curves of Pfotzer and Carmichael and Dymond. Because of the disagreement of the two experimental curves at intermediate depths, it has not seemed worth while to reduce them to an absolute scale, instead, their maxima have been made the same height as that of the calculated curve.

Snyder's multiplication curve differs from that of Carlson and Oppenheimer, and ours in turn from Snyder's, by a shift of the maximum towards smaller depths, the maximum becoming at the same time somewhat higher and narrower. It will be seen from Fig. 2 that the position calculated for the maximum now agrees very well with that observed.

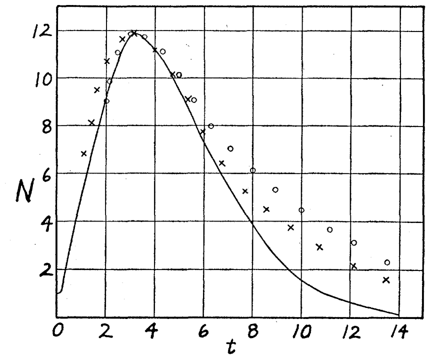

FIG. 2. Multiplication curve for an electron of 7×10^9 v. The circles represent the vertical counter data of Carmichael and Dymond, the crosses those of Pfotzer.

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Scattering and Loss of Energy of Fast Electrons and Positrons in Lead

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A cloud chamber investigation has been made of the scattering and loss of energy in a thin lead lamina (0.13 mm) of fast electrons and positrons (5.0 to 17.0 Mev) produced as secondaries by the gamma-radiation from Li^7+H^1. A comparison of the experimental scattering versus angle in the plane of the chamber with the Mott-Rutherford theory of single scattering shows good agreement above $\theta=13°$. Below this angle the experimental points behave in a manner reasonably consistent with multiple scattering. The dependence of the scattering on energy agrees with the relativistic term $(m_0c^2/W)^2$ in the theoretical scattering expression. Some evidence for an excess scattering at large angles of electrons over positrons has been found. The average loss of energy for two groups of tracks of mean energy 9.0 Mev and 13.5 Mev was found to be 35 Mev/cm and 54 Mev/cm, respectively. These values are roughly 1.5 times the theoretical values, a result in agreement with the findings of Crane and coworkers at Ann Arbor. From the observed scattering it does not seem possible to account completely for this excessive loss of energy on the basis of a longer effective path in the lamina. Two out of 97 tracks in the 9.0 Mev group and nine out of 179 tracks in the 13.5 Mev group lost more than 200 Mev/cm. The small excess over the large radiative losses to be expected theoretically is not statistically significant. No difference in the loss of energy of electrons and positrons was found.

IN connection with the experiments carried out to determine the energy spectrum of the gamma-radiation from lithium bombarded by protons,[1] numerous cloud chamber photographs were secured of the traversal of a thin lead lamina (0.13 mm) by high energy electrons and positrons. We have employed these photographs to investigate the scattering and loss of energy of electrons and positrons in lead; the actual measurements were made by one of us (J. O.).

The cloud chamber and high potential apparatus used in the original experiments have been described in the first reference. Although two stereoscopic photographs were originally taken only the one taken normal to the plane of the cloud chamber was analyzed in these measurements. This simplified the analysis considerably and did not detract significantly from the usefulness of the data in a comparison with theoretical expectations. The second view was employed mainly to check the identification of the incident and emergent portions of a given track.

The normal view was projected onto a screen to size and all those tracks extending from the

[1] Delsasso, Fowler and Lauritsen, Phys. Rev. **51**, 391 (1937).

wall of the chamber nearest the bombarded target to the lead lamina were measured. Thus only those incident tracks falling within 10° of the horizontal plane of the chamber were considered. As these photographs were taken with a magnetic field (2580 gauss) perpendicular to the plane of the chamber, it was possible to determine the energy of a track by a measurement of its radius of curvature. On a large sheet of glossy paper, arcs of circles differing by one-half centimeter radius of curvature were ruled. The paper was then cut along a line passing through the common center of these circles. One edge of each of the two resulting screens was thus normal to the circles drawn on it. These screens were employed to measure the radius of curvature of the incident track, the angle between the normal to the incident track and the lead lamina, the radius of curvature of the emergent track, and the angle in the plane of the cloud chamber between the normals to the incident and emergent tracks. The angles were measured to one degree with a small protractor. From these data the incident and emergent energy, the angle of scattering in the plane, and the length of path in the lead were computed.

Numerous tracks seemed to stop in the lead lamina in the normal view. It was possible to show by use of the other view that many of these merely passed out of the light beam near the lamina. For the stopping of the remainder there is at present no adequate explanation other than that the cloud chamber was not always equally sensitive on the two sides of the lead lamina. We do not feel that sufficient precautions have been taken in these experiments to show definitely the existence of any real stopping in the lead lamina and the results here reported are concerned only with those tracks which emerge from the lead lamina. The stopping if real is definitely not consistent with the observed scattering and loss of energy of the emergent tracks.

Theory of Scattering*

The theory of the scattering of electrons by charged nuclei has been developed on a relativistic quantum mechanical basis by Mott[2] and yields for the probability of scattering into a solid angle $d\Omega$ at the angle θ

$$P(\theta)d\Omega=\tfrac{1}{4}NtZ^2r_0^2((1-\beta^2)/\beta^4)\, f(\theta,\beta,Z)\csc^4\tfrac{1}{2}\theta d\Omega,$$

where N = number of scattering nuclei per cc
t = thickness of scattering lamina
Z = nuclear charge
$r_0=e^2/m_0c^2$
β = velocity of electron/velocity of light
$f(\theta,\beta,Z)=1-\beta^2\sin^2\tfrac{1}{2}\theta\pm\pi\beta Z\alpha\sin\tfrac{1}{2}\theta\cos^2\tfrac{1}{2}\theta+$ terms in $(Z\alpha)^2$
$\alpha=e^2/\hbar c=1/137$.

The term in $Z\alpha$ is positive for electrons and negative for positrons. Champion[3] has found good agreement with this expression for the scattering in nitrogen between 20° and 180° of electrons with $0.83<\beta<0.95$. Neher[4] found the scattering in aluminum of electrons with energies up to 145 kv to be 32 percent greater than the value given by the above expression. For fast electrons passing through in lead, $\beta\to 1$ and $Z=82$, so that

$$f(\theta)=1-\sin^2\tfrac{1}{2}\theta\pm 1.88\sin\tfrac{1}{2}\theta\cos^2\tfrac{1}{2}\theta+\cdots$$

and $$P(\theta)d\Omega=4.4t\Psi f(\theta)\csc^4\tfrac{1}{2}\theta d\Omega,$$

where

$$\Psi=(1-\beta^2)/\beta^4=(1+\eta^2)/\eta^4=1/\beta^2\eta^2$$
$$=(w/(w^2-1))^2\approx(m_0c^2/W)^2 \quad \text{for} \quad m_0c^2\ll W$$

and $\eta=H\rho e/m_0c^2$ = electron momentum in units m_0c

$W=wm_0c^2$ = kinetic plus mass energy of electrons.

In the derivation of this expression the reaction of radiation on the scattering has not been taken into account. The radiation reaction will change the scattering only by an amount of the order of 1/137 and the experiments here described are not critical enough to detect such a small change. The theory of single scattering by a charged

* *Note added in proof:* In the following discussion only the theory of elastic single scattering is treated. In a private communication Professor E. J. Williams has kindly informed us of his most recent computations on plural and multiple scattering. His results make it possible to give a much more complete treatment of the experimental data presented in this paper. A comparison of the data with his results will be given in the near future. In addition, Dr. J. H. Bartlett has informed us that he is now investigating the possibilities of making accurate computations of single scattering by nuclei of large Z.

[2] Mott, Proc. Roy. Soc. **124**, 425 (1929).
[3] Champion, Proc. Roy. Soc. **153**, 353 (1935).
[4] Neher, Phys. Rev. **38**, 1321 (1931). Also see Klarmann and Bothe, Zeits. f. Physik. **101**, 489 (1936) and Skobeltzyn and Stepanowa, Nature **137**, 456 (1936).

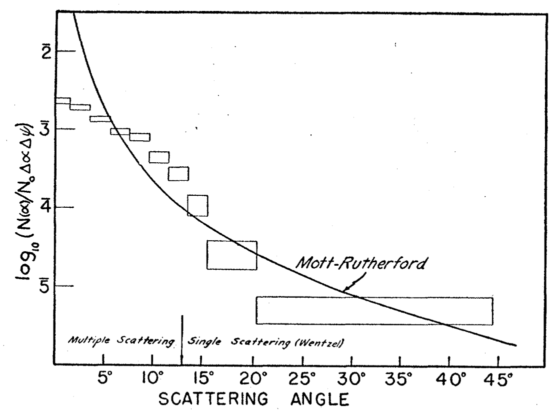

FIG. 1. The scattering in a lead lamina of thickness 0.015 cm of electrons and positrons of average kinetic energy 10.5 Mev as a function of the scattering angle. The vertical spread of the points represents the probable error in the original number of tracks measured in an angle interval denoted by the horizontal spread. The smooth curve represents the Mott-Rutherford theoretical scattering. The quantity $\Delta\alpha$ includes the equivalent angle intervals on both sides of $\alpha=0$.

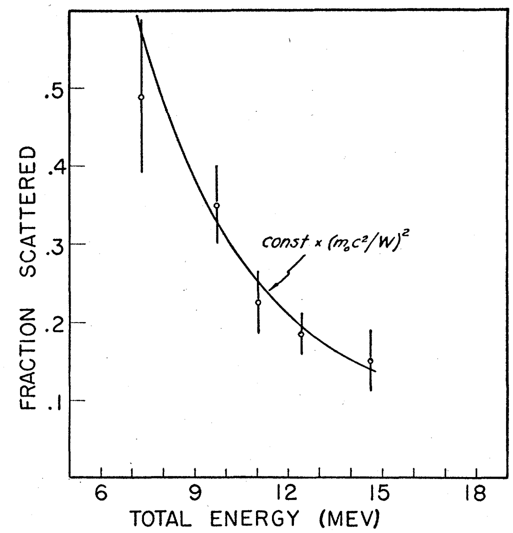

FIG. 2. The fraction of electrons and positrons scattered at an angle greater than 8.5° in the horizontal plane or scattered out of the cloud chamber light beam as a function of energy.

nucleus cannot be strictly compared to the observed scattering because of the screening and scattering by extranuclear electrons and especially because of multiple scattering. The screening effect is small except for very small scattering angles while the scattering by the extranuclear electrons of the lead atom is only 1/82 that due to the lead nucleus. The effect of multiple scattering will be discussed in the comparison with the observed scattering.

The first terms in $f(\theta)$ given above are certainly not accurate for a nucleus with $Z=82$. However, Mott[5] has calculated $f(90°)$ for fast electrons passing through gold and found it to be roughly 3. For angles less than 45°, $f(\theta)$ probably varies between 1.0 and 1.5 for electrons, and between 1.0 and 0.6 for positrons. This is insignificant compared to the variation in the $\csc^4 \frac{1}{2}\theta$ term. It is this essentially classical $\csc^4 \frac{1}{2}\theta$ term as well as the relativistic $(m_0c^2/W)^2$ term which can be tested by our experiments.

In comparing these terms with experiment we have chosen to use the data for electrons and positrons together in order to eliminate statistical fluctuations as much as possible. Although the number of electron tracks is greater than the number of positron tracks, the results should correspond very closely to those to be expected when $f(\theta)$ is assumed equal to unity. The small difference in the scattering of electrons and positrons will be discussed separately.

[5] Mott, Proc. Roy. Soc. **135**, 429 (1932).

THE EXPERIMENTAL SCATTERING

A total of 362 tracks (234 electrons, 128 positrons) with the average of their incident and emergent kinetic energy between 7.5 and 14.5 Mev were analyzed. The mean of this average energy for all the tracks was 10.5 Mev. All of these tracks had emergent lengths greater than 3 cm. An additional 29 tracks (21 electrons, 8 positrons) with incident energy in the same range had emergent track lengths less than 3 cm which precluded accurate scattering or loss of energy measurements. These tracks passed out of the light beam after being scattered in the lead. The light beam was 1.6 cm deep and thus the vertical plane angle of the solid angle in which the scattering was measured was approximately 2 arctan $(0.8/3.0)\approx 30°$. The fraction of tracks scattered into the solid angle bounded by this vertical plane angle and a horizontal plane angle of known interval was found from the measurements. The data so secured cannot immediately be compared with the theoretical expressions as the actual angle of scattering was not measured but only the projection on the horizontal plane. However, the theoretical expression can be used to calculate $Q(\alpha)$, the probability of scattering per unit solid angle with a horizontal plane angle between α and $\alpha+\Delta\alpha$ and a vertical plane angle between $-\frac{1}{2}\Delta\psi$ and $+\frac{1}{2}\Delta\psi$. If $\gamma=\alpha/\Delta\psi$ then for all γ but for α and $\Delta\psi$ not too large we find approximately:

$$\begin{aligned}N(\alpha)/N_0&=Q(\alpha)\Delta\alpha\Delta\psi\\&=P(\alpha)\{\gamma \arctan(1/2\gamma)+2\gamma^2/(1+4\gamma^2)\}\Delta\alpha\Delta\psi,\end{aligned}$$

where $N(\alpha)$ is the number of particles scattered into the interval $\Delta\alpha\Delta\psi$ and N_0 is the number of incident particles. The function in γ is equal to zero for $\gamma=0$ and approaches unity for large γ. In Fig. 1 we have plotted $\log_{10}(N(\alpha)/N_0\Delta\alpha\Delta\psi)$ as a function of α as found experimentally. The vertical spread of the points represents the probable error in the original number of tracks measured in an angle interval denoted by the horizontal spread. The smooth curve represents the corrected Mott-Rutherford expression ($f(\theta)=1$) as given above in which we used the average straight line path length $t=0.015$ as found experimentally and the mean total energy 11.0 Mev. The effects of screening and radiative scattering are too small to be significant in this logarithmic plot.

According to Wentzel's criterion,[6] single scattering should hold for angles greater than 13°. The agreement between the experimental points and the theoretical curve above this angle does not constitute a decisive test of the theory but indicates that the order of magnitude and dependence with angle of Mott's theoretical expression is probably correct. The behavior for small angles is what is reasonably to be attributed to multiple scattering—a lack of tracks at very small angles and an excess before reaching the critical angle for single scattering.

TABLE I. *Comparison of electron and positron scattering.*

	N_0	Number Scattered 8.5° to 14.5°	Number Scattered >14.5°	Ratio Electron/Positron Scattering 8.5° to 14.5° Obs.	8.5° to 14.5° Theor.	>14.5° Obs.	>14.5° Theor.
Electrons	255	36	27	1.2±0.3	1.3	1.6±0.4	>1.5
Positrons	136	16	9				

[6] Wentzel, Ann. d. Physik **69**, 333 (1922).

In order to compare the scattering with energy we have computed the fraction of tracks scattered at an angle greater than 8.5° in the horizontal plane or scattered out of the light beam as a function of energy. This is shown in Fig. 2. The influence of multiple scattering is small above this angle and the increased number of tracks over the number scattered at angles greater than 13° makes the results more reliable. For scattering outside a cone of angle θ, the Mott-Rutherford expression for $W\gg m_0c^2=0.51$ Mev gives

$$N(\theta)/N_0=55t(m_0c^2/W)^2\cot^2\tfrac{1}{2}\theta.$$

We have computed graphically the effective value for $\cot^2\frac{1}{2}\theta$ to be used in this case where the tracks counted lie outside a rectangle subtending 17° horizontally and 30° vertically at the point of scattering and found it to be ~100. A good fit to the experimental points can be secured by the use of the value 150 for $\cot^2\frac{1}{2}\theta$ as shown in Fig. 2. The excess can be quite reasonably attributed to multiple scattering. Fig. 2 indicates that those particles emerging from the lead obey the theoretical scattering dependence on energy (or more fundamentally on momentum) for particles of electronic mass and charge. The agreement is not as good if the particles are assumed to have an $H\rho$ or momentum value given by the curvature and field measurement but a rest mass so much larger than that of electrons that their velocity is appreciably less than that of light. The deviation of theory from experiment becomes marked if the rest mass is assumed to be as high as 5 Mev.

A comparison of the scattering of electrons and positrons should afford a very sensitive test of the quantum-mechanical term in $Z\alpha$ in $f(\theta)$. For not too large angles the ratio of fast electron to fast positron scattering in lead should be

$$f_-/f_+\approx 1+2\pi\beta Z\alpha\sin\tfrac{1}{2}\theta\cos^2\tfrac{1}{2}\theta\approx 1+1.88\theta.$$

TABLE II. *Loss of energy of electrons and positrons.*

Kinetic Energy (Mev) Interval	Kinetic Energy (Mev) Mean	Number of Tracks	Average Energy Loss (Mev/cm) Exp.	Average Energy Loss (Mev/cm) Theor.	Number Losing More than 200 Mev/cm Exp.	Number Losing More than 200 Mev/cm Theor.
6–11	9.0	97	35	26.5	2	1.4
11–16	13.5	179	54	34.5	9	4

This expression will give a minimum value for the ratio of electron to positron scattering at an angle greater than θ and due to the rapid decrease with the $\csc^4 \frac{1}{2}\theta$ term in the scattering law this minimum will be a good approximation to the actual value. In Table I the experimental scattering of electrons and positrons (7.5 to 14.5 Mev) at angles between 8.5° and 14.5° and at angles greater than 14.5° is compared with the theoretical predictions. The existence of an excess electron scattering over positron scattering at large angles is indicated but it must be admitted that the observed numbers are subject to large statistical variations.

Loss of Energy

Because a very thin lead lamina was employed in these experiments the loss of energy of particles traversing the lead could not be accurately measured. In fact the measurements revealed on computation that numerous particles had apparently made small gains in energy. If one assumes the errors of measurement to be symmetrical as to gains and losses, the average loss will still be fairly reliable. The very few particles apparently gaining large amounts of energy were assumed to have actually originated in the far walls of the chamber and to be travelling in the opposite direction to the majority of particles and were not included in the average energy loss calculations. Results for two groups of particles of mean energy 9.0 and 13.5 Mev are given in Table II. The energy loss per cm for both groups is seen to be roughly 1.5 times the theoretical loss computed by adding the collision and radiation losses as given by Heitler.[7] This is in agreement with the original results of Turin and Crane[8] for beta-rays of approximately this energy and is further evidence that high energy beta-rays, Compton recoil electrons and electron or positron members of pairs behave in substantially the same manner in passing through matter. From the observed scattering it does not seem possible to account completely for the excessive loss of energy on the basis of a longer effective path in the lamina. We have also compared the number of tracks losing more than 200 Mev/cm² with the theoretical number expected as computed from

$$P(\delta)=\bar{\delta}E_i(\log\{1-\delta\}),$$

where $P(\delta)$ is the probability of an electron losing a portion of its energy greater than δ when $\bar{\delta}$ is the average fractional energy loss and E_i is the incomplete gamma function of $\log\{1-\delta\}$. This expression is shown to hold for small $\bar{\delta}$ and all δ by Heitler.[7] The theoretical expectations are certainly in agreement with the observed numbers within order of magnitude.

Conclusion

Although some experimental evidence[9] has been found for decided variations from the theoretical expectations for the behavior of fast electrons in passing through matter, the results here reported on the scattering and loss of energy by collisions and radiation certainly cannot be said to disagree with the present relativistic quantum mechanical calculations. It is especially to be emphasized that both scattering and radiation which are closely related even classically seem to be accounted for correctly by the theory. Again the theory of the behavior of fast electrons in passing through matter is fundamentally related to the Klein-Nishina expression[7] for the scattering of gamma-radiation and the Bethe-Heitler-Plesset-Oppenheimer expression[7] for the formation of pairs by gamma-radiation. It has previously been shown in this laboratory[1, 10] that the attenuation coefficients in lead and aluminum of gamma-rays of 6 and 17 Mev are correctly given theoretically. Our results then give no reason for questioning the applicability of calculations based on Dirac's fundamental theory to the behavior of electrons or photons of energies of the order of 10 Mev.

In conclusion we wish to thank Professor C. C. Lauritsen and Dr. L. A. Delsasso (now at Princeton University) in collaboration with whom the original pictures were taken, and Professor Robert Oppenheimer for discussions of the theoretical aspect of these problems, and especially for calling to our attention the possibility of a difference in the scattering of electrons and positrons.

[7] Heitler, *The Quantum Theory of Radiation*, pp. 173, 221.
[8] Turin and Crane, Phys. Rev. **52**, 63 (1937); Phys. Rev. **52**, 610 (1937). See also Ruhlig and Crane, Phys. Rev. **53**, 618 (1938).
[9] Skobeltzyn and Stepanowa, J. de phys. et rad. **6**, 1 (1935); Laslett and Hurst, Phys. Rev. **52**, 1035 (1937).
[10] Delsasso, Fowler and Lauritsen, Phys. Rev. **61**, 527 (1937).