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# The Scattering and Photoelectric Absorption of High Voltage X-Rays in Nitrogen

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A study of the expansion chamber tracks of 810 kv x-radiation passing through nitrogen shows that the angular distribution of total electrons emitted is in good agreement with the Klein-Nishina formula when allowance is made for the presence of photoelectrons. The latter are distinguished from recoils by the criterion of the maximum Compton energy. Electrons having energies greater than the Compton energy for the corresponding angle of emission were classified as photoelectrons while those having smaller energies were either photo- or recoil electrons. The ratio of recoils to photoelectrons is of the order of 100 in contrast to the theoretical value of about 5000. The angle of bipartition for photoelectrons is about 48° for 360 kev and about 20° for 650 kev photoelectrons.

CONSIDERABLE experimental work has been done on the scattering and photoelectric absorption of low voltage x-rays,[1–4] but relatively little work has been done in the region of high voltage x-rays.[5] The latter region is of special interest because it affords an opportunity to test more fully the present quantum theory of the interaction of radiation and matter. According to present theory[6, 7] the photoelectric absorption of high voltage x-rays in light elements is practically zero, while pair production has not begun. Thus, practically all of the absorption should be due to incoherent scattering, and both the angular distribution of the emitted electrons and the absorption cross section should be given by the Klein-Nishina formula.

The present investigation lies in the region of 100–800 kev, and includes a determination of the energy of recoil electrons as a function of the angle of emission and a determination of the angular distribution of photoelectrons as a function of the energy. Also, a preliminary value of $\sigma/\tau$ (the ratio of the number of recoil electrons to the number of photoelectrons) is obtained.

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[3] Bubb, Phys. Rev. **23**, 137 (1924). Compton and Simon, Phys. Rev. **25**, 309 (1925); Proc. Nat. Acad. Sci. **11**, 303 (1925); Phys. Rev. **26**, 289 (1925). Ikeuti, Comptes rendus **180**, 1257 (1925). Nuttall and Williams, Phil. Mag. **1**, 1217 (1926). Kirchner, Physik. Zeits. **27**, 385 (1926); Physik. Zeits. **27**, 799 (1926); Ann. d. Physik **81**, 1113 (1926); Ann. d. Physik **83**, 969 (1927).

[4] Eggleston and Martin, Proc. Roy. Soc. **162**, 95 (1937).

[5] Read and Lauritsen, Phys. Rev. **45**, 433 (1934). Read, Proc. Roy. Soc. **152**, 402 (1935). Cuykendall, Phys. Rev. **50**, 105 (1936).

[6] Klein and Nishina, Zeits. f. Physik **52**, 853 (1929). Heitler, *Quantum Theory of Radiation* (Oxford Press).

[7] Sauter, Ann. d. Physik **11**, 454 (1931).

## Discussion of Theory

### Probability of electron emission

Using relativistic wave equations and Born's approximation which can be applied for light elements, Sauter[7] obtained for photoelectric emission the result

$$\frac{\varphi_k}{\varphi_0}=\frac{3}{2}\frac{Z^2}{137}\left(\frac{\mu}{k}\right)^5(\gamma-1)^{\frac{3}{2}}\left[\frac{4}{3}+\frac{\gamma(\gamma-2)}{\gamma+1}\times\left(1-\frac{1}{2\gamma(\gamma^2-1)^{\frac{1}{2}}}\log\frac{\gamma+(\gamma^2-1)^{\frac{1}{2}}}{\gamma-(\gamma^2-1)^{\frac{1}{2}}}\right)\right], \quad (1)$$

where $\phi_k$ is the cross section for the photoelectric emission of $K$-electrons, $\phi_0=(8\pi r^2)/3$ is the cross section for classical scattering, $Z$ is the atomic number, $\mu$ is the rest energy of an electron, $k$ is the energy of the incident photon, and

$$r=1/(1-\beta^2)^{\frac{1}{2}}=(k+\mu)/\mu$$

is the ratio of the total energy (kinetic energy $+mc^2$) of the electron to the rest energy. Since the $K$ shell contains two electrons and experiment shows that these contribute about 80 percent of the photoelectric effect,[8] one may obtain the approximate total photoelectric cross section by doubling the value obtained from Eq. (3) and multiplying by a factor of 5/4.

The cross section for incoherent scattering can be calculated in a similar manner, except that in this case two light quanta are involved and the process is of the second order. If the energy of the primary quanta is much greater than the binding energy of the electrons, the scattering electrons can be considered as free. By the use of relativistic equations, Klein and Nishina[6] have obtained the following formula for the scattering cross section of free electrons:

$$d\varphi_s=\frac{r_0^2\,d\Omega_\varphi}{2}\frac{k^2}{k_0^2}\left(\frac{k_0}{k}+\frac{k}{k_0}-\sin^2\varphi\right), \quad (2)$$

where $d\phi_s$ is the differential scattering cross section, $r_0$ is the classical radius of an electron, $d\Omega_\varphi$ is the differential solid angle, $k_0$ and $k$ are the energies of the incident and scattered quanta, and $\phi$ is the angle through which a quantum is deflected. If this equation is integrated over all angles, one obtains for the total cross section

$$\frac{\varphi_s}{\varphi_0}=\frac{3}{4}\left\{\frac{1+\alpha}{\alpha^3}\left[\frac{2\alpha(1+\alpha)}{1+2\alpha}-\log(1+2\alpha)\right]+\frac{1}{2\alpha}\log(1+2\alpha)-\frac{1+3\alpha}{(1+2\alpha)^2}\right\}, \quad (3)$$

where $\phi_0$ is again the classical cross section for scattering and $\alpha=k_0/\mu=h\nu_0/mc^2$.

### Angular distribution of recoil electrons

The angular distribution of recoil electrons may be obtained from Eq. (2). This equation as it stands represents the relative number of photons that are scattered into a solid angle $d\Omega_\varphi$ at an angle $\phi$ with the direction of the incident beam. One may now substitute for $d\Omega_\varphi$ the quantity $2\pi\sin\phi\,d\phi$ and obtain the total (relative) number of photons scattered between the angles $\phi$ and $\phi+d\phi$. From the well-known theory of the Compton effect one obtains

$$E_{\text{kin}}=h\nu-h\nu'=h\nu\frac{\alpha\text{ vers }\varphi}{1+\alpha\text{ vers }\varphi};\qquad \therefore\ \frac{h\nu'}{h\nu}=\frac{k}{k_0}=\frac{1}{1+\alpha\text{ vers }\varphi} \quad (4)$$

and with this further substitution for $k/k_0$, Eq. (2) becomes

$$d\varphi_s=r_0^2\pi\sin\varphi\,d\varphi\left(\frac{1}{1+\alpha\text{ vers }\varphi}+\frac{1}{(1+\alpha\text{ vers }\varphi)^3}-\frac{\sin^2\varphi}{(1+\alpha\text{ vers }\varphi)^2}\right). \quad (5)$$

If $\phi$ is expressed in terms of $\theta$ by means of the Compton theory, then Eq. (5) shows the total number of electrons ejected between the angles $\theta$ and $\theta+d\theta$, and this quantity divided by $2\pi\sin\theta\,d\theta$ gives the number of recoil electrons per unit solid angle. The final result is

$$\frac{d\varphi_s}{d\Omega_\theta}=\frac{r_0^2(1+\alpha)^2(2-\sin 2\theta)}{2a^2\cos^3\theta}\times\left[\frac{a}{a+2\alpha}+\left(\frac{a}{a+2\alpha}\right)^3-\frac{4(a-1)}{(a+2\alpha)^2}\right], \quad (6)$$

[8] Rutherford, Chadwick, and Ellis, *Radiations from Radioactive Substances* (Cambridge University Press) p. 464.

TABLE I. *Ratio of total scattering to the photoelectric absorption.*

| $h\nu$ (KEV) | $\sigma/\tau$ |
|---|---|
| 350 | 2260 |
| 510 | 4875 |
| 650 | 8100 |

where $a=(1-\alpha)^2\tan^2\theta-1$. If $d\varphi/d\Omega_\theta$ is evaluated for various angles $\theta$, a graph of the results will show the angular distribution of the recoil electrons in a plane cross section along the incident beam of radiation. Such a curve, computed for 510 kev, is shown in Figs. 6 and 7, where the area under the theoretical curve has been adjusted to be equal to that under the experimental curve so as to refer to equal numbers of tracks in both cases.

Theoretical values of $\sigma/\tau$, or the ratio of the total scattering per atom to the photoelectric absorption, may be obtained by means of Eq. (1), (3)

$$\frac{\sigma}{\tau}=\frac{Z\left(\dfrac{\varphi_s}{\varphi_0}\right)}{\dfrac{5}{2}\left(\dfrac{\varphi_k}{\varphi_0}\right)}=\frac{Z\left[\dfrac{3}{4}\left\{\dfrac{1+\alpha}{\alpha^3}\left(\dfrac{2\alpha(1+\alpha)}{1+2\alpha}-\log(1+2\alpha)\right)+\dfrac{1}{2\alpha}\log(1+2\alpha)-\dfrac{1+3\alpha}{(1+2\alpha)^2}\right\}\right]}{\dfrac{5}{2}\left[\dfrac{3}{2}\dfrac{Z^5}{137}\left(\dfrac{\mu}{k}\right)^5(\gamma^2-1)^{\frac{3}{2}}\left\{\dfrac{4}{3}+\dfrac{\gamma(\gamma-2)}{\gamma+1}\left(1-\dfrac{1}{2\gamma(\gamma^2-1)^{\frac{1}{2}}}\log\dfrac{\gamma+(\gamma^2-1)^{\frac{1}{2}}}{\gamma-(\gamma^2-1)^{\frac{1}{2}}}\right)\right\}\right]}. \quad (7)$$

A few values for nitrogen are shown in Table I. In making comparisons with our experiments, the 510 kev value will be used, since this lies near the maximum[9] of the energy spectrum of the 800 kv x-ray tube.

## EXPERIMENTAL PROCEDURE

The interaction of high voltage x-rays with nitrogen was studied by means of a 6″ automatic cloud chamber. A sylphon tube, the bottom of which was closed with a brass spinning, formed the lower portion of the chamber, and when compressed air was admitted to the brass casting surrounding the sylphon, the sylphon was compressed the desired amount. Then at the proper instant, a valve was opened by an electromagnet, thus allowing the air to escape and the chamber to expand. The expansions and all other events occurring during the operation of the chamber were controlled by a motor-driven commutator arranged so that the expansions occurred every half minute.

Nitrogen was introduced into the chamber by allowing it to bubble slowly through 95 percent ethyl alcohol, and the alcohol that distilled over into the chamber was used to form the condensation on the ions.

An 800 kv x-ray tube was located in the room above the cloud chamber apparatus, and radiation from this tube passed through a $\frac{3}{4}''\times 13''$ hole in a lead cylinder in the three-foot thick concrete floor of that room and then down into a lead tunnel located above the cloud chamber. In this tunnel, the x-rays were further collimated so that the beam passing through the chamber was $\frac{1}{4}''$ in diameter. A solenoid operated lead shutter prevented the radiation from entering the chamber except during a very small interval at the time of the expansion. Soft radiation was removed from the beam of x-rays by a $\frac{1}{2}''$ lead filter, and a special filter located immediately above the chamber removed the $K$-radiation of lead. The x-rays entered the chamber through a Cellophane window $\frac{5}{16}''$ in diameter. In order to accommodate the vertical beam of radiation, the chamber was used in the vertical position. A pair of Helmholtz coils was used to produce a magnetic field so that one could determine the energies of the emitted electrons.

Photographs were obtained by a single Sept camera ($f$: 3.5 lens) placed directly in front of the cloud chamber, and sufficient illumination was provided by two 2000-w Movieflood lamps and a set of hollow cylindrical glass lenses filled with benzene. The photographs were reprojected to

[9] Determined by Mr. John Rose, physicist of the Swedish Hospital, Seattle Washington, by means of total absorption measurements in water and lead. A similar result was obtained by Donald H. Loughridge in preliminary measurements with a crystal spectrograph. The maximum intensity of the x-rays was found to occur near 500 kv.

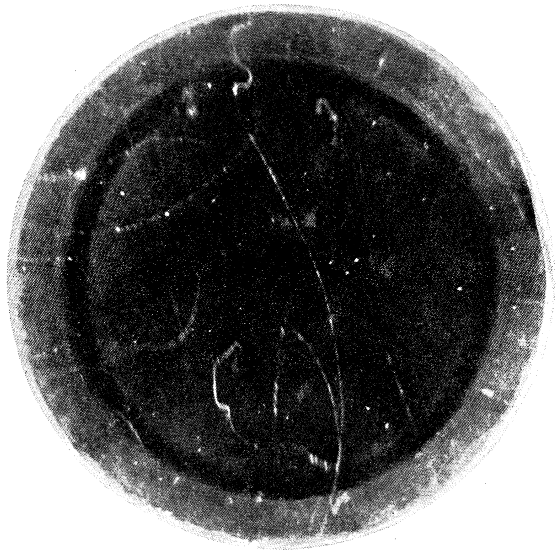

FIG. 1. Photoelectron track 673 kev showing change in curvature due to close collision.

normal size on a ground glass screen and could be viewed easily from the rear. In order to avoid distortion of the tracks, the optical system in the reprojecting apparatus was identical to that with which the photographs were taken.

The current through the Helmholtz coils was measured about every twelve pictures, and the peak voltage across the x-ray tube was kept constant at 810 kv. The magnetic field had been previously calibrated by means of a small search coil, and the x-ray tube peak voltage had been measured with a rotary voltmeter.[10]

The curvatures of the tracks were measured by comparison with a set of curves having known radii of curvature which varied from 1 to 15 cm in steps of 0.1 cm and the initial directions of the tracks were measured by means of a protractor with divisions every 5°.

In order to be satisfactory for measurement, a track must begin within the x-ray beam and not less than $\frac{1}{8}''$ from the walls of the chamber. The track must have a reasonably definite point of origin, a definite initial direction, and a constant radius of curvature. It must also remain within the beam of light, the mean thickness of which was about $\frac{1}{2}''$ ($\frac{3}{8}''$ at the x-ray beam). Some tracks showed definite changes in curvature due to collisions of the electron with other particles (Fig. 1); these tracks could be measured if the curvature remained constant along a sufficient length of track.

## Discussion of Results

### Maximum energy of electrons

If one plots the total number of tracks in each 50 kv interval against the corresponding energy, one obtains the curve shown in Fig. 2. This curve has a definite bend near 600 kv, and the portion above 650 kv is a straight line while that below 550 kv has only a small curvature. If the latter portion of the curve is extended down to the energy axis, the cut-off value is found to be 625 kv which is very near the maximum energy of the Compton recoil electrons. One may thus infer that the electrons are of two types: photoelectrons which have a maximum kinetic energy of 820 kev and recoil electrons with a maximum kinetic energy of 625 kev. Both of these energies are very near the theoretical values of 810 kev and 612 kev.

According to the Compton theory, the energy of a recoil electron is a definite function of its

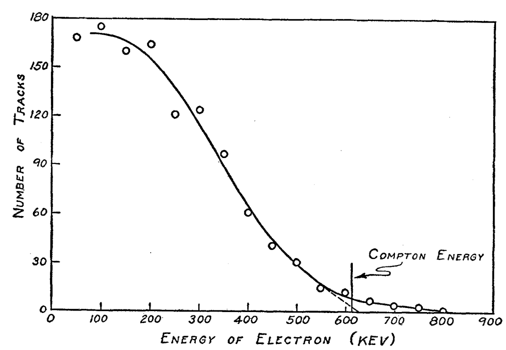


FIG. 2. Total energy distribution.

angle of emission. This relationship has been verified experimentally in the region of 100 kv x-rays and the present data extend this region to 810 kv. The maximum recoil energy for *any particular direction of emission* can be obtained by plotting only these tracks which occur within 5° of that direction (i.e., in a 10° interval) and drawing a curve similar to that in Fig. 2. A number of these curves are shown in Figs. 3, 4, and 5 and the maximum energies for angles up

[10] Henderson, Goss, and Rose, Rev. Sci. Inst. **6**, 63 (1935).

to 52° check the theoretical values within a few percent.

### Angular distribution

If the total number of electrons emitted in each 5° angle is plotted as a function of the angle, one obtains the curve shown in Fig. 6. The smooth curve in the same figure represents the angular distribution of the recoil electrons according to the Klein-Nishina formula. Both curves are for a plane cross section along the incident beam of radiation. Although one is not strictly justified in drawing a curve through each experimental point where the statistical fluctuations may be large, such a curve has been drawn in order to show more clearly the general trend of the experimental results. In Fig. 6 the peak of the experimental curve corresponds rather closely to that of the Klein-Nishina curve, but at 15° the experimental curve rises above the theoretical curve and a little beyond 60° it crosses below again. The sharpness of the experimental peak is to be expected from the experimental set-up. Only one camera was used, and the thickness of the photographic plane was a little less than half an inch. Thus a track which appears to have an initial angle of 0° might actually have an angle of 5° measured in a direction perpendicular to the plane. The tracks with small initial angles should therefore be concentrated near 0°, while tracks with large initial angles should be displaced only slightly toward the smaller angles. The portion of the experimental curve beyond 60° contains only a small number of tracks, and due to statistical fluctuations should not be expected to conform closely with theory. The excessive number of tracks between 15° and 60° however, cannot be explained on the basis of experimental procedure, since the experimental displacement is toward smaller angles instead of larger, and statistical fluctuations should be as much below the curve as above.

In order to explain this excessive emission, there seem to be two alternatives: either the Klein-Nishina formula is not correct in this region or a considerable number of photoelectrons are present. Of these two alternatives, the latter seems the more probable.

According to quantum mechanical theory, the angle of bipartition, or the angle such that equal numbers of electrons are ejected on either side, for photoelectrons is given by the relation[11]

$$\cos\theta_b=\beta=\frac{\{h\nu(h\nu+2mc^2)\}^{\frac{1}{2}}}{h\nu+mc^2}. \quad (8)$$

If $h\nu=510$ kev (or $mc^2$), then $\theta_b=30$ percent, and if $h\nu=100$ kev, $\theta_b=57°$. Since the radiation used in the present experiment ranges from 100 to 800 kv with a maximum intensity near 500 kv, the angle of bipartition is of the right order of magnitude to produce the observed effect.

The initial angle was found by measuring the angle of a line tangent to the track at the point of

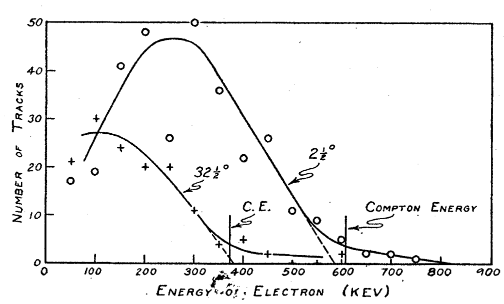


FIG. 3. Energy distribution at 2½° and 32½°.

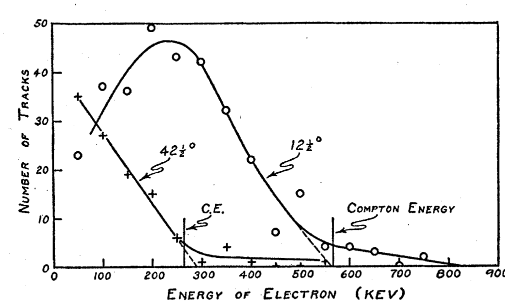


FIG. 4. Energy distribution at 12½° and 42½°.

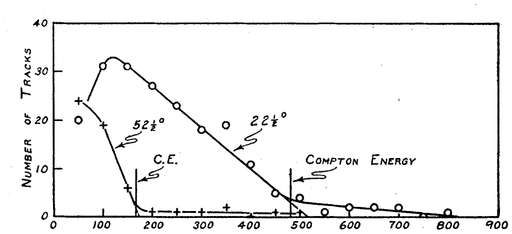


FIG. 5. Energy distribution at 22½° and 52½°.

[11] This equation was derived for small values of $\beta$ and hence is not strictly true when $\beta$ is large, although Sauter has shown that it is nearly correct for values of $\beta$ as high as 0.52.

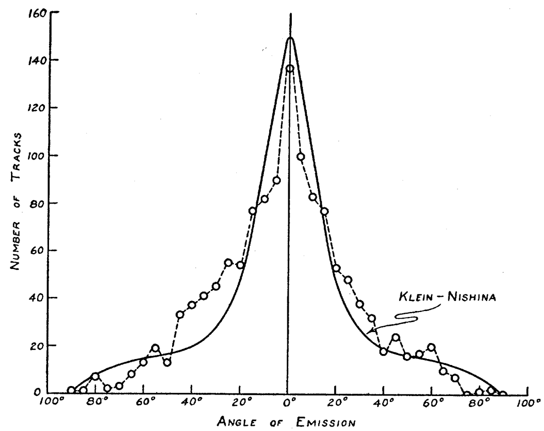


FIG. 6. Angular distribution. (With magnetic field.)

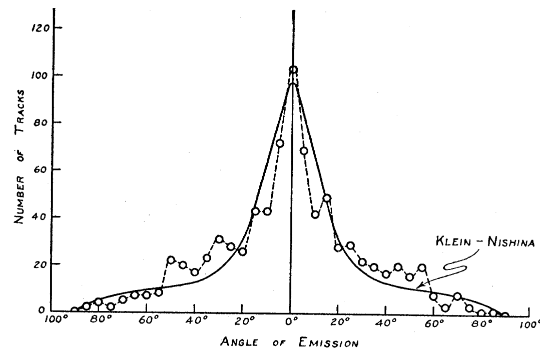


FIG. 7. Angular distribution. (Without magnetic field.)

origin. In order to determine whether the presence of a magnetic field (thus producing curved tracks) caused a systematic error in the measurement of the angles, about 400 photographs were taken without the magnetic field. The tracks were then straight instead of curved and were a little easier to measure. This set of data (821 tracks) is plotted in Fig. 7 and again compared with the Klein-Nishina curve. The experimental curve obtained without the magnetic field is similar to that obtained with the magnetic field, thus showing that no systematic error is present.

The principal difference between the experimental curves in Figs. 6 and 7 is that the shoulder of the latter curve is distinctly lower. In Fig. 6 the excessive emission begins at 15°, while in Fig. 7 it does not begin until 20°. If the excessive emission is due to photoelectrons, this effect may be explained by the experimental conditions under which the two sets of data were obtained. The first set of data was obtained by measuring both the angle and the curvature of a track, while the second set was obtained by measuring the angle only. Since a larger proportion of high energy tracks are satisfactory for measurement of curvature than are low energy tracks, and satisfactory conditions for the measurement of angle are about the same in each case, the second set of data contains a larger proportion of low-energy tracks. The angle of bipartition for low-energy photoelectrons is large, thus tending to spread out the angular distribution curve toward higher angles.

## Maximum value of $\sigma/\tau$

Photoelectrons and recoil electrons can be identified only by their energies. The kinetic energy of a photoelectron (neglecting the binding energy) is equal to the energy of the incident quantum of radiation, while the kinetic energy of a recoil electron is given by the Compton theory. Thus, if monochromatic radiation were used, the photoelectrons and the recoil electrons would be separated into two distinct groups. However, in the present experiment, the incident radiation has a continuous spectrum and such a distinct separation of the two types of emitted electrons is not possible. In this case, one can obtain only a partial separation. The maximum energy of the radiation is 810 kev and from the Compton theory one may compute the corresponding maximum energy of the recoil electrons. Any electron which has a kinetic energy greater than the maximum Compton energy for its particular angle of emission may therefore be classified as a photoelectron, and any electron which has a smaller energy may be either a photoelectron or a recoil electron.

If $T$ represents the total number of emitted electrons and $T_1$ the known number of photoelectrons, then the maximum value of $\sigma/\tau$, or the ratio of the number of recoil electrons to the number of photoelectrons, is given by the equation

$$\sigma/\tau=(T-T_1)/T_1. \qquad (9)$$

The results of the first set of data (about 2600 photographs from which were obtained 966 tracks) indicated the presence of a much larger number of photoelectrons than had been expected from theory. (Eq. (7).) Later the experiment was repeated, and the second set of data (about 1100 photographs from which were obtained 216 tracks) indicated an even lower value of $\sigma/\tau$. Both sets of data were then rechecked, and all tracks that may have been due to photoelectrons were carefully remeasured.

Of the electrons in the first set of data that apparently had energies greater than the maximum Compton energy, 12 were retained as photoelectrons because no reason could be found for classifying them otherwise, 27 seemed to be photoelectrons but were slightly doubtful, and 30 were doubtful. A similar grouping of the second data gave 8, 10, and 8.

In computing the value of $\sigma/\tau$, only the first two of these groups were used, that is, only those photoelectrons about which there can be scarcely any question. In Fig. 8 we have plotted the respective energies of these photoelectrons against their energies and show by the full line the maximum Compton recoil energy as a function of angle. All of the electrons in the first group have measured energies at least 10 percent greater than the maximum Compton energy (well above the probable error), while those in the second group have energies at least 3 percent greater. In both groups, about one-fourth of the electrons have energies more than double the corresponding Compton values.

The number of photoelectrons obtained in this manner does not necessarily give the true proportion, since more of the high energy tracks are satisfactory for measurement than low-energy tracks. In order to overcome this difficulty, in the second set of data, all of the tracks which began within the x-ray beam were counted, but were unsatisfactory for precise measurement. However, if all of these tracks were assumed to be due to recoil electrons, the value of $\sigma/\tau$ would probably be too high. Therefore, in order to obtain an experimental value which would be closer to the true value, the energies of the unmeasured tracks were estimated by inspection and divided into three groups—0–75 kev, 75–375, and 375 kev or more. Since the filtration used ($\frac{1}{2}''$ lead plus a special filter to absorb the $K$-radiation of lead) effectively cut out the radiation below 100 kev, all the electrons in the first estimated group were assumed to be of the recoil type. The second energy range contained a large number of accurately measured tracks and of these only a small proportion were due to photoelectrons, while the third energy range contained a smaller number of accurately measured tracks of which a considerably larger proportion were due to photoelectrons. The number of photoelectrons in the second and third energy ranges of the estimated tracks were then assumed to occur in the same proportion as in the corresponding energy range of the accurately measured tracks. The unmeasured tracks in the first set of data were not counted, but were assumed to occur in the same proportion as those in the second set of data. In these data, the numbers of estimated tracks in the three respective groups were 555, 429, and 50, while the total number of accurately measured tracks was 216.

The maximum value of $\sigma/\tau$ was computed from Eq. (9), where $T$ is the sum of the measured

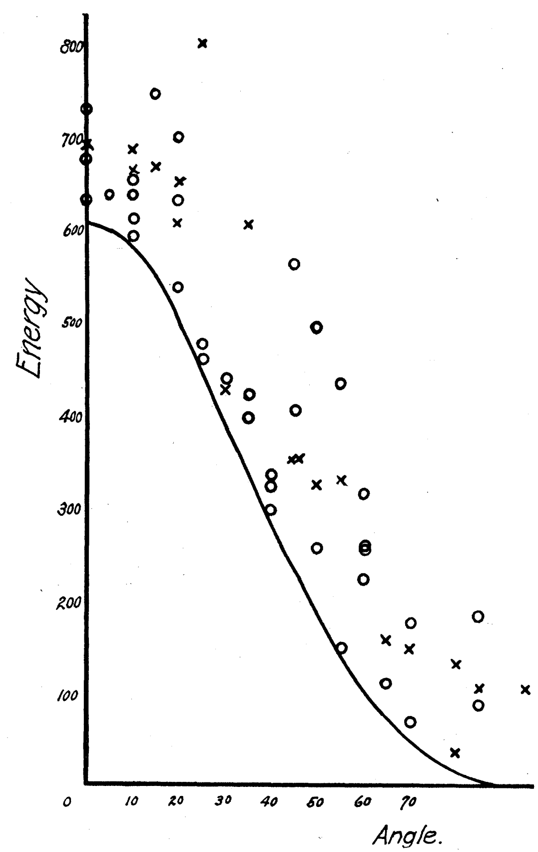


FIG. 8. Energy distribution of photoelectrons. $o$—second group. $x$—first group. Full line curve gives Compton upper energy limit for recoils.

and the estimated tracks and $T_1$ is the sum of the photoelectrons actually counted and those estimated from the tracks which were unsatisfactory for exact measurement. If only those photoelectrons in Group I are considered, the maximum value of $\sigma/\tau$ is 113, while if both groups are considered, the value is 40. Under the assumption that the estimated tracks contain no photoelectrons (all recoil electrons) the corresponding values of $\sigma/\tau$ are 342 and 117. Thus the maximum experimental value of $\sigma/\tau$ for a plane cross section is of the order of 100, which is to be compared with the theoretical value of 4875.

A number of errors are inherent in the experimental procedure, but all of these tend to make the value of $\sigma/\tau$ even smaller. These errors are due to two major causes: the thickness of the photographic plane and the difference between the angular distribution of the photoelectrons and recoil electrons. Since the thickness of the photographic plane is nearly half an inch, the measured tracks do not lie strictly in a plane but rather in a shallow box. The measured angles therefore tend to be a little smaller than the actual angles, and hence the maximum Compton energy for any given electron tends to be too large, thus reducing the number of photoelectrons and making $\sigma/\tau$ too large. Also, the energy which one measures is that due to the component of velocity in the plane of the photograph, and this energy is slightly smaller than the actual energy, which again reduces the number of photoelectrons and makes $\sigma/\tau$ too large.

The difference in angular distribution may be represented by the angle of bipartition. This angle for recoil electrons is always 0°, while that for photoelectrons is given by the relation $\cos\theta_b=\beta$. For 510 kv, $\theta_b$ is approximately 30°. In order to visualize the angular distributions more clearly, consider the diagram in Fig. 9. For purpose of discussion, let the recoil electrons be ejected only within the solid cone $R$ and let the photoelectrons be ejected only between the two larger cones $P$. If now a planar shell cuts through these figures along their common axis, the shell will cut out a volume $r_1$ from the solid cone and a volume $p_1$ between the two other cones. Let $r$ be the total volume of $R$ and $p$ the total volume of $P$. Then from geometrical considerations,

$$r_1/r>p_1/p;\quad \therefore\quad r_1/p_1>r/p.$$

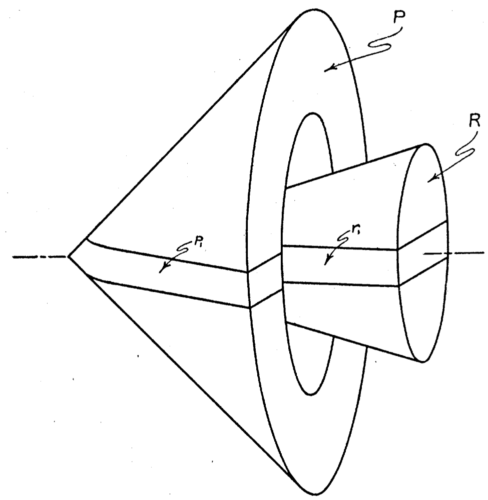


FIG. 9. Illustration of angular distribution. Photoelectron cones, $P$, and recoil cones, $R$.

Therefore the value of $\sigma/\tau$ (proportional to $r_1/p_1$) measured in the planar shell is greater than $\sigma/\tau$ measured in the total volume. As the thickness of the shell increases, $\sigma/\tau_s$ approaches $\sigma/\tau_v$. In the present case, the actual thickness of the shell is nearly half an inch, and hence the measured value of $\sigma/\tau$ is greater than that in the total volume and less than that in a thin shell. Since the theoretical value of 4875 is for the total volume while the experimental value of 100 is for a thick shell, the experimental value must be made smaller in order to be comparable with theory.

The major experimental error in addition to that in measuring the tracks was the variation of the magnetic field. The current through the Helmholtz coils was ordinarily about 25 to 26 amp. and the variation from the mean during a fifteen-second reading was usually less than 0.5 amp., but occasionally the current would drop as much as 1 or 2 amp. However, in a total of more than 300 readings, only 5 times was the current observed to drop as much as 10 percent, and the lowest of these was 15.1 percent (from 25.9 to 22.0 amp.). This would cause an error of about 22 percent in the measurement of the energy. However, even if the measurement of the energy were continuously 25 percent too high, more than half of the photoelectrons would still remain.

Further evidence for a low value of $\sigma/\tau$ is obtained from measurements of the energy alone

(Fig. 2) and measurements of the angle alone (Figs. 6 and 7). In Fig. 2, the slope of the curve beyond 650 kv indicates the presence of a considerable number of photoelectrons at lower energies, while in Figs. 6 and 7 the excessive emission between 10° and 60° also indicates the presence of a considerable number of photoelectrons. These three curves are entirely independent of one another and are consistent in indicating the same result.

### Angular distribution of the photoelectrons

Because of the limited number of photoelectron tracks, one cannot make a precise determination of the angle of bipartition for any given energy. However, one can obtain a good approximation by grouping the tracks into larger intervals of both angle and energy and then comparing the experimental angle of bipartition thus obtained with the theoretical angle corresponding to the mean value of the energy. Such a determination, even though approximate, will extend the region in which these data are known from about 88 kev[4] to 650 kev.

In the present data (see Fig. 10) the photoelectrons were divided into two energy ranges—200 to 500 kev with a mean value of 364 kev (22 tracks), and 500 to 800 kev with a mean value of 650 kev (23 tracks). Photoelectrons with energies below 200 kev were not considered because they were near to the region which had previously been investigated. The interval of angle was taken as 10°, or the sum of two adjacent 5° intervals plotted at the mean. The experimental points plotted in Fig. 10 thus represent at each angle the total number of tracks in the given energy range which have an initial angle in a 10° interval. Since these experimental data are for a planar shell rather than a spherical volume, the relative total number of photoelectrons emitted at each angle may be obtained by multiplying the experimental value at a given angle $\theta$ by the sine of that angle. The relative curves for a spherical volume are shown in the same figure. Since both curves are nearly symmetrical, the angle of bipartition is nearly equal to the angle of the maximum.

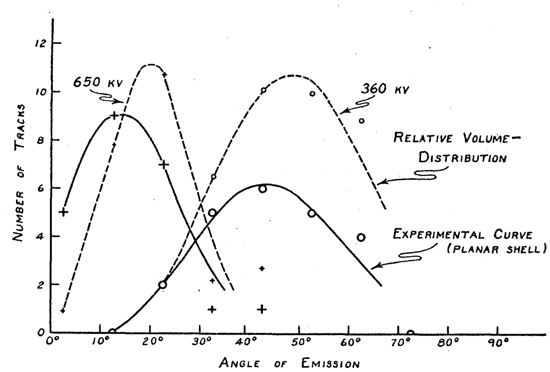


FIG. 10. Angular distribution of photoelectrons.

According to quantum mechanical theory, the angle of bipartition $\theta_b$ for photoelectrons is given to the first order approximation by the expression

$$\cos\,\theta_b=\beta. \tag{10}$$

Sauter[7] has shown that although this expression was obtained by neglecting higher powers of $\beta$, it differs very little from a more exact calculation of $\cos\,\theta_b$ for values of $\beta$ as high as 0.52 (88 kev). By means of a nonrelativistic calculation, the following expression was obtained:[7]

$$\cos\,\theta_b=\frac{v/c}{1+h\nu/2mc^2}, \tag{11}$$

where the velocity $v$ of the photoelectrons is to be computed nonrelativistically. The latter equation has been verified experimentally up to 88 kev,[4] but it cannot be used in the present case where the relativity correction is large.

For 360 kev and 650 kev radiation, the angles of bipartition calculated from Eq. (10) are, respectively, 36° and 26°. These are to be compared with the experimental values of 48° and 20°. Because of the method of separating the photoelectrons, the experimental angle of 48° is probably too high. Only those photoelectrons which have energies greater than the Compton energy can be counted, and with low energy electrons, this method favors large angles. For example, the data include four photoelectrons with an energy of about 250 kev; one of these has an initial angle of 50° and the other three an angle of 60°. There may be other photoelectrons with the same energy at smaller angles, but none can be counted below 45° because the Compton energy at this angle is 236 kev. Thus the apparent angle of bipartition may be shifted toward large angles. On the other hand, the experimental angle of 20° may be too low, because in this case (high energy electrons) nearly all of the electrons are counted, but the thickness of the photographic plane favors small angles.

If one considers the experimental difficulties, the values of 48° and 20° for 360 kev and 650 kev radiation are in reasonably good agreement with theory.

### The Klein-Nishina formula

The present experimental data offer no conclusive evidence in regard to the exactness of the Klein-Nishina formula. Read and Lauritsen found that for a range of 50 to 20 x-units (the wave-length of 500 kev radiation is 24.7 x-units) the *total* absorption coefficients of carbon and aluminum are within 1 percent of the Klein-Nishina value, and they state that their maximum likely error is 3 percent. Since a measure of the total absorption is a measure of the total number of photons removed from the incident beam, it is also a measure of the total number of electrons taking part in the interaction (if one assumes no coherent scattering). Thus, if only recoil electrons are emitted and the Klein-Nishina formula is correct, a measure of the total absorption should check the Klein-Nishina formula exactly. However, if the error in this experiment were as much as 3 percent, three photoelectrons in a total of 100 could be present and still not be detected. This corresponds to a $\sigma/\tau$ value of about 30. Consequently an experiment of this type cannot distinguish between the theoretical value of $\sigma/\tau$ of about 5000 which one gets from Eq. (9), and a value as low as 30. According to the present results, the value of $\sigma/\tau$ for 500 kev x-rays in nitrogen is of the order of 100 and is probably less than this; however, just how much less cannot be determined with the present data. If $\sigma/\tau$ is less than 30, the Klein-Nishina value may be too high.

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# Initial Recombination of Ions

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The probability that a pair of ions of given initial separation will recombine with each other is computed from the laws of Brownian motion, which is the proper procedure whenever the Langevin factor equals unity, as in gases at high pressures. In the absence of forces other than the Coulomb attraction, the probability of escape equals the reciprocal of the Boltzmann factor. This result includes the correlation between temperature and pressure coefficients of the ionization by light particles previously predicted by Compton, Bennett and Stearns, if one allows their basic hypothesis about the laws which govern the initial separation of the ions. The effect of an electric field is to increase the fraction of escaping ions by a factor which in the incipient stage of the effect is proportional to the field intensity and independent of the initial distance, although it depends on the orientation of an ion pair. The predicted increase of the ionization current is a little more than one percent for every 100 volts/cm, which accounts for the observed effects of fields exceeding 1500 volts/cm. A reasonable amount of columnar recombination would help to explain the proportionately greater effects of weak fields. The inferred initial separations of the ions are apparently compatible with present knowledge of electron scattering and attachment.

It stands to reason that an electron, ejected by a photon or by collision with a charged particle, if not removed too far before it becomes attached to a molecule or otherwise slowed down to thermal velocities, may recombine with its parent ion. The possibility of such *initial recombination* was pointed out long ago by Rutherford.[1] In recent years particular attention has been paid to the possible effect of this process upon the ionization of air by particles of low ionizing power (cosmic rays, $\beta$-($\gamma$-) rays).

[1] E. Rutherford, *Radioactivity* (1904), p. 33.

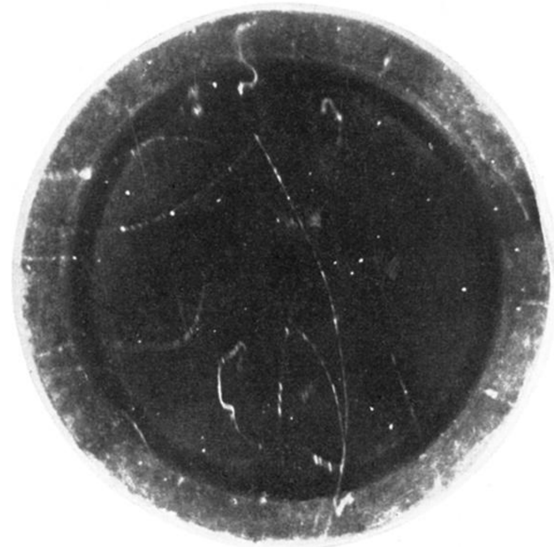

FIG. 1. Photoelectron track 673 kev showing change in curvature due to close collision.