The Stopping Power of Lithium for Low Energy Protons

L. J. HAWORTH* AND L. D. P. KING†
University of Wisconsin, Madison, Wisconsin
(Received March 18, 1938)

The stopping power of lithium for protons as a function of energy has been calculated in the energy range 35 to 400 kv from measurements by Herb, Parkinson and Kerst and by the authors of the yields of 8 cm alpha-particles from thick and thin films of lithium. The absolute scale of units is based on an extrapolation of values given by Mano for the stopping power at energies above 500 kv. Integration of the reciprocals of the stopping power provides, except for a constant of integration which may be estimated, a knowledge of the range law as a function of energy.

ALTHOUGH some work has been done,[1] little direct experimental information is available on the stopping power of substances for protons, etc. at energies much below 500 kv.

Consequently the authors have felt that calculations they have performed, based on the yields of 8 cm alpha-particles from thick and thin films of lithium might prove of interest. The data used were those of Herb, Parkinson and Kerst[2] and of the authors[3] and cover the energy range from 35 to 400 kv.

The number of disintegrations per incident proton from a massive target (thick film) at incident energy E_0 is given by the equation

$$Y(E_0)=N\int_{E_0}^{0}\sigma(E)\frac{dx}{dE}dE,$$

where N is the number of Li^7 nuclei per cm^3, $\sigma(E)$ is the disintegration cross section at energy E and dx/dE is the reciprocal of the stopping power. By differentiation, we obtain

$$\frac{dY}{dE}=N\sigma(E)\frac{dx}{dE} \quad \text{or} \quad \frac{dE}{dx}=\frac{N\sigma(E)}{dY/dE}.$$

$\sigma(E)$ has been computed over the entire energy range in the preceding paper from the experimental thin film yields. In order to calculate dY/dE, $\Delta Y/\Delta E$ was determined graphically from the thick film yield curve (by the use of a large scale log Y *versus* $E^{-\frac{1}{2}}$ curve) for energy intervals of 4 kv. These values were found to represent sufficiently well the values of dY/dE at the midpoints of the energy intervals except at the very lowest energies where it was necessary to apply slight corrections similar to those made by Haworth and King in computing the effective energy of the protons which pass through the thin films. It should be mentioned that the absolute values of the calculated disintegration cross sections were dependent upon a knowledge of the thickness of the thin films used, which thickness was computed by extrapolating data given by Mano[4] on the stopping power of lithium at energies above 500 kv and by applying at a few points a process which was the reverse of that of the present paper.

The values of dE/dx thus obtained are plotted against the energy in Fig. 1, and listed for a few energies in Table I. As discussed in the preceding paper it was necessary to apply corrections to the data of Herb, Parkinson, and Kerst on account of the impurity film which collected on the film surfaces in the time which elapsed between evaporation and the making of the observations. In the case of thick films the stopping power of the impurity film could be estimated from the disagreement, in the absolute values of the yields, with our own observations which were made on fresh films. No such criterion was available, however, in the case of thin films. For this reason two sets of computations were

* Now at the Massachusetts Institute of Technology, Research Laboratory of Physical Chemistry.
† Now at Purdue University.
[1] The best is, perhaps, that of Blackett and Lees on air, Proc. Roy. Soc. **A134**, 658 (1932) and of Parkinson, Herb, Bellamy and Hudson on air and aluminum, Phys. Rev. **52**, 75 (1937).
[2] Herb, Parkinson and Kerst, Phys. Rev. **48**, 118 (1935).
[3] See the preceding paper.
[4] G. Mano, Ann. d. Physik **1**, 407 (1934).

made. In the first of these no correction whatever was made. In the second the same correction was applied as in the case of thick films (3 kv at 175 kv). As the values of $\sigma(E)$ seemed to agree with our own better in the first than in the second case it is probable that the true value lies somewhere between. The values of $\sigma(E)$ were adjusted to give agreement between all sets at 175 kv. The slight difference in the values of the stopping powers at this energy is due to a difference in the values of dY/dE between the two sets of observers.

The results were computed as though for a film of constant stopping power (ΔE constant). Included for comparison in Fig. 1 (squares) are values computed by King and Haworth for a film of constant thickness ($\Delta E/\Delta R$ of their paper). The staggering is due to the fact that they were calculated directly from the data without any smoothing, whereas the present calculations were based on values read from smoothed curves.

The dashed curve of Fig. 1 was obtained from that of Livingston and Bethe[5] for air, after approximate adjustment was made for the relative stopping power of lithium and air according to Mano.[4] The greater steepness of our curve at low energies may be, perhaps, the result of the effect of nonuniformity of the thin films used. As shown in the preceding paper this would

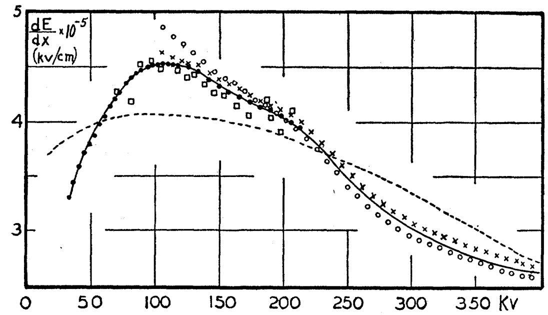

FIG. 1. The stopping power as a function of energy. Legend: Solid dots, Haworth-King, from smoothed yield curves; squares, Haworth-King, from measurement of $\Delta E/\Delta R$; crosses, H. P. K. no correction to thin film yields; circles, H. P. K. thin film voltages lowered (3 kv at 175 kv); dashed curve Livingston-Bethe, reduced to lithium according to Mano.

TABLE I. *Our own values have been used up to 200 kv and above that energy the mean of the two sets from Herb, Parkinson and Kerst. The last figure in each case is added only for the sake of internal consistency.*

E (KV)	dE/dx (KV/CM $\times 10^{-5}$)	RANGE (CM $\times 10^4$)
36	3.40	1.80
40	3.57	1.93
50	3.86	2.19
60	4.02	2.44
70	4.20	2.68
80	4.32	2.93
90	4.47	3.16
100	4.54	3.38
125	4.51	3.92
150	4.34	4.50
175	4.18	5.06
200	4.05	5.68
250	3.46	7.00
300	3.02	8.56
350	2.79	10.38
400	2.62	12.28

[5] M. S. Livingston and H. A. Bethe, Rev. Mod. Phys. 9, 245 (1937).

result in too small values for $\sigma(E)$ to an increasing extent as the energy is lowered. The same effect would, of course, be exerted on the computed values of the stopping power.

Another error producing the same sort of effect results from the straggling of the protons in passing through the lithium. In calculating ΔY we have used the relationship $\Delta Y = Y(E_0) - Y(E')$ where $E' = E_0 - \Delta E$ and we have assumed that ΔY is the yield produced by protons all of which lose energy ΔE in the process. Actually, however, when the mean energy loss is ΔE there is a distribution of energies so that the above equation should be replaced by $\Delta Y = Y(E_0) - Y_0(E)$ where $Y_0(E)$ is the yield that would be produced by an incident proton beam having just the distribution of energies that our beam has after suffering a mean energy loss ΔE. Now for symmetrical straggling $Y_0(E) > Y(E')$ since $d^2Y/dE^2 > 0$. Then

$$Y(E_0) - Y(E') > Y(E_0) - Y_0(E)$$

and we have overestimated ΔY and consequently underestimated dE/dx. This effect will be pronounced only at the lowest energies.

In order to calculate the order of magnitude of this effect let us consider as a simple example that when the mean energy loss is ΔE there are two equally intense groups of protons of energies $E_0 - \Delta E + \delta E$ and $E_0 - \Delta E - \delta E$, respectively. Then

$$Y_0(E) = \tfrac{1}{2}[Y(E_0 - \Delta E + \delta E) + Y(E_0 - \Delta E - \delta E)].$$

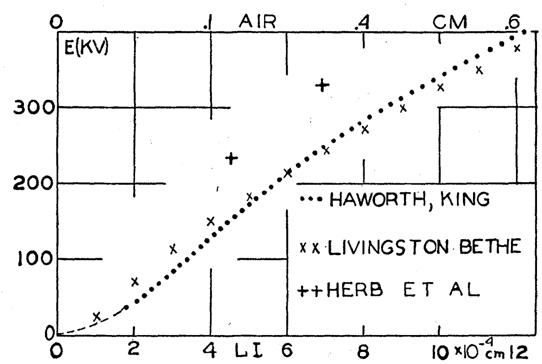

FIG. 2. Range-energy curve.

From an equation of Livingston and Bethe[6] it is calculated that the mean square deviation from the mean energy loss for the case $E_0=42$ kv, $\Delta E=4$ kv is approximately 3 kv^2 or δE is 1.7 kv. As an extreme case we will use $\delta E=2$ kv. Then $E'=38$ kv; $E_0-\Delta E+\delta E=40$ kv; $E_0-\Delta E-\delta E=36$ kv. To obtain the various Y's it is sufficiently accurate for this purpose to use our thick film yield curve. We find, in units of 10^{-12} disintegrations per incident proton: $Y(36)=1.46$; $Y(38)=2.27$; $Y(40)=3.42$; $Y(42)=5.01$. Then

$$Y(42)-Y(38)=5.01-2.27=2.74,$$
$$Y(42)-Y_0(E)=5.01-\tfrac{1}{2}(3.42-1.46)=2.57$$

or a difference of about six percent. A similar calculation at 70 kv shows a difference of 2.5 percent, etc.

[6] Reference 5, Eq (788).

Because of the indefiniteness no corrections were applied for this effect. It seems safe to say, however, that the errors on this account cannot at the most exceed five or six percent even at the very lowest voltages and that they decrease rapidly as the voltage is increased.

In Fig. 2 is plotted a range-energy curve obtained by numerical integration of a smooth curve of dx/dE against E. For this purpose calculations were made from the solid curve of Fig. 1. The unknown integration constant, representing the range below 36 kv, was approximated by extrapolating the stopping power curve to zero energies before taking the reciprocals. This checked well with values arrived at by extrapolating the range curve itself without any assignment of the integration constant.

For comparison the curve of Blackett and Lees[1] (as corrected by Livingston and Bethe[5]) for air has also been plotted. It is seen that this curve, as does the dashed curve of Fig. 1 indicates a maximum stopping power at a somewhat lower energy than in our own case. Measurements of Parkinson, Herb, Bellamy and Hudson[1] are also included. Their data show a somewhat shorter range than do those of Blackett and Lees.

The authors are deeply grateful to Professor G. Breit for much valuable advice. We are also indebted to Messrs. G. Ragan and R. Syrdal for assistance in some of the numerical calculations. One of us (L. J. H.) acknowledges with gratitude the receipt of a grant from the Wisconsin Alumni Research Foundation during part of the time that the work was being done.