The Excitation Function for the Disintegration of Li^7 Under Bombardment by Low Energy Protons*

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Accurate measurements were made of the yields of 8 cm alpha-particles from thick and thin films of lithium bombarded by homogeneous beams of protons in the energy range from 38 to 210 kv. Careful analyses are described whereby the thin film data were reduced to the values to be expected from films of infinitesimal thickness, thus allowing a relative calculation of the excitation function. From values of the stopping power by other observers the absolute cross section of the lithium nucleus for this process was calculated approximately as a function of the energy. The curves were extended to 400 kv by applying corrections to previously published data of Herb, Parkinson and Kerst.

MANY investigators[1–9] have studied the yield of 8.4 cm alpha-particles arising from the disintegration of Li^7 by protons in the reaction $Li^7+H^1\rightarrow He^4+He^4$. At low energies, however, little work has been done with metallic lithium as a target material and practically none with thin films. A notable exception is the thick and thin film work of Herb, Parkinson and Kerst[5] in the energy range from 100 to 400 kv. Preliminary work, of which this is an extension, has been done on thick metallic films in the energy range 45 to 200 kv by Heydenburg, Zahn and King.[8] Other determinations at low energies mostly concern themselves with the yields from thick films of lithium compounds.

It is impossible to calculate the excitation function from thick film data alone since the law of penetration of the particles into the target material is not accurately known. Furthermore the yields from thick films of compounds cannot readily be converted into equivalent yields from thick metallic films since, as Mano,[10] Livingston and Bethe,[11] and others have shown, the ratio of the penetrations of the protons in two different materials is not independent of the energy.

APPARATUS

The protons used in these experiments obtained their energy by passing down a three foot vertical accelerating tube to which were applied potentials generated by a Cockcroft and Walton[12] type voltage quadrupler. Potentials were determined by measuring, with a standard resistance and potentiometer, the currents through a corona-free resistance unit similar to that described by Hafstad, Heydenburg and Tuve.[9] Detailed descriptions of the voltage generator, the proton source, the accelerating tube and the voltmeter (together with the method of calibrating it) are given elsewhere,[13] but a brief description of the capabilities of the apparatus will be included here.

A steady potential in excess of 300 kv is available. Small fluctuations (of the order of a few tenths of one percent) were continuously compensated by a manually operated rheostat in the primary circuit of the high voltage trans-

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[1] J. D. Cockcroft and E. T. S. Walton, Proc. Roy. Soc. **A137**, 229 (1932).
[2] E. O. Lawrence, M. S. Livingston and M. G. White, Phys. Rev. **42**, 150 (1932).
[3] M. L. Oliphant and Lord Rutherford, Proc. Roy. Soc. **A141**, 259 (1933).
[4] M. C. Henderson, Phys. Rev. **43**, 98 (1933).
[5] R. G. Herb, D. B. Parkinson and D. W. Kerst, Phys. Rev. **48**, 118 (1935).
[6] L. R. Hafstad and M. A. Tuve, Phys. Rev. **48**, 306 (1935).
[7] H. D. Doolittle, Phys. Rev. **49**, 779 (1936).
[8] N. P. Heydenburg, C. T. Zahn and L. D. P. King, Phys. Rev. **49**, 100 (1936).
[9] L. R. Hafstad, N. P. Heydenburg and M. A. Tuve, Phys. Rev. **50**, 504 (1936).
[10] G. Mano, J. de phys. et rad. **5**, 628 (1934); Ann. d. Physik **1**, 407 (1934).
[11] M. S. Livingston and H. A. Bethe, Rev. Mod. Phys. **9**, 245 (1937).
[12] J. D. Cockcroft and E. T. S. Walton, Proc. Roy. Soc. **A136**, 619 (1932).
[13] L. J. Haworth, L. D. P. King, C. T. Zahn and N. P. Heydenburg, Rev. Sci. Inst. **8**, 486 (1937).

former so that the average deviation of the potential from the desired value was probably negligible. The 60-cycle ripple present is of the order of two percent between extremes.

Very steady, sharply focused, atomic ion beams are available in intensities up to 12 microamperes. By employing the molecular ion beams of mass 2 and mass 3, proton intensities equivalent to 40 and 60 microamperes respectively may be obtained. These relatively intense molecular ion beams are quite useful in disintegration experiments at low energies where the disintegration probability is very low. The inhomogeneity of the proton energies is somewhat increased by the fact, discussed in detail in reference 13, that they do not all originate at points of the same potential in the ion source. This effect is of importance, however, only when very intense beams of atomic ions are used and is negligible in the greater part of the present experiment.

For most of the observations the voltmeter multiplier consisted of a 15-section unit of resistance 3×10^9 ohms. However, in the case of the five points of lowest energy in the thin film studies a smaller unit of six sections was used. These multipliers have been carefully calibrated in various ways and are believed to be accurate to 0.1 percent relatively and perhaps 0.3 percent absolutely when proper corrections are made for temperature variations.[14]

Details of the disintegration chamber, etc. are shown in Fig. 1. The beam, after being deflected in the magnetic analyzer, is focused and aligned by observing the fluorescence on the quartz plate Q. After proper adjustment of the beam, Q is removed from its path by rotating the ground joint J and the beam is allowed to strike the target T. The latter consists of a film of lithium deposited by evaporation from the furnace F onto a thick nickel plate. T may be rotated by a second ground joint G, a stop being provided to assure the proper position for either evaporation or bombardment. Rotation of G after the target strikes the stop gives, by means of the screw S, a horizontal motion to T so that different parts of the nickel plate may be used for the target backing. The usable portion is 1.25 inches in length.

The system for recording the alpha-particles consists of the ionization chamber I, a Dunning[15] type linear amplifier, a scale of four thyratron counter and a mechanical counter. A cathode-ray oscilloscope and a loudspeaker are useful accessories. The solid angle included in the measurements is determined by the aperture d in the ionization chamber. Such an aperture 4.76 mm in diameter was carefully measured and the solid angle computed geometrically. A second, much larger, aperture covered with a wire grid was used when the counting rate was low. It was calibrated empirically by comparison with the small aperture.

Proton currents were measured by observing the deflection of a galvanometer attached to the target chamber which served as a Faraday cage. A visual average of the deflection was made and recorded over each thirty seconds. The escape of secondary electrons was prevented by a small

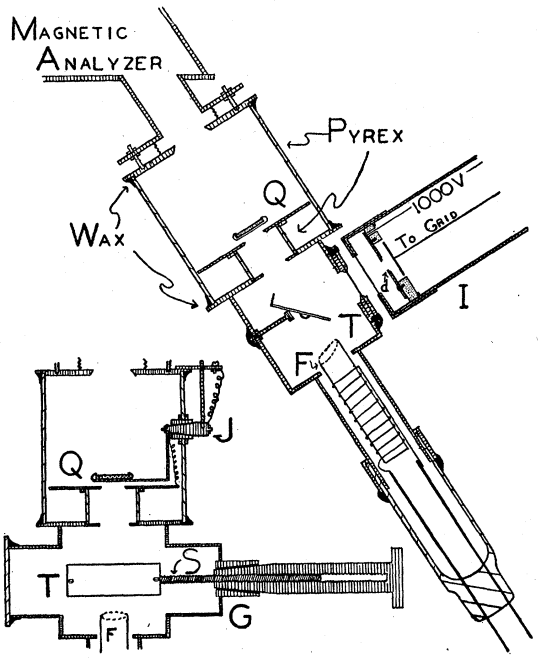

FIG. 1. The disintegration chamber, etc. Both views are normal to the proton beam.

[14] It should be mentioned that the data taken with the smaller resistance unit were obtained before it was realized that there was a dependence on temperature so that no temperatures were recorded. However, as will be shown later, it is believed that in this particular case any error arising from this source is negligible.

[15] Dunning, Rev. Sci. Inst. 5, 387 (1934).

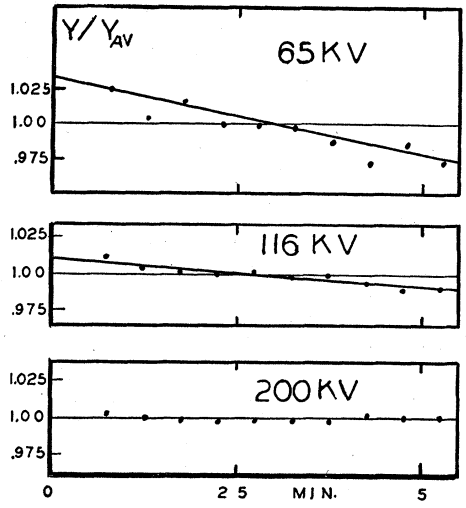

FIG. 2. Thick film fatigue curves. The individual yields have been divided by the average yield throughout the entire five minutes. The ordinate 1.00 represents the average yield.

electromagnet. In general the currents were very steady; the maximum fluctuation in a thirty second interval was usually not greater than one percent.

EXPERIMENTAL METHODS AND RESULTS

Thick film measurements

It was found, in agreement with other observers, that the yield from massive films showed a steady decrease with time after evaporation of the film. The relative effect was greatest at the lowest proton energies. This was interpreted as being due to the formation of a thin film of oil or other contamination which slowed down the protons so that their energy on reaching the lithium became less as the layer became thicker. After bombardment a brown spot appeared where the beam had struck the target, due, probably, to a carbonization of the surface impurity.[16] The fatigue in the yield did not seem, however, to be affected by the bombardment itself to any appreciable extent.

Because of this effect only very fresh lithium surfaces were used. The lithium furnace was kept hot continuously. At the beginning of a set of experiments several minutes were allowed for the deposition of a thick coating on the target. The target was then turned into position for bombardment, the beam turned on and, after thirty seconds allotted for making any minor adjustments, the measurements were begun. These continued for five minutes during which the counter and the current galvanometer were read every thirty seconds. A period of three or four minutes was then devoted to evaporating a new layer of lithium over the old. The proton beam was checked meanwhile and the voltage changed if desired. Another five minute run was then taken and so on.

It was necessary after a few days to remove and clean the target and, less often, the mica window. A careful washing in distilled water sufficed in both cases.

In addition to the precautions taken to ensure fresh surfaces, corrections were applied for the fatigue occurring during the five minutes required to take the data for each layer. At each energy the yields for each thirty second interval were calculated and plotted as a function of the time after evaporation (see Fig. 2). The resulting curve was extrapolated to zero time and the intercept taken as the true yield. Reference to Fig. 2 indicates that the fatigue effect is greater the lower the energy. It was determined graphically that the variation of the effect with energy was similar to that to be expected from a film of impurity of stopping power about 0.2 kv at 115 kv incident energy.

TABLE I. *Thick film yields $Y(E)$ and collision cross sections $\sigma(E)$. These values were read from smooth curves. See the text for a discussion of possible errors.*

E (kv)	$Y(E)$ (DISINTEGRATIONS PER PROTON)			$\sigma(E)$ (CM²)		
	H.K.	H.P.K. (UNCORR.)	H.P.K. (CORR.)	H.K.	H.P.K. (UNCORR.)	H.P.K. (CORR.)
36	1.46×10^{-13}			2.60×10^{-31}		
40	3.42×10^{-13}			5.64×10^{-31}		
50	1.87×10^{-12}			2.38×10^{-30}		
60	6.50×10^{-12}			6.61×10^{-30}		
70	1.77×10^{-11}			1.53×10^{-29}		
80	3.92×10^{-11}			2.91×10^{-29}		
90	7.71×10^{-11}			4.90×10^{-29}		
100	1.35×10^{-10}	1.10×10^{-10}	1.30×10^{-10}	7.50×10^{-29}	7.75×10^{-29}	8.20×10^{-29}
125	4.20×10^{-10}	3.73×10^{-10}	4.17×10^{-10}	1.68×10^{-28}	1.70×10^{-28}	1.77×10^{-28}
150	9.83×10^{-10}	9.0×10^{-10}	9.82×10^{-10}	3.00×10^{-28}	3.04×10^{-28}	3.08×10^{-28}
175	1.96×10^{-9}	1.84×10^{-9}*	1.97×10^{-9}	4.73×10^{-28}	4.73×10^{-28}	4.78×10^{-28}
200	3.46×10^{-9}	3.20×10^{-9}	3.42×10^{-9}	6.73×10^{-28}	6.78×10^{-28}	6.65×10^{-28}
250		8.15×10^{-9}	8.43×10^{-9}		1.10×10^{-27}	1.06×10^{-27}
300		1.67×10^{-8}	1.69×10^{-8}		1.51×10^{-27}	1.46×10^{-27}
350		2.90×10^{-8}	2.95×10^{-8}		1.94×10^{-27}	1.85×10^{-27}
400		4.64×10^{-8}	4.71×10^{-8}		2.35×10^{-27}	2.25×10^{-27}

* Dr. Herb has asked us to call attention to an error in this value as listed in the table in conjunction with Fig. 3 of their paper. The value given was much too high to be consistent with their data, the error having arisen when reading the value from a smooth curve.

[16] R. L. Stewart, Phys. Rev. **45**, 488 (1934).

The final thick film yields, after correction for probe voltage, fatigue, etc. are given in Table I and in Fig. 3, curves A and B, where the log of the yield[17] is plotted as a function of $E^{-\frac{1}{2}}$. Experimental points are plotted in the figure, but in the table are given values read at standard energies from a smooth curve. The results of other observers are also plotted for comparison. Those of Doolittle,[7] at energies below 75 kv, were obtained from a target believed to be LiOH. The observed values were multiplied by four in an attempt to compensate for the stopping power of the oxygen and hydrogen atoms. Such a method is not very accurate since the binding energies of the electrons as well as their number becomes of consequence at these very low energies. This probably accounts for the lack of agreement with our own results.

The high energy end of the curve is shown on a larger scale in curve B together with the results of Herb, Parkinson and Kerst[5] (hereafter referred to as H. P. K.). There is a marked disagreement with their values as published (circles) which is greatest at the lowest energies. However, in their work the lithium target was not usually used until some time (often a day or more) after evaporation so that there was ample time for the formation of a relatively thick impurity layer. The crosses represent an attempt to correct for this by assuming the presence of a foreign film of stopping power 3 kv at 175 kv incident energy. As in our own case adjustment was made for the variation with incident energy of the stopping power of the impurity film. The assumption made is not unreasonable in view of our own experience with old films.[18] Furthermore, a few experimental points taken by H. P. K. with very fresh surfaces are found to agree better with the corrected than with the original curve. The agreement of the corrected results with those of the present authors is very good over the whole range of energies in which they overlap; both the yield curves and their slopes agree to about one percent except at the very lowest energies, where H. P. K. do not consider their results very accurate. No explanation offers itself for the discrepancy, at the higher energies, between the results of H. P. K. and those of Hafstad, Heydenburg and Tuve[9] which are shown approximately by the dotted curve.

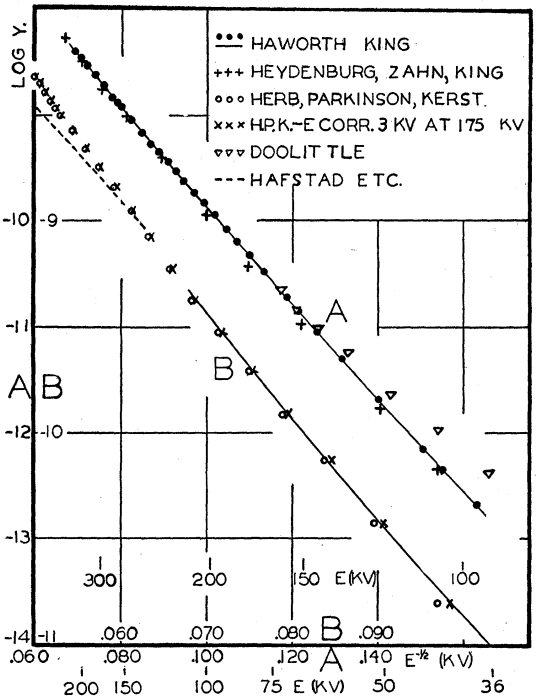

FIG. 3. Thick film yield curves. *Erratum:* In curve B the ordinates indicated are one unit too large negatively.

Thin film measurements

The requirement that the surface be relatively fresh necessitated the use of a great many thin films, about 400 in all. These were formed by evaporation onto the nickel backing for a fixed time (30 seconds) with a definite heating current through the winding of the lithium furnace. By moving the target horizontally it was possible to deposit and use four films before it became necessary to remove and clean the nickel.

In spite of the utmost precautions it was impossible to obtain equal thicknesses for all films. For this reason it was necessary to observe the yield from each film at some standard reference energy(116 kv was chosen) as well as at the voltage under consideration. The following procedure was adopted. Observations were taken for five minutes as in the case of the thick films. The voltage was then changed to the reference

[17] Throughout this paper the term "yield" is understood to refer to the number of disintegrations per incident proton.

[18] It should be mentioned that H. P. K. used a mercury diffusion pump and liquid-air trap, whereas an Apiezon oil pump without trap was used in the present work.

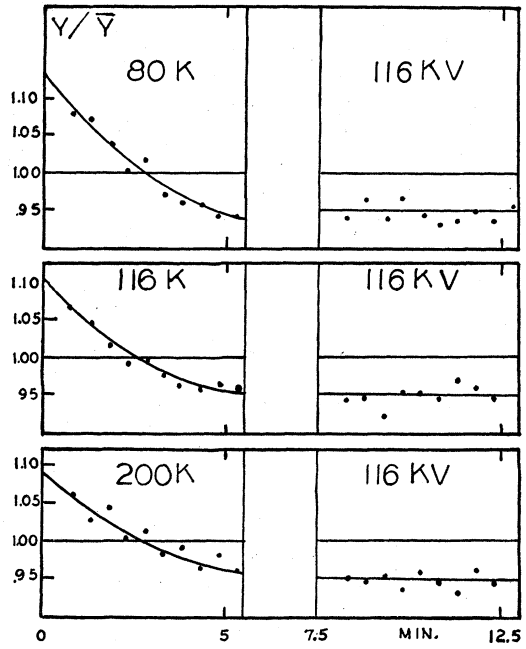

FIG. 4. Thin film fatigue curves. In each case $\bar{Y}$ represents the average yield, over the first five minutes, from a fresh film bombarded at the energy in question. The yields for the reference energy are plotted on the right.

value and the beam readjusted and refocused. Great care was necessary that the beam strike exactly the same spot on the target at both energies.[19] A small electromagnet at the top of the magnetic analyzer served to give lateral adjustment. Two minutes were alloted for the adjusting process, after which observations were made for five minutes at the reference energy.

The yields for a few characteristic energies are plotted as functions of time in thirty second intervals in Fig. 4. It may be seen that the yields decreased very rapidly during the first few minutes and assumed later a more nearly constant value. It is believed that this fatigue arose from two causes: (1) the surface impurity previously discussed and (2) an actual thinning of the film due to evaporation caused by the heating effect of the beam. To this evaporation is attributed the rapid decrease in yield during the first few minutes. That it did not continue indefinitely was doubtless due to the formation of the impurity layer on the surface. The surface impurity has its greatest relative effect at low energies, whereas a given change in thickness by evaporation should produce its greatest relative change in the yields at high energies (since the yield does not fall off so rapidly with penetration). A careful study was made in which the surface impurity was considered the same as in the case of thick films. The residual fatigue after correcting for the impurity was indeed found to be higher at the higher energies to an extent about that to be expected if the decrease in thickness were the same in all cases. (The beam intensity had been adjusted in such a way that the energy density on the surface was always approximately the same.) It was also found that the effect of the fatigue on the yield at the reference energy was essentially independent of the proton energy used during the first five minute period. This was established by the above described features of the fatigue and by the following additional facts. (1) Films evaporated successively and presumably of approximately the same thickness gave nearly equal yields at the reference energy regardless of the energy used during the first period. (2) Reversal of the usual order of bombardment yielded the same results as the regular method, when calculations to be described later were correctly carried out. (3) In a number of cases a film was bombarded for a short time at the reference energy, following which the regular procedure was carried out at a spot slightly removed from the first one. The ratio of the yields at the reference energy from the two spots was found to be independent of the energy used during the first five minutes on the second spot.

The combined effect of the impurity layer and the evaporation was such as to decrease the average yield at the reference point to a value five percent lower than it would have been had the film not been previously bombarded. It was found from thick film studies that the average absorbing power of the impurity film during a second five minute period was approximately 0.5 kv, or 0.3 kv greater than during the first period. This 0.3 kv change will account for about 1 percent in the 5 percent fatigue effect, leaving 4 percent to be attributed to evaporation.

The necessary corrections for fatigue thus

[19] Neglect of this precaution at one time gave rise to a spurious "hump" in the thin film yield curve. This effect was present at the time of a report given before a Chicago meeting of the American Physical Society, Nov., 1936.

consist of: (1) a 0.15 to 0.2 kv decrease in the energies at the experimental points; (2) a 4 percent increase in the yields and a 0.5 kv decrease in the energies at the reference point. The corrections applied to the reference point have, however, only a very slight effect on the relative thin film yields calculated from the data.

Evaluation of the thin film data

The purpose of a thin film yield curve is to make it possible to calculate the disintegration cross section of the atomic nucleus, at least relatively. A true thin film (or excitation) curve should have ordinates proportional to this cross section at each energy. If a beam of protons of incident energy E_0 passes through a film of thickness Δx, the yield $y(E_0)$ is given by

$$y(E_0)=N\int_0^{\Delta x}\sigma(E)dx, \qquad (1)$$

where N is the number of Li^7 nuclei per cm^3 and $\sigma(E)$ is the disintegration cross section. It is obvious that if the proton energy remained constant throughout the film the yield would be given by

$$y(E_0)=N\sigma(E_0)\Delta x.$$

In the case of infinitesimally thin films this would be true to a high degree of approximation and with such films a reduction to constant thickness could be made by dividing the various values of $y(E_0)$ by the corresponding yields $y(R_0)$ at the reference energy. Actually, however, the films used were of finite thickness and the reduction in energy in passing through them was appreciable. The above approximation is therefore not sufficiently accurate. Proper treatment of the data makes it possible, however, to obtain values for $\sigma(E)\times$ const. at each energy E as follows.

Let the energy loss of the protons in passing through the film be ΔE. Then we may say

$$y(E_0)=N\int_0^{\Delta x}\sigma(E)dx=N\sigma(E_e)\Delta x, \qquad (2)$$

where E_e is some energy in the interval E_0 to $E_0-\Delta E$. Then the various $y(E_0)$ will, when reduced to a constant film thickness, be proportional to the true cross section at the corresponding energies E_e.

As a first approximation to the true values for E_e it was decided to use the mean energy $\bar{E}=E_0-\Delta E/2$. In order to determine the values of ΔE it was necessary to resort to the thick film curve. Let the thick film yield at incident energy E_0 be $Y(E_0)$. Of this, an amount $y(E_0)$ is the yield from a surface layer of thickness Δx equal to that of a thin film which yields $y(E_0)$ at the same incident energy. The difference $Y(E_0)-y(E_0)$ is equal to the thick film yield at incident energy $E'=E_0-\Delta E$. In order to find ΔE it is only necessary to find, on the thick film yield curve, the energy E' at which $Y(E')=Y(E_0)-y(E_0)$ and obtain $\Delta E=E_0-E'$ by subtraction. This was done for all cases, including the reference point. The values of E_0 and $y(E_0)$ were obtained by averaging over the several films used at the experimental energy in question. The various effective energies E_e were then calculated to a first approximation from the relationship $E_e\cong\bar{E}=E_0-\Delta E/2$.

The yields were then corrected for film thickness. To a first approximation this could be done by simply dividing $y(E_0)$ by the corresponding yield $y(R_0)$ at the reference energy. However, the values of $\bar{R}$, the mean reference energy, were not all exactly the same for two reasons: (1) although the voltmeter current was always the same, R_0 varied slightly because of differences in temperature and probe voltage; (2) different film thicknesses affected $\bar{R}$ through ΔR. It was necessary, therefore to apply corrections to the various $y(R_0)$. Each $y(R_0)$ was reduced to the value it would have had at some standard $\bar{R}$. This was accomplished as follows. A number of films for which $\bar{R}$ was practically the same were selected and, a curve of $y(E_0)/y(R_0)$ against energy was plotted in the neighborhood of the reference point. One hundred fourteen kv was chosen as the standard value of $\bar{R}$ and each $y(R_0)$ was corrected to this standard by forming the product $y(R_0)\times z(114)/z(\bar{R})$ where $z(114)$ and $z(\bar{R})$ are the respective ordinates at 114 kv and $\bar{R}$ on the curve just described.

A standard film was chosen of such thickness that the absorbing power would be 4 kv (approximately the average for all films) at an incident energy of 116 kv. By inspection of the thick film yield curve it was found that such a film would give a yield of 4.65×10^{-11} disinte-

grations per proton at $E_0=116$ kv, $\bar{E}=114$ kv. The final corrected values of the yields were then given by

$$\bar{y}(\bar{E})=\frac{y(E_0)}{y(R_0)}\times\frac{z(\bar{R})}{z(114)}\times 4.65 \times 10^{-11} \text{ disintegrations per proton.} \quad (3)$$

A plot of $\bar{y}(\bar{E})$ against $\bar{E}$ gives a first approximation to the desired excitation curve. In order to determine the accuracy of the approximation it is necessary to investigate the extent to which $\bar{E}$ differs from E_e, the true effective energy. By Eq. (2) the true E_e will be the energy such that

$$\sigma(E_e)=\frac{\int_0^{\Delta x}\sigma(E)dx}{\Delta x}$$

or, in more usable form

$$\sigma(E_e)=\frac{\int_{E_0}^{E_0-\Delta E}\sigma(E)(dx/dE)dE}{\int_{E_0}^{E_0-\Delta E}(dx/dE)dE}. \quad (4)$$

Values of dx/dE were calculated in arbitrary units as follows. ΔE and the corresponding ΔR are measures of the stopping powers of a given thin film at incident energies E_0 and R_0 respectively, so that

$$\frac{\Delta E}{\Delta R}=\frac{\Delta E/\Delta x}{\Delta R/\Delta x}$$

is a measure of the stopping power of that film at incident energy E_0 in terms of the stopping power at incident energy R_0. The various values of $\Delta E/\Delta R$ (corrected for the variation in $\bar{R}$) are thus measures of $\Delta E/\Delta x$ in arbitrary units. Numerical and graphical tests were applied to the curve having $\Delta x/\Delta E$ as ordinates and $\bar{E}$ as abscissae which showed that to a high degree of approximation (much better than one percent)

$$\frac{\Delta R}{\Delta E}=k\frac{\Delta x}{\Delta E}=k\left(\frac{dx}{dE}\right)^{20} \text{ at } E=\bar{E}.$$

[20] That this approximation is a highly accurate one can be surmised in another way as follows. Assume that in the narrow energy range ΔE a sufficiently accurate approximation to the range law is given by the relationship $x=A+CE^n$ where A and C are constant over ΔE. Then

$$\Delta x=C[(\bar{E}+\Delta E/2)^n-(\bar{E}-\Delta E/2)^n]$$
$$=Cn\bar{E}^{n-1}\Delta E[1+(1/24)(n-1)(n-2)(\Delta E/\bar{E})^2+++],$$

whence at $E=\bar{E}$

$$(dx/dE)=[\Delta x/\Delta E][1-(1/24)(n-1)(n-2)\times(\Delta E/\bar{E})^2+++]=(\Delta x/\Delta E)(1-\delta+++).$$

Actually, in our case n varies between 1.5 and 0.5 at different parts of the curve. Thus $(1/24)(n-1)(n-2)$ lies between -0.01 and $+0.03$. The highest value in any case for $(\Delta E/\bar{E})^2$ was for the case $\bar{E}=36$, $\Delta E=4.3$ kv, or $(\Delta E/\bar{E})^2=0.01$ (approximately). Thus we see that δ is of the order of 10^{-4} or less and may be considered negligible.

[21] Professor Breit has suggested the following method of checking this point. From the curve for $\bar{y}(\bar{E})$ it is found that approximately $\sigma(E)=Ce^{-k/(E)^{\frac{1}{2}}}$ where, in the low energy range, k has the approximate value 88 when E is measured in kilovolts. Furthermore dx/dE varies so slowly in comparison with $\sigma(E)$ that we shall consider it constant over the thin film for purposes of this discussion. Then Eq. (4) becomes

$$\sigma(E)=\frac{\int_{\Delta E/2}^{\bar{E}+\Delta E/2}\sigma(E)dE.}{\Delta E}$$

Expanding about $\bar{E}$ and integrating, we obtain

$$\sigma(E_e)=\sigma(\bar{E})+\frac{(\Delta E)^2}{24}\sigma''(\bar{E})++\cdots.$$

Furthermore, letting $E_e=(\bar{E})+\delta E$ and expanding about $\bar{E}$ we obtain

$$\sigma(E_e)=\sigma(\bar{E})+\sigma'(\bar{E})\delta E++\cdots.$$

Comparison of the two above values for $\sigma(E_e)$ yields

$$\delta E=\frac{(\Delta E)^2}{24}\frac{\sigma''(\bar{E})}{\sigma'(\bar{E})}++\cdots.$$

Or, by substitution of the assumed expression for $\sigma(E)$

$$\delta E=\frac{(\Delta E)^2}{48\bar{E}}(K\bar{E}^{-\frac{1}{2}}-3).$$

In the case of the lowest energy which we used $\bar{E}=36$ kv, $\Delta E=4.3$ kv, whence

$$\delta E=0.11 \text{ kv},$$

in fairly good agreement (below experimental error from other sources at these energies) with the directly calculated correction.

Curves of (dx/dE) at $E=\bar{E}$ and $\bar{y}(\bar{E})$ were then plotted as functions of $\bar{E}$ and were considered (except for proportionality constants) sufficiently good approximations to the true curves for dx/dE and $\sigma(E)$ to furnish a criterion for the accuracy of the relationship $\bar{E}\cong E_e$. Values at corresponding energies read from these curves were substituted in Eq. (4) and graphical and numerical calculations (in which the proportionality constants cancel) were performed which showed that $\bar{E}$ differs appreciably from E_e only at the very lowest energies. It is approximately 0.15 kv too low at 36 kv and practically correct at all energies above 70 kv.[21]

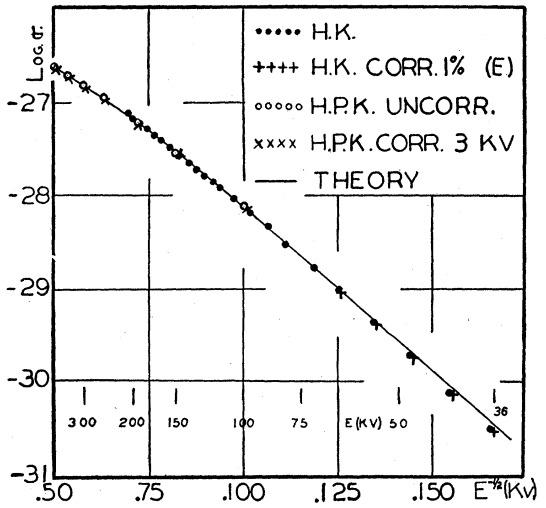

FIG. 5. The excitation cross section. All sets brought into agreement at 175 kv. The plus signs show the effect of lowering the voltage one percent at both the experimental and the reference energies. The theoretical curve is for the case $L=1$, $U=23$ kv.

The indicated corrections were applied to $\bar{E}$ to give the true E_e. A plot of $\bar{y}(E_e)$ then gave our final curve for the excitation function in terms of the yield from a film of stopping power 4 kv at 114 kv.

Disintegration cross section

An approximate calculation of the thickness of our standard film and hence of the absolute cross section was made as follows. Livingston and Bethe[11] give for σ_a, the stopping cross section of air, at 175 kv the value 14.9×10^{-18} kv cm^2. From figures given by Mano[10] it is estimated that the stopping cross section of Li relative to air at the same energy is 0.565. Thus the stopping power of Li at 175 kv is given approximately by

$$dE/dx=0.565\sigma_a N'=0.565\times14.9\times10^{-18}\times4.67\times10^{22}=3.94\times10^{5}\text{ kv/cm}$$

where N' is the *total* number of Li atoms per cm^3. From our curve of $\Delta E/\Delta R$ it is found that the energy absorbed in our standard film at 175 kv is 3.7 kv. Hence $\Delta x=3.7/(3.94\times10^5)=9.4\ 10^{-6}$ cm. As the stopping cross section of air and the ratio of Li to air are probably more accurately known at higher energies, the data of H. P. K. (the thin film curve was standardized to ours at 175 kv) were used at 400 kv to perform a similar calculation. This yielded the result $\Delta x=8.35\times10^{-6}$ cm.

A value of 8.8×10^{-6} cm was then chosen for use in future computations. From this thickness and the thin film excitation curve it is possible to calculate approximate absolute values of $\sigma(E)$ from the equation

$$\bar{y}(\bar{E}_e)=N\sigma(E_e)\Delta x.$$

In this case $N=4.31\times10^{22}$, the number of Li^7 nuclei per cm^3.

Experimental values of $\sigma(E)$ thus computed are plotted in Fig. 5[22] and listed in Table I.

The results of Herb, Parkinson and Kerst are also given. Two corrections to their data as published were considered. (1) The energies assigned to the respective yields were shifted so as to represent the average energy of the protons in the films rather than the incident energy. This was done in a manner analogous to that used on our own data. (2) The effect of a possible surface film was considered. Considerable uncertainty exists as to how large this second correction should be. Consequently the results are given for two cases. In the first no correction was applied. In the second the same correction as in the case of

[22] As previously mentioned the data for the five points of lowest energy in Fig. 5 were obtained when the small voltmeter multiplier was used without making temperature observations. Both thick and thin film measurements were made with this multiplier at all energies up to 115 kv. After the installation of the larger multiplier all of these were repeated with the exception of the five points in question. Because of the very large number of films required to obtain sufficient counts at these energies it was believed not to be worth while to repeat these. An average temperature was assumed, which was based on temperature observations in a great many cases after the effect was discovered. Any error involved is believed to be negligible for the following reasons. (1) Thick film measurements obtained more or less simultaneously with the thin film measurements in question agreed well with those obtained later with the larger multiplier in the energy range from 36 to 115 kv. The same was true of thin film measurements in the higher part of this energy range. (2) The maximum temperature variation observed was such as to allow a maximum uncertainty in the energy of only 0.5 percent. (3) Any voltage error due to temperature uncertainty was largely nullified by the use of a reference point. For example, at 36 kv any error present would be reduced by a factor of four on this account. It should be mentioned in this connection that these points were obtained by the use of molecular ion beams so that the voltmeter currents were of the same order at both experimental and reference points. In fact, at energies 36 and 55 kv they were exactly the same. The plus signs of Fig. 5 represent the effect of assuming a one percent error in voltage at both the experimental and the reference points. Such an error is believed to be outside the possible limits, at least in so far as temperature is concerned.

the thick films (3 kv at 175 kv) was applied. It is seen that the first gives results which seem to agree best with our own. However there was undoubtedly a film present and the seeming agreement without a correction is probably fortuitous. It is possible that the slight lack of agreement at the low energies when a correction is applied is the result of stray counts. Dr. Herb has informed us that they had considerable difficulty on this account. It should be mentioned that the two sets are separately adjusted to agree with our own at 175 kv.

Possible errors

The errors involved in the present experimental work may involve both the yields and the voltages. Errors in the counting process itself are believed to be negligible in so far as their effect on the smooth yield curves is concerned, although it is true that at the lowest energies the number of counts observed, particularly in the case of thin films was not great enough to prevent some statistical fluctuation in the individual points. The absolute scale of yields in the case of thick films is believed correct to one percent.

Of greater importance are the possible errors involving the voltage. The mean energy of the incident protons is believed to be known in every case to within two or three tenths of one percent (except, perhaps, in the case discussed in reference 22). However, some error is introduced by the inhomogeneity of the proton beam. In the case of a symmetrical distribution about the mean energy, the inhomogeneity would result in slightly too high values of the yields since the second derivatives of the yield curves are positive. This effect would be worst at the lowest energies. As mentioned earlier, the total inhomogeneity is of the order of two percent which at 40 kv results in a spread of 0.8 kv in the incident energies. By assuming as an extreme case that the beam consists of two equally intense components 0.8 kv apart it was calculated that this would introduce an error of less than one percent in the yields. This was not corrected for since it was within the limit of other experimental errors, corrections for which could not be computed.

Two additional possible sources of error are inherent in the use of films of finite thickness. Our development of the data was based on the assumptions that the films were of uniform thickness across the area struck by the beam and that there was no straggling of the protons in passing through the films. Nonuniformity of the individual films might introduce serious errors at low energies where the yield is a very rapid function of the energy. In this case nuclei in the lower layers of the thicker spots would contribute fewer disintegrations than those nearer the surface so that the yield from a given amount of lithium would be lower if it were "bunched" in a few spots than if it were uniformly distributed. At the reference point, on the other hand, this effect would be much less marked since the dependence of cross section on energy would not be so great. Thus the relative yields at the low energies would be too small. This effect would be magnified by the fact that too small a value for $y(E_n)$ results in an underestimation of ΔE and consequently too high a value for E_e. The magnitudes of the possible errors on this account are hard to estimate, but they may amount to several percent at the lowest energies and are probably the most important in the entire experiment.

The straggling of the protons in passing through the thin film will somewhat increase the value of $y(E_0)$. For, since the second derivative of $\sigma(E)$ is positive, the extra yield from those protons whose average energy through the film is greater than $\bar{E}$ will more than compensate for the deficit from those whose average energy is less than $\bar{E}$. There is, however, a compensating factor. There will be straggling to the same extent in the thick films as in the thin so that, after passing through a surface layer of thickness Δx equal to that of the thin film, there will be the same distribution in proton energies as on emergence from the thin film. Then we may say

$$Y(E_0)=y(E_0)+Y_0(E),$$

where $Y_0(E)$ is the thick film yield which would be produced by a beam having an energy distribution just that of the protons after passing through the layer Δx. Now, since $d^2Y/dE^2>0$,

$$Y(E'')=Y_0(E)>Y(E')$$

for a case of symmetrical straggling. Here E' is the mean energy of emergence from Δx and $Y(E')$ is the measured thick film yield at incident

energy E'. Then our measured value of ΔE is given by

$$\Delta E_{\text{meas.}} = E_0 - E'' < E_0 - E'$$

and is too small. Thus the measured E_e will be greater than had there been no straggling so that the increase in thin film yield introduced by the straggling is accompanied by a compensating increase in the energy assigned to that yield. By making various simple but exaggerated assumptions as to the straggling and graphically integrating $\sigma(E)(dx/dE)$ between the proper limits it was found that the two increases correspond almost exactly (the net effect on σ being less than 1 percent). In these integrations account must be taken of the fact that dx/dE is different for each degree of straggling.

We may summarize by stating our belief that the thick film yield curve (of the authors) is accurate to perhaps one or two percent (in the yields) over the entire range of energies covered; that the cross section curve (of the authors) is accurate relatively to about the same extent at the higher energies; but that errors of a few percent may enter in the latter at the lowest energies, principally because of the possibility of nonuniformity in the individual films. The absolute values of the cross section are, of course, only approximate.

The corrected values calculated from the data of H. P. K. may involve somewhat larger errors but it is difficult to make an estimation.

Comparison with theory

Ostrofsky, Breit and Johnson[23] have calculated theoretically the disintegration cross section of the lithium nucleus for this process for various values of the depth of the potential well. The smooth curve of Fig. 5 represents their calculations for the case $L=1$, $U=23$ Mev which seems to give the best agreement. Adjustment has been made arbitrarily to bring the values into agreement at 175 kv.

The absolute values of $\sigma(E)$ as here plotted are somewhat larger than those arrived at by Ostrofsky, Breit and Johnson who used the data of H. P. K. in a somewhat different manner. Assuming the $\frac{3}{2}$ power dependence of proton range on energy and a constant value of 0.547 for the stopping power of lithium relative to air they computed, from the range measurements given by Rutherford, Chadwick and Ellis on oxygen, air, etc., the value of k_1 in the equation $x=k_1E^{\frac{3}{2}}$ where x is the range. Inserting this in the equation

$$Y(400) = N\int_0^{400} \sigma(E)\frac{dx}{dE}dE = Nk_2\int_0^{400} y(E)\frac{dx}{dE}dE$$

and using for $Y(E)$ and $y(E)$ the experimental values of H. P. K. they computed numerically the right hand integral and evaluated k_2 by a comparison with $Y(400)$. Values of $\sigma(E)$ were then found through the relationship $\sigma(E) = k_2y(E)$. In this way they arrived at a value for $\sigma(400)$ some 30 percent lower than that of the present calculations. The difference is caused principally by their assumption of the law of range which, as they have pointed out, makes their computations somewhat in error. The corrections applied to the data of H. P. K. are of little moment in this connection, being, in fact, in such a direction as to reduce the difference in the absolute values.

It is seen that the relative agreement of experiment and theory is quite good except at the very lowest energies. The discrepancy is in the direction to be expected from errors in the data due to nonuniformity of the thin films.

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[23] Ostrofsky, Breit and Johnson, Phys. Rev. **49**, 22 (1936).