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Čerenkov Radiation

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Electrons of two million volts energy from an electrostatic generator were used to investigate the properties of the asymmetric radiation discovered by Čerenkov. This radiation is produced when electrons traverse a material medium with a velocity greater than the velocity of light in that medium. It was found for several solids and liquids that the direction of the emission of the radiation is accurately expressed by the relation, $\cos\theta = 1/\beta n$, and that the intensity maximum is quite sharp. The nature of the radiation from all solids and liquids investigated was found to be continuous and identical in appearance. The radiation apparently extends with increasing intensity from the infra-red to the ultraviolet absorption limit of the medium in which it is produced. Rough quantitative measurements of its intensity indicate that one 1.9 million volt electron in being brought to rest in water produces 40 quanta in a wave-length range 4000A to 6700A. These results are in good agreement with the classical explanation of the phenomenon given by Frank and Tamm.

INTRODUCTION

ČERENKOV[1] in 1934 reported the existence of a visible radiation from pure liquids and solids when electrons traverse the medium with a velocity greater than that of light in the medium. The radiation was observed to be unusual in several ways, and its characteristics were such as to indicate that the physical processes responsible for its production were not any of the usual ones associated with atomic or molecular changes. It seems to result rather from a cooperative phenomena between the incoming electrons and an assemblage of atoms. In his original investigation Čerenkov showed that the radiation was not fluorescent, and that it was polarized with the electric vector parallel to the direction of the electron beam. Later work by Čerenkov[2] showed that the radiation had the unusual property of being emitted asymmetrically and that its intensity depended only on the refractive index of the medium.

Frank and Tamm[3] have given a theoretical explanation of this phenomenon which is entirely classical, but which is in agreement with the qualitative observations of Čerenkov. A striking feature of this theory is that it describes a new process for the production of radiation.

In his experiments Čerenkov used both β-rays and Compton electrons produced by gamma-rays from radioactive substances. These are, of course, not "monochromatic" sources of electrons, and furthermore they produce only a very feeble radiation which made it difficult to obtain results of a quantitative nature. In the investigation reported here the high speed electrons were obtained from an electrostatic generator and accelerating tube. This equipment produces a well-collimated homogeneous beam of electrons of about 10 microamperes at potentials up to 2.0 million volts. With this, fairly strong sources of radiation could be obtained which allowed a considerable extension of the work of Čerenkov.

THEORY

In the theoretical investigation of this phenomenon Frank and Tamm treated the radiation as due to an electron passing through a medium of refractive index n with a variable velocity v. Their work consisted in a purely classical derivation of the relation between the current density of the moving electron and the electrical and magnetic field strengths of the emitted radiation. The solution of the resulting differential equation, for $\beta n < 1$, results in a damped radiation whose intensity decreases exponentially with distance, so that the radiation is not observable. The situation is similar to the penetration of light into the second medium in the case of total reflection. The solution for $\beta n > 1$ gives spherical waves which are continually being emitted by the moving electrons and constructive interference (see Fig. 1) between these rays gives rise

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[1] P. A. Čerenkov, C. R. Acad. Sci. USSR **8**, 451 (1934).
[2] P. A. Čerenkov, C. R. Acad. Sci. USSR **14**, No. 3 (1937); Phys. Rev. **52**, 378 (1937).
[3] I. Frank and Ig. Tamm, C. R. Acad. Sci. USSR **14**, No. 3 (1937).

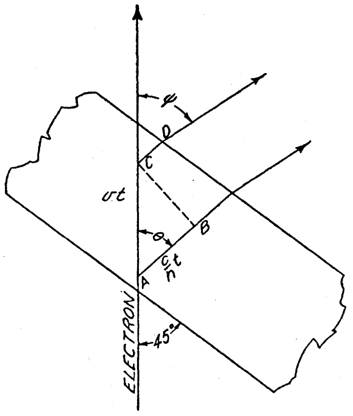

FIG. 1. Diagram showing interference between rays produced by a moving electron in a medium which leads to the relation, cos $\theta = 1/\beta n$.

to an observable radiation.

Equation (1) was obtained by these authors and expresses

$$W = \frac{e^2 l}{c^2} \int_{(\beta n > 1)} \omega d\omega \left(1 - \frac{1}{\beta^2 n^2}\right) \qquad (1)$$

the total energy W radiated by the electron as a function of the length of path l, the velocity ratio β, the frequency of the emitted light ω, and the refractive index n.

It should be noted that this result is independent of all physical properties of the medium with the exception of the refractive index. The theory also shows that the radiation should be continuous, that it should extend from the infrared to the ultraviolet absorption limits of the medium in which it is produced, and that its intensity should vary inversely with the cube of the wave-length.

It is to be understood that the electron in its passage through the medium gradually loses nearly all its energy through ionization and excitation processes, and that the resulting acceleration is responsible for the Čerenkov radiation. The process may be compared to the classical explanation of the continuous x-ray spectrum. It differs in that the acceleration which the electron suffers is in comparison much smaller, and the energy lost by the electron which does not appear as radiation is in comparison much larger. A calculation of the total energy lost by a fast electron ($\beta \sim 1$) through radiation as given by (1) was shown by Frank and Tamm to be only several kilovolts, a negligible amount compared to losses by other causes.

The theory also implies that the radiation emitted by the electron along its path is coherent and its asymmetrical character is due to interference of the light produced at points along the electron's path. This conclusion leads directly to Eq. (2)

$$\cos \theta = 1/\beta n, \qquad (2)$$

where θ is the direction of emission of the asymmetrical radiation measured with reference to the direction of the electron beam, and n and β have their usual significance. The derivation of this relation may be seen by referring to Fig. 1. For any two light rays, AB and CD, to be in phase after leaving the medium, the light ray produced first must travel a distance $AB = ct/n$ while the electron travels the distance $AC = vt$. Equating these in terms of the angle of emission θ yields $vt \cos \theta = ct/n$ which reduces immediately to Eq. (2). This relation holds for all the rays produced in the medium no matter what their separation may be. The verification of this relation, which follows, is then essentially a proof of the fact that the radiation produced by an electron is coherent.

EXPERIMENTAL

In general the aim of this investigation was to obtain additional experimental information about this radiation, and to compare the results with

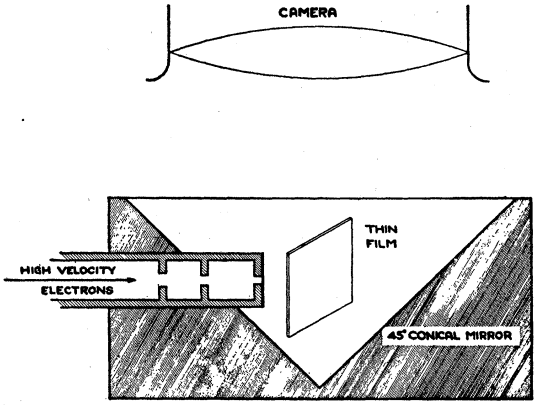

FIG. 2. Diagram of apparatus to show angular distribution of Čerenkov radiation.

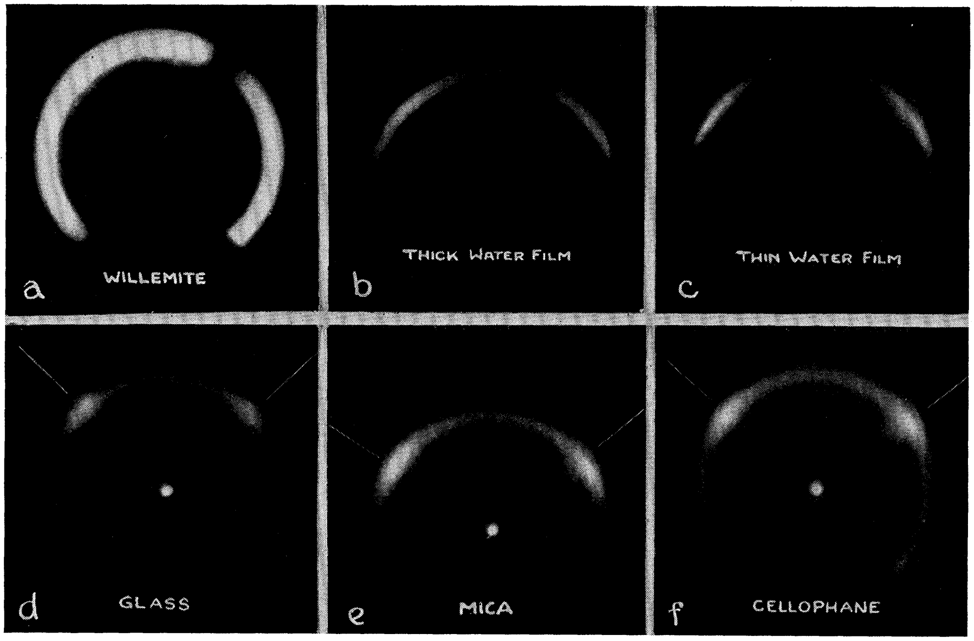

FIG. 3. Photographs of the radiation after reflection from a conical mirror showing the angular distribution of intensity. (*a*) powdered willemite; (*b*) 0.1 cm film of water; (*c*) 0.03 cm film of water; (*d*) 0.006 cm glass; (*e*) 0.002 cm mica; (*f*) 0.002 cm Cellophane. Lines indicate angles calculated from Eq. (2).

the theory of Frank and Tamm. The work was therefore divided into three parts whose purposes were the following: (1) to investigate quantitatively the direction of emission of the radiation from several solids and liquids and to compare these results with the values obtained from Eq. (2); (2) to determine the spectroscopic character of the radiation; (3) to determine in absolute units the intensity of the radiation and compare these results with the theoretical predictions of Eq. (1).

I. Direction of emission of the radiation

The apparatus used to investigate the direction of emission and asymmetrical character of the radiation is given in Fig. 2. Thin films of the substances under investigation were placed in the center of the conical mirror in the path of the electron beam which had been collimated by three 0.5 mm circular aperatures and the radiation produced in the film was then reflected by the conical mirror and photographed with a conventional camera. This optical system is such that if radiation is emitted in all directions from the film a circular ring is obtained on the photographic plate. The system was tested for optical uniformity by photographing the radiation induced in a fluorescent substance, which should show a uniform angular distribution of intensity. For this purpose a small rectangular fluorescent screen of willemite was placed at the center of the conical mirror and bombarded with electrons. Fig. 3a resulted and shows a uniform intensity distribution, as expected, except for the gaps in the ring which are due to the shadows cast by the film holder and electron beam windows.

With this arrangement the asymmetrical character of the Čerenkov radiation from several solids and liquids was investigated. The liquid films were produced by flowing the liquid over a copper plate in which was drilled a 2 mm hole. Surface tension maintained a fairly uniform film over the hole whose thickness was altered by varying the thickness of the copper plate. Figs. 3b and 3c show the directions of emission of the Čerenkov radiation from films of water 0.1 cm and 0.03 cm thick, which were set normal to the electron beam. Both show a decided tendency towards asymmetry, and it is to be noted that the sharpness of the intensity maxima increases

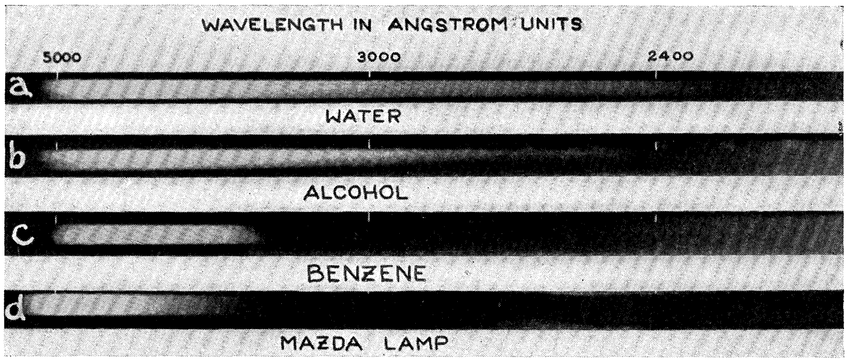

FIG. 4. Spectra of Čerenkov radiation produced in liquids by high velocity electrons. 1 mm slit, exposures 10 minutes.

with decreasing film thickness. Thus one is led to believe that the breadth of the intensity maxima is due to a considerable extent to scattering of the electron beam in the film. No definite conclusion as to the limiting sharpness of the intensity maxima is possible, as a reduction in the thickness of the film always resulted in an increased sharpness, to the point where the films became too thin to give sufficient intensity. From this it is concluded, however, that the intensity maxima from a perfectly collimated beam of electrons probably would result in very sharp intensity maxima.

Quantitative measurements of the angle of emission of the radiation were made only with solids because the liquid films used did not have optical surfaces sufficiently good to give reliable results. Sheets of glass, mica, and Cellophane were investigated. For these substances it was found necessary to set the sheet at a large angle to the electron beam so that the Čerenkov radiation produced inside the sheet would not be totally reflected at the surface.

Figures 3d, 3e, and 3f were obtained by setting the sheet of the material under investigation at an angle of 45° to the incident electron beam (Fig. 1) and, with a current of about 1 microampere, exposing the plate for 10 seconds. The sheet was then rotated to an angle of 45° on the other side of the electron beam and the exposure repeated on the same plate. The lines on each figure mark the position of the calculated maxima as given by Eq. (2), after correction was made for refraction at the surface of the sheet. The relationship used to obtain the theoretical value of the angle of emission outside the medium for sheets set at 45° to the electron beam was $\psi=45°+\sin^{-1} n\cdot\sin(\theta-45°)$ where ψ is the angle of emission outside the medium and θ is the angle of emission inside the medium as given by the relation $\cos\theta=1/\beta n$. The comparison between the observed and calculated angles may be found in Table I. The agreement is quite good and what discrepancy there is can be explained by a slight unavoidable bending of the thin sheets used.

II. Spectroscopic character of the radiation

The Čerenkov radiation was bluish white in color and was identical in appearance for all the solids and liquids investigated. With thick samples of liquid the intensity was sufficient to obtain spectrograms with moderate dispersion. The source of the radiation was a very thin walled quartz bulb, 0.5 cm in diameter, containing the liquid, which was supported in the electron beam. The spectra of this radiation from several liquids were photographed with a quartz spectrograph having a dispersion of 30A/mm at 3500A. A rather wide slit was used as it was found desirable to reduce the exposure

TABLE I. *A comparison of the observed and computed angles of emission of the Čerenkov radiation from sheets of various substances set at 45° to the electron beam.*

MEDIUM	THICKNESS	n	OBS. ANGLE	COMPUTED ANGLE
Mica	0.002 cm	1.59	53° 30′	52° 10′
Glass	0.006 cm	1.47	45° 15′	46° 30′
Cellophane	0.002 cm	1.54	50° 0′	49° 22′

time to a few minutes. Figs. 4a, 4b, and 4c are reproductions of the spectra of the radiation obtained from water, alcohol and benzene. Fig. 4d is the spectrum of a tungsten lamp taken with the same instrument and slit for the purpose of comparison. In every case the spectrum was found to be continuous and to extend from the long wave limit of sensitivity of the plate approximately to the ultraviolet absorption limit of the medium in which it was produced. It is to be noticed also that the intensity of the Čerenkov radiation at short wave-lengths is relatively stronger than the radiation from the tungsten lamp. The radiation was also inspected visually with a glass spectroscope (dispersion about 45A/mm). Even with a very narrow slit, no indication of structure was observed. This result seems to confirm that the radiation is continuous as stated in the theory of Frank and Tamm. It seems also likely that the radiation extends throughout the region where the condition $\beta n > 1$ holds.

III. Intensity of the radiation

As an additional test of the validity of Eq. (2) a determination of the absolute intensity of the radiation was made. The method consisted essentially in focusing a part of the radiation from a 0.5 cm bulb of water, produced by 1.9 Mev electrons upon a photoelectric cell (RCA 868) whose absolute sensitivity curve in microamperes per watt was provided by the manufacturers. In order to convert the photoelectric current obtained into watts emitted by the source per microampere of beam-current several assumptions of an approximate nature were necessary. The most serious was one concerning the angular distribution of the radiation emitted by the bulb. Reasonable assumptions, however, led to the conclusions that 9×10^{-4} watt of radiation between 4000A and 6700A resulted from each microampere of 1.9 Mev electrons. Expressed in another form each electron of this energy produced 40 photons between 4000 and 6700A in being stopped by the liquid. The value of W obtained from Eq. (2) upon introducing appropriate values for the limits of ω, and the range of electrons in water, is 22×10^{-4} watt per microampere which is twice the experimental value. This result indicates an agreement as far as order of magnitude is concerned and the discrepancy is perhaps not significant in view of the uncertainties in the assumptions that were necessary.

While investigating the Čerenkov radiation it was suggested that there might be some connection between this radiation and one investigated by Cohn[4] and others.[5] Cohn observed under certain conditions a visible radiation emanating from the anodes of x-ray tubes. This radiation was reported as bluish-white, evidently similar in color to the Čerenkov radiation, but differing from it in that it was unpolarized. To establish the fact that the Čerenkov radiation and that observed by Cohn were distinct phenomena, metal foils, about 0.001 cm thick, of platinum, silver, copper, and aluminum were irradiated with electrons of 1.95 Mev energy. All metals exhibited a feeble radiation which was bluish-white in character, including silver which has a refractive index less than unity throughout the visible region. The radiation, therefore, cannot result from the Frank-Tamm process which requires that $\beta n > 1$, and this together with the fact that the radiation from metals is not polarized shows that the two phenomena are distinct.

In conclusion it may be stated that the experimental results reported here are in complete agreement with the classical explanation as developed by Frank and Tamm. It would be expected, however, that at very short wave-lengths a determination of the intensity would result in a deviation from the classical theory in much the same way that the classical theory of Rayleigh-Jeans fails at short wave-lengths.

The authors are indebted to Dr. E. Guth for many helpful suggestions and discussions while the work was in progress.

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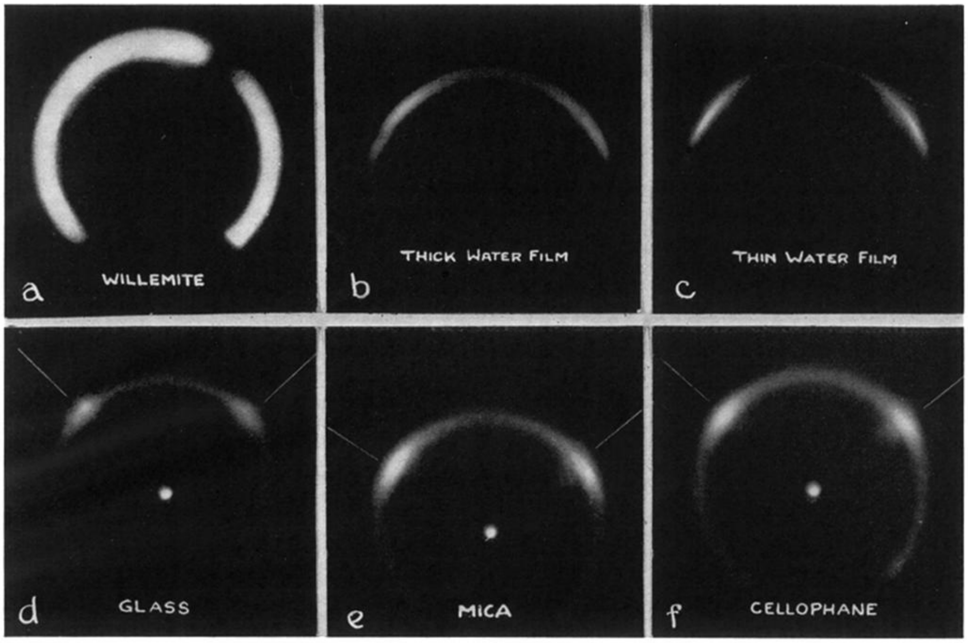
[4] Willi M. Cohn, Zeits. f. Physik **72**, 302 (1931).

[5] J. E. Lilienfeld, Physik. Zeits. **20**, 280 (1919).



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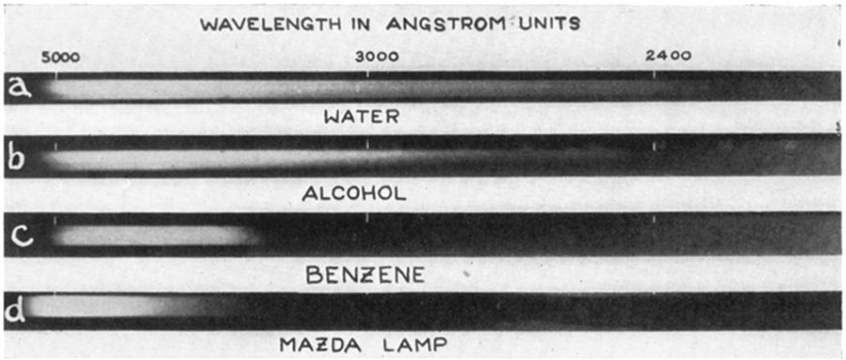

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