

The Angular Distribution of Resonance Disintegration Products*

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For resonance symmetry considerations alone are found to give definite results on the angular distribution of the products of nuclear reactions. Formulae are developed for general coupling and Russell-Saunders coupling. It is found that the distribution is axially symmetric about the direction of the incident particle. It is also symmetric about the equatorial plane. The expression for the distribution is usually restricted to a small number of terms in an expression in spherical harmonics. These results are applied to the reactions: $D^2 + D^2 \rightarrow H^3 + H^1$; $B^{11} + H^1 \rightarrow Be^8 + He^4$, $Li^7 + H^1 \rightarrow 2He^4$, and $Li^6 + H^2 \rightarrow Li^7 + H^1$. For Russell-Saunders coupling the distribution for these reactions is uniquely determined by the assignment of the quantum number to the states involved.

§1. INTRODUCTION

THE spatial distribution of the products of a nuclear disintegration has already been investigated experimentally for several reactions. The results were in some cases unexpected, e.g., for $H^2 + H^2 = H^3 + H^1$ and for $B^{11} + H^1 = 3He^4$ large deviations from spherical symmetry were found. From these angular distributions important conclusions can be drawn about the mechanism of the nuclear reaction concerned. E.g., the symmetry of all observed distributions about the "equatorial plane" (plane perpendicular to the motion of the incident particle) gives strong evidence for the validity of the Bohr mechanism in the reactions studied (§3). It seems worth while to investigate theoretically, in a more quantitative way, how the angular distribution is connected with the angular momenta of the particles involved in the reaction.¹ To give a general treatment of this problem would require a specific nuclear model. However, the symmetry about the equatorial plane mentioned above shows that in all probability we are dealing with resonance disintegrations, and that only one level of the compound nucleus is important (§3). Under this assumption significant conclusions can be reached without detailed assumptions. In this paper we shall restrict our considerations to resonance disintegrations.

The state of the compound nucleus is charac-

terized by its total angular momentum J and its parity. There are strict selection rules for both of these quantities. In particular the parity rule determines the permissible values of the relative orbital momentum l of the initial particles causing the reaction. We shall designate in the following by S the spin of the incident particle P and by i the spin of the initial nucleus A , although the theory is, of course, symmetrical in the two. In all practical cases the incident particle is very light (proton, neutron, deuteron, α -particle) and therefore has even parity. The rule for the parity can thus be formulated as:

I. The relative orbital momentum l of the initial particles must be even if the initial and compound nucleus have the same parity, and l must be odd, if they have opposite parity.

The angular momentum selection rule is:

$$\text{II. } |i+s+1|_{\min} \leq J \leq i+s+l,$$

$$\text{where } |i+s+1|_{\min}$$

is the smallest value which the resultant of the three vectors i , s , l can take. Corresponding rules hold for the final particles. Ordinarily, the knowledge of J and the parity alone does not determine the angular distribution completely. Additional assumptions must be made about the coupling of the various angular momenta. For light nuclei it is probably a good approximation to assume Russell-Saunders coupling.² Orbital momentum and spin are separately conserved. The compound nucleus has then definite values of L (total orbital momentum) and S (total spin). Similarly the

² Oppenheimer and Serber, *Phys. Rev.* **53**, 636 (1938).

* Parts of this paper were submitted in a thesis for a doctor's degree at Cornell University.

¹ Reinsberg (*Zeits. f. Physik* **108**, 189 (1938)) has published a paper on this subject. His results are, however, based on incorrect selection rules.

total momentum i of the initial nucleus is composed of an orbital part λ and a spin part σ . The spin s of the incident particle forms a resultant S with σ , and the intrinsic orbital momentum λ is coupled with the orbital momentum of relative motion l to form L . Then L and S are coupled to give J . In "allowed" transitions the total orbital momentum and total spin of the final nuclei, L' and S' are equal to L and S , respectively. The calculation becomes somewhat complicated since, for initial particles alone, four pairs of vectors must be considered, *viz.* $(\lambda\sigma)i$, $(\lambda l)L$, $(s\sigma)S$, $(L S)J$ (cf. §2). There is considerable simplification if the intrinsic momentum i of the initial nucleus is due entirely to spin,³ as is probably the case for all nuclei up to Li⁶. In this case, i and s form a resultant S , and S and l are compounded to give J . This case will be treated in detail in §2 of the present paper.

If Russell-Saunders coupling cannot be assumed, we have to sum the result over all possible values of S , taking interference into account. A particularly simple case arises again when there is no trace of the Russell-Saunders coupling left, i.e., when the probability of each value of S is proportional to its statistical weight. The quantitative results will then, in general, be different from Russell-Saunders coupling or intermediate coupling. Most of the qualitative results are, however, independent of the coupling assumed.

§2. CALCULATIONS

As already mentioned we shall assume that only one resonance level of the compound nucleus is involved. Then the cross section per unit solid angle for scattering through the angle θ , φ is given by:⁴

$$I(\theta, \varphi) = \frac{\pi^2 \lambda_p}{(W - W_c)^2 + \frac{1}{4} \Gamma_c^2} \left| \sum_{l'l'M} e^{i(\delta_l + \delta_{l'})} (2l+1)^{\frac{1}{2}} \times H_{l\sigma m_s i m_i}^{JM} H_{l'm_l' s' m_s' i' m_i'}^{*JM} P_{l'm_l'}(\theta, \varphi) \right|^2, \quad (1)$$

where λ_p is the wave-length of the incident particle divided by 2π , l , l' the relative orbital momenta of the initial and final particles, m_l'

the projection of l' on the direction of motion of the incident particle, i , s , m_i , m_s , the spins and their projectors for the initial particles, i' , s' , m_i' , m_s' the same for the final particles, J , M the total angular momentum of the compound state and its projection, H the matrix element between the states as shown by the indices, δ_i the phase shift introduced by the potential between the particles, W , W_c the energy of the system and compound level, respectively, Γ_c the width of the compound level, P_{lm} the normalized spherical harmonic.

The angular distribution is most conveniently discussed if the intensity is expanded in spherical harmonics:

$$I(\theta, \varphi) = \frac{\pi^2 \lambda_p^2}{(W - W_c)^2 + \frac{1}{4} \Gamma_c^2} \sum_{pq} A_{pq} P_{pq}(\theta, \varphi). \quad (2)$$

By the customary Fourier method we find:

$$A_{pq}^{m_i m_s m_i' m_s'} = (2i+1)(2s+1) \int \left| \sum_{l'l'M} e^{i(\delta_l + \delta_{l'})} \times (2l+1)^{\frac{1}{2}} H_{l\sigma m_s i m_i}^{JM} H_{l'm_l' s' m_s' i' m_i'}^{*JM} P_{l'm_l'}(\theta, \varphi) \right|^2 \times P_{pq}^*(\theta, \varphi) \sin \theta d\theta d\varphi. \quad (3)$$

Since in experiments one is not interested in the result for specific orientations of the spins of the particles, we only need the above expression averaged over the magnetic quantum numbers, *viz.*:

$$A_{pq} = \frac{1}{(2i+1)(2s+1)} \sum_{m_i m_s m_i' m_s'} A_{pq}^{m_i m_s m_i' m_s'}. \quad (3a)$$

Bethe and Placzek⁵ have shown that this is facilitated by expressing the matrix elements as given above, as linear combinations of matrix elements between states of such symmetry that the matrix elements are independent of the magnetic quantum numbers. This is accomplished by expressing the wave functions describing the initial and final states as a linear combination of wave functions characterized by a definite resultant angular momentum. In accord with our considerations in §1, we consider the resultant J to be formed in two stages, *viz.*:

³ A momentum entirely due to orbital motion would also give simple results.

⁴ Bethe, Rev. Mod. Phys. 9, 105 (1937).

⁵ Bethe and Placzek, Phys. Rev. 51, 450 (1937).

$i+s=j$, $j+l=J$, and $i'+s'=j'$, $j'+l'=J$. We have:

$$H_{l'm_l's'm_s'i'm_i}^{JM} = \sum_{j'm'} C_{l'm_l'j'm'}^{JM} C_{s'm_s'i'm_i}^{j'm'} H_{l's'i'j'}^J, \quad (4)$$

where the C 's are the coefficient common to all problems in which two angular momenta are combined to form a resultant and are defined:

$$C_{lmism_s}^{jm} = \int \psi^*(lsm) \psi(l_mism_s) d\tau. \quad (5)$$

When (4) is introduced (3a), the summations over the spin orientations m_i m_s m_i' m_s' can be carried out at once by virtue of the orthogonality relations for the C 's:

$$\sum_{m_s m_i} C_{sm_s i m_i}^{jm} C_{sm_s i m_i}^{j^* m^*} = \delta_{jj^*} \delta_{mm^*}. \quad (6)$$

We obtain

$$\begin{aligned} A_{pq} = & \sum_{l'l'i'} e^{i(\delta l + \delta l' - \delta l^* - \delta l'^*)} H_{lsij}^J H_{l^*sij}^{*J} H_{l's'i'j'}^{*J} \\ & \times H_{l^*s'i'j'}^J (2l+1)^{\frac{1}{2}} (2l^*+1)^{\frac{1}{2}} \sum_{mMm'l'} C_{lojm}^{JM} C_{l^*ojm}^{JM} \\ & \times C_{l'm_l'j'm'}^{JM} C_{l^*m_l'^*j'm'^*}^{JM} A_{l^*i',p}^{l'} C_{l^*m_l'^*p,q}^{l'm_l'}. \quad (7) \end{aligned}$$

Here we have used the relation

$$\int P_{pq}^* P_{l^*m_l'^*p,q}^* P_{l'm_l'p,q} = A_{l^*i',p}^{l'} C_{l^*m_l'^*p,q}^{l'm_l'}. \quad (8)$$

The summation over the remaining magnetic quantum numbers is effected with the aid of the theorem (cf. Appendix):

$$\sum_{m_l, m} C_{lmijm}^{JM} C_{l^*m_l'^*j'm'^*}^{J^*M^*} C_{lm_l'p,q}^{l^*m_l'^*} = B_{lJ, j'p}^{l^*J^*} C_{JMpq}^{J^*M^*}, \quad (9)$$

giving

$$\begin{aligned} A_{pq} = & \sum_{l'l'i'} [\text{part independent of } p \text{ and } q] \\ & \times A_{l^*i',p}^{l^*} B_{l^*J, j'p}^{l^*J} B_{Jl, j'p}^{Jl^*} C_{lopq}^{l^*o}. \quad (10) \end{aligned}$$

This formula is independent of the coupling and will be used in §3 for discussing qualitative results. Since, however, it involves summations

over j , j' , and the matrix elements will depend on specific nuclear properties, it can in general not be explicitly evaluated. This difficulty is removed if the intrinsic orbital momenta of the initial and final nuclei λ and λ' are both zero, in which case $j=j'=S$ —the intrinsic spin of the compound state.

If Russell-Saunders coupling holds this difficulty can be circumvented if instead of (4) one uses the following:

$$\begin{aligned} H_{lmism_s i m_i}^{JM} = & \sum_{m_\lambda m_\sigma M L M_S} C_{\lambda m_\lambda \sigma m_\sigma}^{i m_i} C_{lm \lambda m_\lambda}^{LM L} \\ & \times C_{sm_s \sigma m_\sigma}^{SM S} C_{LM L SM S}^{JM} H_{lLS}^J. \quad (11) \end{aligned}$$

If (11) is substituted into (3), although no formal summation over the magnetic quantum numbers has been obtained, one can write the results of the averaging in the form

$$A_{pq} = \sum_{l, l' l^* l'^*} F(H) \times f(p, q), \quad (12)$$

where f is determined only by the spatial symmetry and may be calculated by direct summation over the magnetic quantum numbers, and F depends upon the specific nuclear properties but is independent of p , q .

Equations (10), (12) determine the angular distribution completely if JLS and l are known. The first three are determined by the compound level concerned; l is usually fairly restricted by selection rules; at least for all reactions to which the theory will be applied in §4, the selection rules restrict the sum in (12) to one term. Except for the special case $\lambda=\lambda'=0$, the summation over j, j' makes (10) the less useful, although it is slightly more general.

§3. DISCUSSION

We shall base the discussion of the result on Eq. (10). First of all, we note that the distribution is axially symmetric, since the coefficient $C_{lopq}^{l^*o}$ vanishes if $q \neq 0$. This result also follows from more general considerations since after averaging over the spins, the direction of the incident beam is the only characteristic direction.

Second, the distribution must be symmetric about the equatorial plane. This results from the

parity selection rule (I in §1). According to this rule, the outgoing wave can contain either only even or only odd orbital momenta l' . Therefore, l' and l^{*} are either both odd or both even, and from the definition of A in (8) it follows that p must be even, i.e., the angular distribution of the outgoing particles contain only even spherical harmonics, which means symmetry about the equatorial plane.

The condition for the validity of this rule is obvious. Either only one compound level must be involved (resonance disintegration) or, if several levels contribute, they must all have the same parity.

The number of terms in the expression in spherical harmonics (2) is very limited for any given nuclear process. The maximum value of p is determined by the fact that all the coefficients A , B , and C in (10) will vanish if p is greater than the sum of the corresponding upper and lower indices. For example if the particles are incident on s waves, the products are spherically symmetric even though p waves or even higher are present in the outgoing waves. Similarly disintegrations produced by p waves can have only $p=0$ and 2 in the intensity.⁶

Even with an incident wave of nonvanishing l , the angular distribution will be spherically symmetrical if $J=0$, and generally p must be $\leq 2J$. In the case of Russell-Saunders coupling (Eq. (12)) it follows in a similar way that $p \leq 2L$.

The interference between terms of different l , l^* (or l' , l^{*}) is similarly restricted by the fact that $|l-l^*| \leq p$, $|l'-l^{*}| \leq p$.

One of the applications of the angular distributions will be the determination of the quantum numbers of compound states. In discussing actual reactions, it should of course be remembered that slow incident particles will in general only reach the nucleus when they have small orbital momentum l , i.e., H^J_{lLS} is small for large l . On the other hand there is no such restriction for the outgoing particles which are usually fast.

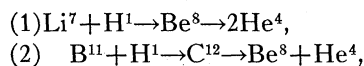
§4. DISCUSSION OF EXPERIMENTAL RESULTS

The distribution has been investigated experimentally by Neuert⁷ for the α -particles of the

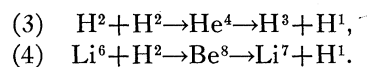
⁶ I wish to thank Professor E. Teller for pointing out these restrictions for the special cases and suggesting their generality.

⁷ Neuert, *Physik. Zeits.* **38**, 122 (1937).

reactions:



and for the protons of the reactions:



In the reactions (1) and (4) the distributions were isotropic, and in (2) and (3) the distributions are equatorially symmetric as is predicted for resonance disintegrations and have distinct minima at 90° . Bethe⁸ has shown qualitatively how one may interpret these results. We shall summarize his arguments.

Reaction (4) is dismissed by noting that at low energies of the incident deuterons only s waves are effective in producing the disintegration and we have already noted this results in isotropy.

Reaction (1) is somewhat more complex. Since the α -particles obey Bose statistics and have no spin their relative motion must be symmetric, hence the parity of the final state is even, since the internal parity of an α -particle is even. Therefore, the initial state must also be even. But the internal parity of Li^7 is probably odd so that the relative motion must be odd and $l=1$. (This relation of $l=1$ may be connected with the observed low probability of this reaction.⁹) We would therefore expect anisotropy unless for the state of the Be^8 we have $J=0$. This value of J is therefore assumed.

The second reaction has been studied by Williams *et al.*,¹⁰ and the excitation function of the homogeneous group of long range alphas, which are the ones for which the angular distribution has been investigated, showed a maximum at 180 Kev with a width of about 10 Kev. This corresponds to an excited level of C^{12} at about 16 Mev. This same level is associated with γ -ray emission, and was tentatively assigned $J=1$, odd parity.¹¹ Because of the rapidly decreasing penetration into the nucleus of the waves with increasing l , one expects that $l=0$ will be the dominant factor and one would find an isotropic distribution, unless some selection rule prevents

⁸ Bethe, *Rev. Mod. Phys.* **9**, 215 (1937).

⁹ Goldhaber, *Proc. Camb. Phil. Soc.* **30**, 561 (1934).

¹⁰ J. H. Williams, W. H. Wells, J. T. Tate, and E. L. Hill, *Phys. Rev.* **51**, 434 (1937).

¹¹ Bethe and Livingston, *Rev. Mod. Phys.* **9**, 276 (1937).

$l=0$ from giving rise to reaction. Since the distribution has been found anisotropic, we shall assume this to be the case. The simplest assumption is that the excited level of C^{12} is even. Then since the parity of the ground state of B^{11} is odd, only odd relative momenta will cause the reaction. Similarly since the parities of both Be^8 and He^4 are even, the relative momenta will be even. Further since both particles have zero spin, the relative momentum will be the same as the total angular momentum of the excited state. $J=0$ would lead to spherical symmetry so we must take $J=2$. We shall then classify the excited state of C^{12} as 1D_2 even. This assignment is not contradictory to the γ -transition since for nuclei dipole and quadrupole transitions are almost equally probable. This assignment is also in agreement with the fact that the product particles must form a singlet state, since both Be^8 and He^4 have zero spin. The ground state of B^{11} is thought to be ${}^2P_{3/2}$, odd. With these assignments and supposing that L - S coupling holds rigorously, we find that the angular distribution is given by

$$I(\theta) = 1 + 3 \cos^2 \theta. \quad (13)$$

This gives a greater anisotropy than that observed experimentally by Neuert. His results seem to be best represented by

$$I(\theta) = 1 + 1.8 \cos^2 \theta. \quad (14)$$

If we return to expression (11) we find that when the spin and orbital momenta are not conserved separately the same assignments would lead to the more general result,

$$I(\theta) = A(1 + 3 \cos^2 \theta) + B \sin^2 \theta, \quad (15)$$

where A and B are both positive constants whose relative values depend upon the strength of the interaction between spin and orbital momenta. We see, however, that the assumption of L - S coupling gives rise to the greatest anisotropy so that a partial relaxation of the spin selection rule would tend to bring the predicted and experimental results into agreement. After the completion of these calculations, Oppenheimer and Serber¹² have investigated the reaction $B^{11} + H' = Be^8 + He^4$ much more thoroughly. They conclude, with the same arguments as given here,

¹² Oppenheimer and Serber, Phys. Rev. **53**, 636 (1938).

that the C^{12} level is even and has probably $J=2$, but they consider 5 different possibilities for this state (${}^1D_2, {}^3P_2, {}^3D_2, {}^3F_2, {}^3F_4$) and 6 possibilities for the ground state of B^{11} (${}^2P_{1/2, 3/2}, {}^2D_{3/2, 5/2}, {}^2F_{5/2, 7/2}$). Of these, only very few give an angular distribution compatible with Neuert's experiments, and none of these permits an explanation of the selection rule forbidding the production of long range alphas by protons with $s=0$ and of the occurrence of capture γ -rays along with the long range alphas.

The third reaction is probably not of the type to which our theory can be applied. Its excitation function has been studied and there is little evidence that it is of resonance type, but is well fitted by the Gamow function. Then too for such light nuclei the Bohr scheme is questionable. Calculations by Schiff¹³ show that direct interaction between the initial and final wave functions already gives a larger cross section than observed. At the same time he has found a much smaller departure from a spherical distribution than is reported experimentally. If, however, one ignores these facts one can carry out the calculations on the basis of this theory. First the action of $l=0$ is excluded by supposing that the excited state has odd parity. Feenberg predicts such a state near the disintegration energy of the alpha-particle. Since the parities of all particles are even, the relative orbital momentum is most probably 1. Because the deuterons obey Bose statistics the odd orbital momentum necessitates a total spin 1 (triplet state). This is compatible with the spins of the outgoing particle ($\frac{1}{2}$ each). The compound state must therefore be a 3P_1 or 3P_2 state ($J=0$ is excluded because of the anisotropy). Since the ground state of all particles considered are S states $\lambda=\lambda'=0$, and Eq. (10) may be used with $j=1, S'=1$, and we obtain for

$$\begin{aligned} J=1: & I(\theta) = 1 + \cos^2 \theta, \\ J=2: & I(\theta) = 21 \cos^2 \theta + 13. \end{aligned} \quad (16)$$

The experimental results of Neuert are represented by

$$I(\theta) = 1 + 0.7 \cos^2 \theta. \quad (17)$$

This agreement would be satisfactory if it were not for the questionable assumptions which underlie the application of a resonance theory to

¹³ Schiff, Phys. Rev. **51**, 783 (1937).

this reaction. This reaction also proceeds in an alternate manner to give neutrons and He³. One would perhaps expect the distribution of neutrons to be the same as that for the protons. This point does not seem to be settled experimentally, for although Kempton, Maasdorf, and Brown report it as such, more recently Oliphant at a conference reported a spherical distribution of the neutrons. If this latter should be the case one must suppose that mechanism of the reaction to be quite different. Two possibilities suggest themselves—that the *s* waves are responsible, or that the state of the compound nucleus has total angular momentum zero, either of which would lead, according to our results, to a spherical distribution. Since the yields of neutrons and protons are

about the same and low, one is inclined to believe that *p* waves are primarily responsible—hence the intermediate state, one of total momentum zero.

It might finally be noted that only those reactions which have low yields would be expected to have an anisotropic distribution, since to have anisotropy *p* waves or those with larger orbital momentum must be primarily responsible. One should therefore investigate the distribution of the products of low yield reactions to find other distributions of interest.

In conclusion I wish to acknowledge the aid and valuable suggestions which Professor H. A. Bethe has given me in connection with this problem.

APPENDIX

In order to carry out the summation we utilized several theorems which will now be established.

The product of two angular functions can, because of the completeness of the system of such functions, be expressed:

$$\psi_{lm_l}(\theta_1)\psi_{sm_s}(\theta_2) = \sum_{jm} A_{lm_lsm_s}^{jm} \psi_{lsm}(\theta), \quad (18)$$

where Θ_1, Θ_2 refer to all the angle variables of ψ_l, ψ_s , respectively, and Θ contains all the variables in both. If we operate on both sides of the expression (18) with the momentum operator¹⁴ ($J_x + iJ_y$) \hbar we get:

$$\begin{aligned} [(l-m_l)(l+m_l+1)]^{\frac{1}{2}} \psi_{lm_l+1}(\theta_1)\psi_{sm_s}(\theta_2) + [(s-m_s)(s+m_s+1)]^{\frac{1}{2}} \psi_{lm_l}(\theta_1)\psi_{sm_s+1}(\theta_2) \\ = \sum_{jm-1} [(j+m)(j-m+1)]^{\frac{1}{2}} A_{lm_lsm_s}^{jm-1} \psi_{jm}(\theta). \end{aligned}$$

If we express the terms on the left in the manner of (18) and equate coefficients we find:

$$[(l-m_l)(l+m_l+1)]^{\frac{1}{2}} A_{lm_l+1sm_s}^{jm} + [(s-m_s)(s+m_s+1)]^{\frac{1}{2}} A_{lm_lsm_s+1}^{jm} = [(j+m)(j-m+1)]^{\frac{1}{2}} A_{lm_lsm_s}^{jm-1}.$$

If this is compared with the recursion relation for the *m* dependence of the C^{15} we find that it is the same so that one may write

$$A_{lm_lsm_s}^{jm} = a_{ls}^j C_{lm_lsm_s}^{jm}.$$

In particular:

$$P_{lm}(\theta, \varphi) P_{l^*m^*}(\theta, \varphi) = \sum_{LM} A_{ll^*}^L C_{lm_l^*m^*}^{LM} P_{LM}(\theta, \varphi)$$

so that

$$\int P_{LM}^* P_{lm} P_{l^*m^*} = A_{ll^*}^L C_{lm_l^*m^*}^{LM}.$$

¹⁴ Condon and Shortley, *Theory of Atomic Spectra*, p. 43.

¹⁵ Reference 14, p. 74.

If we multiply $\psi_{lsjm}(\theta_1, \theta_2) = \sum_{m_s m_l} C_{lm_s m_s}^{jm} \psi_{lm_l}(\theta_1) \psi_{sm_s}(\theta_2)$ by $\psi_{pq}(\theta_1)$.

By using the multiplication theorem, we get :

$$\sum_{l'j'm'} a_{lj, sp}^{l'j'} C_{j m p q}^{j' m'} \psi_{l' s j' m'}(\theta_1, \theta_2) = \sum_{l' m_l' m_l m_s} C_{l m_l m_s}^{jm} b_{lp}^{l'} C_{l m_l p q}^{l' m_l'} \psi_{l' m_l'}(\theta_1) \psi_{sm_s}(\theta_2).$$

If now we multiply by

$$\psi_{l^* s j^* m^*}(\theta_1, \theta_2) = \sum_{m_l m_s} C_{l^* m_l m_s}^{j^* m^*} \psi_{l^* m_l}(\theta_1) \psi_{sm_s}(\theta_2)$$

we get because of orthogonality

$$a_{lj, sp}^{l^* j^*} C_{j m p q}^{j^* m^*} = \sum_{m_l m_s} b_{lp}^{l^*} C_{l m_l m_s}^{jm} C_{l^* m_l^* m_s}^{j^* m^*} C_{l m_l p q}^{l^* m_l^*}.$$

By combining the *b* and *a* we get

$$B_{lj, sp}^{l^* j^*} C_{j m p q}^{j^* m^*} = \sum_{m_l m_s} C_{l m_l m_s}^{jm} C_{l^* m_l^* m_s}^{j^* m^*} C_{l m_l p q}^{l^* m_l^*}.$$

The Static Interaction of Charged Particles

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The energy of the field due to two charges as described by the field equations of the unquantized Born-Infeld theory is calculated to the first approximation in the case of stationary charges not subject to other external fields. The method of representing the field as the superposition of plane waves in a very large cavity is applied to the field equations expanded in a power series in the constant $1/b^2$ of the Born theory. For the first term this leads to the usual Coulomb energy. The process is then extended to

calculate the second term from insertion of the coefficients arising in the first term evaluation. In this case it is found necessary to perform threefold averages in place of the previous single averages, since three of the radiation oscillators describing the field enter into combination. The resulting energy is $e_1 e_2 / r [1 - (1/b^2)(e_1^2 + e_2^2) / 8r^4]$. The nature of the series expansion makes the method invalid in the region near 10^{-13} cm since there the series becomes divergent.

THE derivation of the energy of the field due to point charges from Maxwell's equations suggests that a somewhat similar procedure might be applied to arrive at the corresponding relation for field equations which are slightly different from those of Maxwell. Despite the persistent difficulties arising in the nonclassical, as well as in the classical theories, the belief that some such departure is necessary

seems to be well founded. Whether the alteration should be introduced as a semi-classical concept before quantization, or arise in the process of quantization is not yet clear. In any case, it seems probable that for large fields or small distances the classical equations of Maxwell must be modified by the addition of a small nonlinear part. It is the purpose of the present discussion to calculate the interaction