## The Velocity of Sound in Liquid Helium

J. C. FINDLAY, A. PITT, H. GRAYSON SMITH AND J. O. WILHELM McLennan Laboratory, University of Toronto, Ontario, Canada (Received July 8, 1938)

The velocity of sound in liquid helium under its vapor pressure has been determined by an ultrasonic method over a temperature range from 4.22°K to 1.76°K. From the measurements obtained, the compressibility of the liquid helium has been computed. At temperatures not too close to the  $\lambda$ -point, the values are in fair agreement with those estimated from thermodynamical considerations, but the expected discontinuity at the  $\lambda$ -transition has not been observed.

## INTRODUCTION

A S a part of a general program of obtaining as much information as possible about the physical properties of liquid helium, the velocity of sound has been measured in liquid He I and He II. This provides an experimental determination of the compressibilities of these liquids under their vapor pressures, and of the possible change in compressibility at the  $\lambda$ -transition.

According to the relations developed by Ehrenfest<sup>1</sup> for a so-called phase change of the second order, there should be a discontinuous change in compressibility at the  $\lambda$ -point, and therefore a change in the velocity of sound. Von Laue<sup>2</sup> has raised objections to Ehrenfest's treatment of this type of transition, but Rutgers and Wouthuysen<sup>3</sup> have shown that in a transition which extends over a small but finite range of temperature without a discontinuous absorption of latent heat, Ehrenfest's relations should still be valid. From these relations, the slope of the  $\lambda$ -line on a pressure temperature diagram should be given by

$$(dp/dT)_{\lambda} = -\Delta\alpha/\Delta K_T, \tag{1}$$

where  $\Delta \alpha$  is the change in the coefficient of expansion, and  $\Delta K_T$  the change in the isothermal compressibility, in going from He II to He I. However, the measurements to be described in the present paper show that there is very little change, if any, in the velocity of sound at the  $\lambda$ -point.

### EXPERIMENTAL METHOD

The method of measuring the velocity of sound in liquids by means of ultrasonic waves has been extended to condensed gases by Pitt and Jackson,<sup>4</sup> who obtained values in liquid oxygen and liquid hydrogen, but were unable to obtain detectable standing waves in liquid helium, with its high compressibility and small density. After the method had been modified so that it was sufficiently sensitive to give measurements in gases, it was found satisfactory for the liquid helium.

The apparatus, shown in Fig. 1, consisted of a quartz crystal B, sputtered on both sides with



FIG. 1. Arrangement of ultrasonic crystal and reflector.

<sup>&</sup>lt;sup>1</sup> P. Ehrenfest, Proc. Kon. Akad. van Weten. Amsterdam **36**, 153 (1933). (Leiden Comm. Suppl. 75b.)

<sup>&</sup>lt;sup>2</sup> M. von Laue, Physik. Zeits. 35, 945 (1934).

<sup>&</sup>lt;sup>8</sup> A. J. Rutgers and S. A. Wouthuysen, Physica 4, 515 (1937).

<sup>&</sup>lt;sup>4</sup> A. Pitt and W. J. Jackson, Can. J. Research **12**, 686 (1935).



FIG. 2. Crystal-controlled ultrasonic generator circuit.

aluminum, which was held by means of a spring  $S_2$  between a brass ring  $E_1$  and a disk  $E_2$ . With this arrangement, the crystal was clamped at its edges. The crystal, in a fiber shell F and a metal sheath, was supported by a German silver tube G, which served as the grounded lead from  $E_1$ , and also as a shield for the lead T from  $E_2$ . A reflector R was mounted at one end of a thin German silver tube, the other end of which was held by means of the spring  $S_1$  against a ball and socket joint at the end of a micrometer screw A.

The crystal which generated the ultrasonic waves was driven at a very constant frequency by means of a generator whose frequency was controlled by a second crystal. In Fig. 2, A represents the controlling crystal, whose signal is amplified and imposed on B, the crystal in the liquid. The amplitude of vibration of B is indicated by the galvanometer G, connected in

TABLE I. Preliminary results which show the reliability of the apparatus and method.

SUBSTANCE	FRE-	Tempera-	VELOC-	Compara-
	QUENCY	ture	ITY	tive
	KC/SEC.	°C	M/SEC.	Value
Ether	669 1338	22.3 21.4	984 1010	
Liquid Oxygen	669 1338	$   \begin{array}{r}     13.0 \\     -182.9   \end{array} $	1045 908	1024 <sup>1</sup> 912 <sup>2</sup> 903 <sup>3</sup>
Helium Gas	1338	-182.9	540	5594
	1338	-268.9	104	102 <sup>5</sup>

<sup>1</sup> At 15°C (I. C. T.). Independent measurements in ether are not consistent because of the variability of its composition.
<sup>2</sup> Pitt and Jackson, reference 4 in text.
<sup>3</sup> Debye-Sears method; R. Bär, Nature 135, 153 (1935).
<sup>4</sup> W. H. Keesom and A. van Itterbeek, Kon. Akad. van Weten. Amsterdam 33, 440 (1930). (Leiden Comm. 209a.)
<sup>6</sup> W. H. Keesom and A. van Itterbeek, Kon. Akad. van Weten. Amsterdam 34, 204 (1931). (Leiden Comm. 213b.)

the plate circuit of a 57 detector tube. As the reflector was moved through intervals of onehalf wave-length, sharp maxima were observed in the galvanometer deflection, which were due to the setting up of standing waves in the liquid column. A much more vigorous response was obtained by driving the sonic crystal at a frequency close to its natural resonance.

The micrometer screw was calibrated by means of a highly accurate comparator. The driving frequency, the second harmonic of the frequency of crystal A (Fig. 2), was determined by the method of obtaining audio-beats with the carrier waves of various broadcasting stations. The fundamental was determined to be 669 kc, the harmonic 1337.9 kc. Preparatory work was done in ether, benzene, liquid oxygen, and helium gas. A few results of these preliminary tests are shown in Table I, together with values obtained by other experimenters for comparison.

## MEASUREMENTS IN LIQUID HELIUM

When determining the wave-length in liquid helium at various temperatures, readings of the micrometer screw were taken at every fifth maximum of the galvanometer deflection over a range of 25 to 100 half wave-lengths. The wavelength for the frequency used—1338 kc—was of the order of 0.16 mm. The velocities calculated from these measurements are shown plotted against temperature in Fig. 3, and values derived from the original smoothed graph are given in Table II.



FIG. 3. Velocity of sound (meters per sec.) in liquid helium.

As a check on the measurements at different temperatures, the reflector was set at a fixed distance from the crystal while the temperature was altered. The galvanometer fluctuations then gave the change in the number of half-waves between the reflector and the crystal. From this the change in velocity was calculated and found to agree very well with the values obtained by moving the reflector at constant temperature. As the  $\lambda$ -point was crossed, the galvanometer movement was erratic, and attempts to determine by this method whether there was a change in velocity at the  $\lambda$ -point were unsuccessful.

The temperature was controlled by the vapor pressure of the helium, which was maintained constant within 1 mm of mercury. Fluctuations of temperature would not cause errors in the measurement of wave-length greater than 0.03 percent in He I or 0.2 percent in He II. If allowance is made for the possible error in measuring the displacement of the reflector, the results

TABLE II. The velocity of sound and the compressibility of He I and He II at various temperatures.

Temperature °K	Velocity M/sec.	Compressibility c.g.s. units
He I		
4.22	179.8	$2.474 \times 10^{-8}$
4.0	189.2	2,170
3.6	206.5	1.722
2.5	223.3	1.382
2.20	221.2	1.402
HeII		-
2.18	221.7	1.392
2.0	225.3	1.355
1 76	231.4	1.285



FIG. 4. Adiabatic compressibility of liquid helium, deduced from velocity measurements.

should be correct within 0.2 percent in He I and 0.4 percent in He II. The graph shows that practically all the results are within this limit of error.

## Compressibility of Liquid Helium

The measured values of the velocity are about 12 percent lower than the values predicted from the formula

$$W = 1/(K\rho)^{\frac{1}{2}}$$
.

However, the adiabatic compressibility at low pressures can only be estimated roughly from the published data on pressure, density, and specific heat. There seems to be no reason to suppose that, in He I at least, the usual formula for the velocity of sound does not hold. Values for the adiabatic compressibility have therefore been deduced from the measured velocities. These are shown plotted against temperature in Fig. 4, and values taken from the graph are given in Table II.

The estimated value for the change in expansion coefficient at the  $\lambda$ -point,  $\Delta \alpha$ , is 0.0499, while the value for  $(dp/dT)_{\lambda}$  is  $-82 \times 10^6$  c.g.s. units per degree.<sup>5</sup> Hence from relation (1), the change  $\Delta K_T$  in the isothermal compressibility should be  $-6.2 \times 10^{-10}$  c.g.s. unit, which represents a decrease of 5 percent in the estimated values. Since the calculated value of  $\gamma$  is about

<sup>&</sup>lt;sup>5</sup> Values derived from experimental results of W. H. Keesom and Miss A. P. Keesom, Proc. Kon. Akad. van Weten. Amsterdam **36**, 482 and 612 (1933), (Leiden Comm. 224*d*, *e*); Physica **1**, 128 (1933), (Leiden Comm. Suppl. 76*b*.)

1.005 for both He I and He II near the  $\lambda$ -point, the relative change in the adiabatic compressibility should be very nearly the same as that in the isothermal value. This should cause an increase in the velocity of sound of about 2.5 percent. However it will be seen from Fig. 3 that the change at the  $\lambda$ -point, if any, is not more than 0.5 percent. It will be seen, further, that the measured velocity is a maximum in He I at 2.5°K. Such a maximum is not predicted by calculations based on the published data for liquid helium, and there may be some significance in the fact that the increase from the value in He II at the  $\lambda$ -point to the maximum in He I is approximately equal to the increase predicted by Ehrenfest's relations.

#### WAVE FORM

As the reflector was moved, there appeared in addition to the main maxima of the galvanometer

deflection, smaller secondary peaks. These peaks maintained the same positions relative to the primary maxima as the reflector was moved, and were the same for all liquids used, with the exception of liquid He II. Hence they could not be attributed to different velocity or frequency components. It is thought that they are due to some discontinuity in the activity of the crystal. In liquid helium, there was a distinct change in the form of these secondary peaks on passing from He I to He II. However, their form remained constant throughout the motion of the reflector in both liquids, and hence the measurements of the wave-length should not be affected. This change in wave form may be related in some way to the peculiar physical properties of He II.

In conclusion, the authors wish to thank Dr. E. F. Burton for his kindly interest and guidance in this research.

#### **OCTOBER 1, 1938**

#### PHYSICAL REVIEW

VOLUME 54

# The Inelastic Scattering of Slow Electrons From a Silver Single Crystal

JOHN C. TURNBULL\* AND H. E. FARNSWORTH Brown University, Providence, Rhode Island (Received July 26, 1938)

The energy distribution of electrons inelastically scattered from a (111) face of a silver single crystal has been studied by the method of magnetic deflection. At about 45° incidence primary electrons are regularly reflected into the analyzer, and diffraction beams are observed at primary energies of 7.7, 23.2, and 83.2 ev. The structure in the energy distribution shows two discrete loss peaks at 3.9 and 7.3 ev, respectively, in general agreement with that found by Rudberg for polycrystalline silver. However, the relative intensities of the two discrete loss peaks depend on both the primary voltage and the target angle in the neighborhood of the diffraction beams, while Rudberg has found that for polycrystalline targets the peaks are inde-

#### INTRODUCTION

 $\mathbf{P}^{\mathrm{REVIOUS}}$  measurements<sup>1</sup> of the energy distribution of electrons scattered from an

pendent of these variables. The energy distribution curves are also distorted by an extra inelastic scattering which accompanies the elastic scattering of the diffraction beams, and which extends down to an energy loss of 10 to 15 ev. Thus, maxima are observed in the curves giving the amount of inelastic scattering as a function of the primary energy for the constant values of energy loss. The maxima occur at secondary energies equal to the critical voltages of the diffraction beams. This indicates the existence of a double process consisting of inelastic scattering of the incident electrons followed by diffraction of the scattered electrons by the crystal lattice.

outgassed metal target in high vacuum have shown that certain discrete energy loss peaks are superposed on the general background of inelastic scattering. For polycrystalline silver there are two such peaks at 3.9 and 7.8 ev

<sup>\*</sup> Part of a dissertation presented for the degree of Doctor of Philosophy at Brown University.

<sup>&</sup>lt;sup>1</sup> Rudberg, Phys. Rev. **50**, 138 (1936); **45**, 764 (1934); Proc. Roy. Soc. **A127**, 111 (1930); K. Svenska Vet. Akad.

Handl. 7, 1 (1929). Haworth, Phys. Rev. 48, 88 (1935); 37, 93 (1931); 42, 906 (1932).