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Transition Effects of Cosmic Rays in the Atmosphere

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The calculation of the multiplication of the soft component of the cosmic rays in the upper atmosphere is refined by use of the exact high energy radiative formulae of Bethe and Heitler, and the resulting diffusion equations are solved by the method of Snyder. Comparison with the vertical counter work of Pfotzer, and of Carmichael and Dymond, and with the ionization chamber data of Bowen, Millikan, and Neher, shows that the calculated position of the maximum of the multiplication curve is now in good agreement with that observed. The latter comparison also indicates that the penetrating component is at least largely of secondary origin.

I

THE general features of the variation of cosmic-ray intensity in the upper atmosphere have been explained by Carlson and Oppenheimer[1] and by Bhabha and Heitler[2] on the basis of the multiplication of high energy electrons and γ-rays in their passage through matter. Since the appearance of these papers, Bowen, Millikan, and Neher[3] have obtained ionization curves at different latitudes which give a fairly good picture of the energy distribution of incoming particles, as well as of the multiplication of particles in definite energy ranges, and Carmichael and Dymond[4] have extended the vertical counter work of Pfotzer.[5] On the other side, Snyder[6] and Landau and Rumer[7] have so far developed the solution of the multiplicative diffusion equations that one can now give quite accurately the theoretical multiplication curve. Since a comparison of the observed and calculated curves may be expected to give valuable information as to the source and production of the penetrating component of the cosmic rays, it seems worth while at the present time to give the results of the theoretical calculations in as precise a form as possible, and to make a preliminary comparison with experiment. The latter is necessarily rather rough, since a detailed interpretation of the ionization chamber work is limited by our lack of knowledge of the angular distribution of the incoming particles, while the counter experiments have so far been subject to such large statistical fluctuations that it has not been worth while to obtain latitude difference-curves.

II

Snyder, in his solution of the diffusion equations, followed Carlson and Oppenheimer in

[1] Carlson and Oppenheimer, Phys. Rev. **51**, 220 (1937).
[2] Bhabha and Heitler, Proc. Roy. Soc. **159**, 432 (1937).
[3] Bowen, Millikan and Neher, Phys. Rev. **52**, 80 (1937); **53**, 217, 855 (1938).
[4] Carmichael and Dymond, Nature **141**, 910 (1938).
[5] Pfotzer, Zeits. f. Physik **102**, 23, 41 (1936).
[6] Snyder, Phys. Rev. **53**, 960 (1938). See also Iwanenko and Sokolow, Phys. Rev. **53**, 910 (1938). An error in writing appeared in Snyder's formulae (the correct formulae were used in his calculations, however). The right-hand side of (23) and (24) should read $s\mu C(y, s-1)$.
[7] Landau and Rumer, Proc. Roy. Soc. **166**, 277 (1938).

using simplified formulae for the differential cross sections for the production of γ-rays and pairs. It has recently been pointed out by Landau and Rumer that the equations are not essentially complicated by introduction of the more exact cross sections given by Bethe and Heitler,[8] since these do not destroy the one feature of the diffusion equations which is exploited in obtaining a solution, namely that, except for ionization terms, they are homogeneous in the energy. The changes in Snyder's treatment entailed by use of the more exact expressions for the cross sections are of a purely formal nature, and it will only be necessary here to summarize the results. The physical basis of the calculation is in no way altered; for a more detailed discussion of the questions involved the reader is referred to Snyder's paper.

The screening constant which appears in the differential cross sections has been recalculated by Professor J. H. Bartlett and we are indebted to him for informing us of his results. He finds, for the probability an electron of energy E_0 will radiate a γ-ray of energy E in a distance dx,

$$P_\gamma dEdx=\frac{4\alpha Z^2r_0^2N}{EE_0^2}\left[\ln\left(\frac{191}{Z^{\frac{1}{3}}}\right)(E_0^2+E_1^2-\tfrac{2}{3}E_0E_1)+\frac{31}{90}E_0E_1\right]dEdx, \quad (1)$$

where $E_1=E_0-E$. The probability a γ-ray of energy E produces an electron of energy E_0 and a positron of energy $E_1=E-E_0$ is

$$P_pdE_0dx=\frac{4\alpha Z^2r_0^2N}{E^3}\left[\ln\left(\frac{191}{Z^{\frac{1}{3}}}\right)(E_0^2+E_1^2+\tfrac{2}{3}E_0E_1)-\frac{31}{90}E_0E_1\right]dE_0dx. \quad (2)$$

If we introduce a new unit of length,

$$t=4\alpha Z^2r_0^2N\ln(191/Z^{\frac{1}{3}})x,$$

(1) and (2) become, for air,

$$P_\gamma dEdt=(E_0^2+E_1^2-\tfrac{3}{5}E_0E_1)dEdt/EE_0^2, \quad (3)$$

$$P_pdE_0dt=(E_0^2+E_1^2+\tfrac{3}{5}E_0E_1)dE_0dt/E^3. \quad (4)$$

[8] See Heitler, *The Quantum Theory of Radiation*, pp. 170, 198.

The ionization energy loss per unit distance may be written

$$dE_{\text{ion}}/dt=\beta. \quad (5)$$

Following Snyder, we can obtain solutions of the diffusion equations which result from (3), (4), and (5) in terms of double contour integrals. Let $N(t,E)$ be the number of charged particles of energy *greater* than E at depth t, and $\gamma(t,E)$ the number of γ-rays of energy E per unit energy at t. The solution which represents an incident electron of energy E_0, accurate to order β/E_0, is

$$N(t,E)=e^{-\sigma t}\partial Z_1(t,E)/\partial t,$$

$$\gamma(t,E)=e^{-\sigma t}Z_2(t,E)/E,$$

$$Z_1(tE)=-\frac{1}{4\pi^2}\int_C\frac{dy}{y}\left(\frac{E}{E_0}\right)^{-y}\int_S ds\frac{\Gamma(-s)\Gamma(y+s)}{\Gamma(y)}\times\left(\frac{E}{\beta}\right)^{-s}\frac{K_\mu(y,s)e^{\mu(y)t}-K_\nu(y,s)e^{\nu(y)t}}{\mu(y)-\nu(y)}. \quad (6)$$

Here

$$\begin{matrix}\mu\\ \nu\end{matrix}=-\tfrac{1}{2}[A(y)-\sigma]\pm\tfrac{1}{2}\{[A(y)-\sigma]^2+4B(y)C(y)\}^{\frac{1}{2}},$$

$$\sigma=23/30,$$

$$A(y)=\frac{7}{5}[\psi(y)+\gamma]-\frac{1}{10}\left[9-\frac{4}{y+1}-\frac{10}{y+2}\right],$$

$$\psi(y)=d\ln\Gamma(y+1)/dy,\quad \gamma=0.577\cdots,$$

$$B(y)=\frac{2}{5}\left[\frac{5}{y+1}-\frac{7}{y+2}+\frac{7}{y+3}\right],$$

$$C(y)=\frac{1}{5}\left[\frac{7}{y}-\frac{7}{y+1}+\frac{5}{y+2}\right].$$

TABLE I.

y	a	k	b	H
0.2	1.916	2.29	2.62	0.290
0.4	1.496	1.138	2.01	0.362
0.6	1.265	0.584	1.74	0.388
0.8	1.122	0.236	1.64	0.399
1.0	1.000	0.000	1.60	0.399
1.2	0.881	−0.175	1.58	0.384
1.4	0.765	−0.301	1.54	0.362
1.6	0.672	−0.397	1.47	0.333
1.8	0.576	−0.470	1.39	0.302
2.0	0.492	−0.525	1.28	0.273
2.2	0.422	−0.567	1.17	0.252
2.4	0.358	−0.604	1.06	0.236
2.6	0.304	−0.629	0.95	0.220
3.0	0.223	−0.667	0.75	0.187

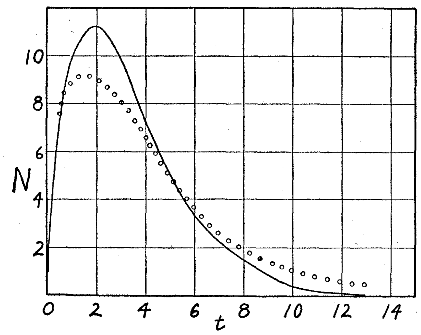

FIG. 1. Multiplication curve for an electron of 11×10^9 v, averaged over all directions of incidence. The circles give the San Antonio-Madras difference curve of Bowen, Millikan and Neher.

The difference equation satisfied by $K_\mu(y, s)$ is

$$\{\mu[A(y+s)-A(y)]-[B(s+y)C(s+y) -B(y)C(y)]\}K_\mu(y, s)=s\mu K_\mu(y, s-1),$$

and the normalization is such that $K_\mu(y, 0)=1$. The corresponding relations for $K_\nu(y, s)$ are obtained by replacing μ by ν.

The function $Z_2(t, E)$ differs from (6) by a factor $(s+y)C(s+y)$ in the integrand.

Snyder has shown that, for $t>\frac{1}{2}$, the total number of charged particles at thickness t is given by a relation of quite simple form:

$$N=He^{kt+y\epsilon}/(1+bt)^{\frac{1}{2}}, \quad t=(\epsilon y-1)/a, \qquad (7)$$

with $\epsilon=\ln(E_0/\beta)$. Here

$$H(y)=K_\mu(y, -y)\mu/(2\pi)^{\frac{1}{2}}(\mu-\nu),$$

$$k=\mu-\sigma, \quad a=-yd\mu/dy, \quad b=y^2d^2\mu/dy^2.$$

The values of k, a, b, and H are given in Table I. For air t is measured in units of 0.39 m water equivalent, $\beta=95$ Mev.

The depth at which the number of particles reaches a maximum is given quite well, for $\epsilon>3$, by

$$t_{\max}=\epsilon-\tfrac{1}{2}(1.6\epsilon+1)/(\epsilon-1). \qquad (8)$$

The maximum number is nearly equal to the number at $y=1$, so we may write

$$N_{\max}=0.4E_0/\beta[1+1.6(\epsilon-1)]^{\frac{1}{2}}. \qquad (9)$$

III

We shall first compare the theoretical multiplication curve with the San Antonio-Madras difference curve of Bowen, Millikan, and Neher. From the energy distribution determined by these authors, we estimate that the mean energy of the incoming particles in the relevant band is about 11×10^9 electron volts. Calculating the number of particles as a function of depth from (7), and averaging over all directions of incidence of the incoming particles, under the rough assumption that their intensity is independent of direction, we obtain the curve shown in Fig. 1. The circles represent the measurements of Bowen, Millikan, and Neher. The scale of the experimental curve is determined from Bowen, Millikan, and Neher's integration of the ionization curve; the areas under the calculated and observed curves are the same. It should be remarked that the ratio of the maximum heights of the two curves is approximately independent of the assumed initial energy, E_0, since the number of incident particles must be taken inversely proportional to the assumed E_0, while the number of particles at the maximum is approximately proportional to E_0, as one sees from (9).

The discrepancy in the curves at larger depths must be attributed to the penetrating component. At 5 m ($t=13$) this discrepancy is a factor 5, at 7 m ($t=18$) a factor 50. The fall of the observed curve below the calculated one in the neighborhood of the maximum is to be expected if γ-rays or electrons are in some part absorbed in the production of a nonmultiplying penetrating component. This interpretation is supported by the fact that at 5 m about 0.4 penetrating particles are present for each incident particle. If these were primary particles the electron multiplication curve would have to be reduced by a factor 0.6, which would lead to definite disagreement with the experimental multiplication factor of 9, This seems a strong argument that the penetrating component cannot be entirely of primary origin, that penetrating particles are produced in the atmosphere.

The vertical counter work of Pfotzer and of Carmichael and Dymond was carried out at high latitudes (Pfotzer, $\lambda=49°$; Carmichael and Dymond, $\lambda=88°$). At these latitudes the mean energy of the incident particles is about 7×10^9 v. In Fig 2 we give the multiplication curve calculated for this energy, and the experimental curves of Pfotzer and Carmichael and Dymond. Because of the disagreement of the two experimental curves at intermediate depths, it has not seemed worth while to reduce them to an absolute scale, instead, their maxima have been made the same height as that of the calculated curve.

Snyder's multiplication curve differs from that of Carlson and Oppenheimer, and ours in turn from Snyder's, by a shift of the maximum towards smaller depths, the maximum becoming at the same time somewhat higher and narrower. It will be seen from Fig. 2 that the position calculated for the maximum now agrees very well with that observed.

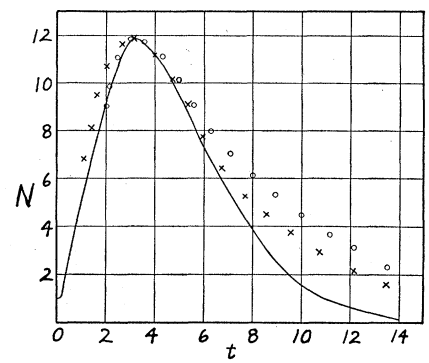

FIG. 2. Multiplication curve for an electron of 7×10^9 v. The circles represent the vertical counter data of Carmichael and Dymond, the crosses those of Pfotzer.

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Scattering and Loss of Energy of Fast Electrons and Positrons in Lead

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A cloud chamber investigation has been made of the scattering and loss of energy in a thin lead lamina (0.13 mm) of fast electrons and positrons (5.0 to 17.0 Mev) produced as secondaries by the gamma-radiation from Li^7+H^1. A comparison of the experimental scattering versus angle in the plane of the chamber with the Mott-Rutherford theory of single scattering shows good agreement above $\theta=13°$. Below this angle the experimental points behave in a manner reasonably consistent with multiple scattering. The dependence of the scattering on energy agrees with the relativistic term $(m_0c^2/W)^2$ in the theoretical scattering expression. Some evidence for an excess scattering at large angles of electrons over positrons has been found. The average loss of energy for two groups of tracks of mean energy 9.0 Mev and 13.5 Mev was found to be 35 Mev/cm and 54 Mev/cm, respectively. These values are roughly 1.5 times the theoretical values, a result in agreement with the findings of Crane and coworkers at Ann Arbor. From the observed scattering it does not seem possible to account completely for this excessive loss of energy on the basis of a longer effective path in the lamina. Two out of 97 tracks in the 9.0 Mev group and nine out of 179 tracks in the 13.5 Mev group lost more than 200 Mev/cm. The small excess over the large radiative losses to be expected theoretically is not statistically significant. No difference in the loss of energy of electrons and positrons was found.

IN connection with the experiments carried out to determine the energy spectrum of the gamma-radiation from lithium bombarded by protons,[1] numerous cloud chamber photographs were secured of the traversal of a thin lead lamina (0.13 mm) by high energy electrons and positrons. We have employed these photographs to investigate the scattering and loss of energy of electrons and positrons in lead; the actual measurements were made by one of us (J. O.).

The cloud chamber and high potential apparatus used in the original experiments have been described in the first reference. Although two stereoscopic photographs were originally taken only the one taken normal to the plane of the cloud chamber was analyzed in these measurements. This simplified the analysis considerably and did not detract significantly from the usefulness of the data in a comparison with theoretical expectations. The second view was employed mainly to check the identification of the incident and emergent portions of a given track.

The normal view was projected onto a screen to size and all those tracks extending from the

[1] Delsasso, Fowler and Lauritsen, Phys. Rev. 51, 391 (1937).