physical scale, and the new scale of atomic weights are used in the evaluation.

From a careful consideration of the possible sources of error resulting from the measurement of the interval, it is felt that the value of e/m as determined from the Dα and Hα interval cannot be greater than 1.7583 nor smaller than 1.7575 $\times10^7$ e.m.u./gram.

The mass of the electron in atomic weight units is found to be $5.4902\pm0.0005\times10^{-4}$.

Acknowledgment

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A Method of Calculating Fluctuations

Raymond Ellickson*
University of Chicago, Chicago, Illinois
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A general method for the solution of fluctuation problems is described. A generating function for the differential equations that are satisfied by the probabilities in question is defined and the problem is reduced to the determination of the generating function. The general method for its determination is given and the results are applied to fluctuation problems in pair-production and radioactive disintegration. The method of finding the mean, mean square, and standard deviation from the generating function without reference to the form of the distribution itself is applied to the examples. The problem of fluctuations in chain and branch radioactive disintegrations is solved without restrictions on the size of the disintegration constants involved.

Introduction

In experiments on radioactivity, cosmic rays, energy loss of electrons, etc., fluctuations play an important part. In some cases it is relatively easy to take these fluctuations into account, for example, those that follow the Poisson law. The Poisson law can be derived from the more general binomial law[1] if the restriction is made

* Now at Brooklyn Polytechnic Institute.

[1] L. I. Schiff, Phys. Rev. **50**, 88 (1936).

FIG. 1. Interferometer fringes of H and D obtained with a 3-mm étalon spacing.