energy production is

$$\epsilon=410\rho c_H{}^2\tau^2e^{-\tau} \text{ ergs/gram sec.} \quad (35)$$

The density at the center of the sun[14] is about 80, the hydrogen content $c_H\approx 0.35$, therefore $\rho c_H{}^2 \sim 10$. Table II gives the energy evolution due to the proton-proton reaction at various temperatures. With a hydrogen content of 35 percent, the central temperature of the sun is about $20\cdot 10^6$ degrees. At this temperature, the energy production is about 2.2 ergs/g sec. This is of the same order as the observed average energy production of the sun (2.0 ergs/g sec.). Thus we come to the conclusion that the *proton-proton combination gives an energy evolution of the right order of magnitude for the sun.*

For a quantitative comparison, it must be remembered that both the temperature and the density of the sun decrease fairly rapidly from the center outwards, and that the rate of reaction decreases with both of these quantities. The average energy production per gram will thus be considerably smaller than that at the center, perhaps by a factor of 10. Since our calculations are rather accurate, it seems that there must be another process contributing somewhat more to the energy evolution in the sun. This is probably the capture of protons by carbon (reference 2).

For many problems, it is necessary to know the temperature dependence of the energy production $\epsilon$. It is convenient to express this dependence as a power law, $\epsilon\sim T^n$. Then (cf. (35))

$$n=d \log \epsilon/d \log T=\tfrac{1}{3}(\tau-2). \quad (36)$$

At 20 million degrees, this gives in our case $n=3.5$. This is large enough to make the point source model of stars a rather good approximation. On the other hand, it is too slow a dependence on temperature to explain, with temperatures of the order $2\text{–}4\cdot 10^7$ degrees, the very high rates of energy production found in very heavy stars. However, we believe that our process is the principal source of energy in stars lighter than the sun.

[14] Stromgren, Ergebn. d. exakt. Naturwiss. **16**, (1937).

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# On the Production of Heavy Electrons*

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The barytron theory of nuclear forces shows a close correspondence to the theory of the electromagnetic field. Estimates based on this analogy are given for the probabilities of the processes leading to actual production of barytrons. A comparison with the experimental evidence regarding the occurrence of barytrons in the cosmic radiation shows that the theoretical cross sections are too small to explain the properties of the hard component. The difficulty is increased by the short lifetime of the barytrons which is estimated to be of the order $\sim 10^{-8}$ sec. from radiative $\beta$-decay. Therefore no simple picture of barytron production in terms similar to radiation theory can be given. This failure, however, indicates only the inapplicability of perturbation calculations, but does not constitute an actual disproof of the link between nuclear forces and cosmic radiation.

## Introduction

At present there are two different lines of evidence for the existence of a new particle, the barytron,[1] of electronic charge (positive and negative) and of a mass $\mu$ about 200 times the electronic mass $m$.

From cosmic-ray investigations there can be little doubt that a large percentage of the hard component, which itself constitutes the greater part of all radiation from sea level downwards, can be neither electrons nor protons[2] but must

* A preliminary report on this paper has been given by L. W. Nordheim and E. Teller at the Washington Meeting of the American Physical Society, April 1938.

[1] This name was proposed at the Washington Meeting of the American Physical Society, April 1938.

[2] Anderson and Neddermeyer, Phys. Rev. **51**, 884 (1937); Street and Stevenson, Phys. Rev. **51**, 1005 (1937).

consist of rays of intermediate mass. An actual determination of the mass[3] from ionization-range-curvature measurements has only been possible for a few rays but they all show, within the rather wide limits of error, masses about 200 m.

The other line is purely theoretical and arises from the attempts to build up a field theory of nuclear forces similar to the connection of the Coulomb law and the electromagnetic field. As is well known such a theory was proposed by Heisenberg and others[4] on the basis of Fermi's theory of $\beta$-decay. According to this theory a heavy particle (proton or neutron) can emit or absorb an electron-neutrino pair much as a charged particle can emit or absorb light quanta. Such a scheme leads to an interaction between heavy particles due to a "playing ball" effect, which implies the emission of the "field particles" (electrons and neutrinos) by one heavy particle and subsequent reabsorption by another. With the $\beta$-field as determined from radioactive decay, however, grave difficulties arose.[5] The interaction between heavy particles is far too small for distances of the known range of nuclear forces ($\sim 10^{-13}$ cm) but diverges at small separations at least as $r^{-5}$ because of the simultaneous action of two transmitting particles. This behavior can be avoided only by rather artificial modifications which cannot be formulated in a consistent and relativistically invariant way.[6]

It was noted by Yukawa[7] as early as 1935, before the experimental evidence on a new particle, that a part of these difficulties could be avoided by assuming the interaction field to be associated with single particles of finite rest mass (i.e. barytrons).

It has been suggested by Yukawa[8] and several other authors[9] that the new cosmic-ray particles are identical with those introduced in the field theory of nuclear forces. In order to see, whether such an interpretation is possible on the basis of the "nuclear force field" theory now under discussion, it is necessary to investigate conditions under which the barytrons can actually be created.

The only method available for this task at the present time is a correspondence treatment on lines similar to those used in ordinary radiation theory. As a logical preliminary step for the discussion of the connection of cosmic-ray phenomena and nuclear forces we have tried therefore:

Firstly, to work out as far as possible the consequences of such a correspondence treatment.

Secondly, to deduce from observational data the cross sections for barytron production necessary to account for their actual occurrence in the atmosphere.

The result so far has been a negative one, i.e., the estimated cross sections come out too small by a factor larger than 10 even in the most favorable case (barytron production by photons).

We show this in Part I by rather general and qualitative arguments which should apply to any field theory which can be treated in the same way as ordinary radiation theory and in Part II for a special form of the nuclear field (the one initially proposed by Yukawa) for which the calculations can be carried out explicitly.

This failure of the correspondence treatment does not imply necessarily that such a connection between nuclear forces and cosmic-ray phenomena is ruled out altogether. It may mean only that it is not legitimate to extrapolate the nuclear field theory in its present state as developed for the low energy phenomena of nuclear forces and $\beta$-decay to the higher energy range of cosmic radiation. In the formalism of the theory such a failure will manifest itself in the occurrence of divergences in the application of perturbation theory. It is known that such divergences already occur in ordinary quantum electrodynamics, though the omission of the divergent terms leads in that case to reasonable results in agreement with experiment. From our investigation it seems then necessary to conclude that this expedient does not work for the nuclear field theory and that it is not possible, therefore,

[3] Compare the discussion by Corson and Brode, Phys. Rev. **53**, 773 (1938); Williams and Pickup, Nature **141**, 836 (1938).

[4] Compare Nordsieck, Phys. Rev. **46**, 234 (1934); Tamm and Iwanenko, Nature **133**, 981 (1934).

[5] Compare f.i. the report by Bethe and Bacher, Rev. Mod. Phys. **8**, 82 (1936).

[6] For such an attempt compare Camp, Phys. Rev. **51**, 1046 (1937) and Nordheim, Nordheim, Oppenheimer and Serber, Phys. Rev. **51**, 1037 (1937).

[7] Yukawa, and others, Proc. Phys. Math. Soc. Japan **17**, 58 (1935); **19**, 1034 (1937); **20**, 1 (1938).

[8] Yukawa, Proc. Phys. Math. Soc. Japan **19**, 712 (1937).

[9] Oppenheimer and Serber, Phys. Rev. **51**, 1113 (1937); Bhabha, Nature **141**, 117 (1938).

to give a simple correspondence picture for the production of barytrons. It is not impossible, however, that the barytron production can be described on lines indicated by Heisenberg[10] in his discussion of field theories containing a fundamental length.

## Part I. Possible Processes and Orders of Magnitude

### Section 1. General properties of barytrons

An interaction between heavy particles due to the exchange of one single particle with finite rest mass gives rise to a potential, the space dependence of which is given by

$$I(r)=(G^2/r)\exp(-r/\lambda\!\!\!^-_0);\quad \lambda\!\!\!^-_0=\hbar c/\mu, \tag{1}$$

where $G$ is a constant of the dimension of a charge and $\lambda\!\!\!^-_0$ the "Compton wave-length" of the barytron.[11] This interaction corresponds to a screened Coulomb field. The proportionality to $r^{-1}$ is the same as in the Coulomb case because only one particle has to be transmitted; the screening is due to the finite rest mass in contrast to light quanta. To understand this factor one might say that it represents essentially the probability distribution of finding a barytron at distance $r$ from a proton or neutron when there is not energy enough actually to create it.

Equation (1) gives the order of magnitude[12] and range for nuclear forces for $\mu\sim M/10$ and

$$A=G^2/\hbar c\sim\mu/M\sim 1/10, \tag{2}$$

where $A$ can be considered as the analog to the fine structure constant $\alpha=1/137=e^2/\hbar c$ in ordinary radiation theory. The fact that $A\ll 1$ gives some hope that a correspondence treatment of the nuclear field has some meaning. It signifies that higher order processes (i.e. those involving a greater number of barytrons) should be of lesser importance than those of lowest order. We shall therefore study only these. In case the value (2) should turn out not to be small enough to ensure this convergence it would be, of course, not possible to derive any conclusions by the standard methods of perturbation theory.

In order to have a simultaneous explanation of radiative $\beta$-decay it is necessary to assume that the barytrons have electronic charge (positive and negative), obey Einstein-Bose statistics, and that they can disintegrate spontaneously into an electron and a neutrino. The lifetime can be estimated in the following way: The probability of finding a barytron in the neighborhood of a neutron-proton pair in a nucleus is of the order 1/10 (ratio of binding energy per particle and rest energy of the barytrons). The lifetime $\tau_N$ of a nucleus subject to $\beta$-decay should be therefore $\sim 1/10$ of the lifetime of a free barytron $\tau_B$ if just the energy is available to create one. Now in Fermi's theory $\tau_N$ is proportional at least to $\epsilon_0^{-5}$, where $\epsilon_0$ is the available energy (maximum energy of electrons). Therefore $\tau_B/\tau_N(\epsilon_0)\sim(\mu/M)(\epsilon_0/\mu)^5$. The lifetime of light elements of $\epsilon_0\sim 5m$ is of the order 100 sec. and therefore the lifetime of a barytron should be $\sim 10^{-8}$ sec. This estimate will be verified for the model discussed in Part II but it apparently should not depend sensitively on special assumptions regarding the mechanism.

The limited lifetime would lead to the conclusion that the barytrons have to be produced as secondaries in the atmosphere itself. As $\tau_B$ is so extremely small (a barytron of energy $10^{10}$ ev $=100\mu$ would only have a free path $l\cong c\cdot\tau_B\cdot 100$ cm $\sim 300$ m) the barytrons must be created very near to the place where they are observed or their connection with the $\beta$-decay has to be given up altogether.

On basis of these conceptions the most probable mechanisms for the actual production of barytrons are discussed in the next two sections.

### Section 2. Production of barytrons by protons or neutrons

This mechanism is quite analogous to the emission of light quanta by charged particles. A proton carrying with it a "nuclear force field" can actually emit barytrons when passing with sufficient energy through another nuclear force field, e.g. of a nucleus at rest. If the theory converges in the sense discussed in Section 1 the

[10] Heisenberg, Zeits. f. Physik **101**, 533 (1936).

[11] We denote the rest energies (i.e., mass $\times c^2$) of electrons, barytrons and protons by $m$, $\mu$, $M$, respectively.

[12] For a short range potential, zero binding energy for the two particles problem (deuteron) results when approximately

$$\int_0^\infty I(r)dr\cong\hbar^2/M; \tag{1a}$$

(comp. Bethe and Bacher, reference 5, p. 109). Inserting (1) into (1a) the relation (2) is obtained. The actual value for $A$ might be slightly larger than $\mu/M$.

emission of a single barytron should be the most probable process and representative of the whole effect.

Because of the resemblance of $I(r)$ to a Coulomb field a very rough estimate of the order of magnitude of this effect can be obtained by a simple "translation" of the ordinary formula of radiative emission. The dictionary is:

| Radiation | Nuclear force field theory |
|---|---|
| $\alpha = e^2/\hbar c = 1/137$; | $A = G^2/\hbar c = \mu/M = \alpha 137\mu/M \sim 1/10$; |
| $r_0 = e^2/m$; | $R_0 = {}^2G/M = r_0(137\mu/M)\cdot\mu/M \sim 0.075(\mu/M)\cdot r_0 \sim 7.5\times 10^{-3} r_0$; |

$$\rho_0 = e^2/\mu = r_0 m/\mu \sim 5\cdot 10^{-3} r_0 \sim R_0.$$

The cross sections of ordinary radiative processes for high energies are of the order of magnitude $Z^2\alpha r_0^2$, where $Z$ is the atomic number. From this we obtain for example for the ratio of the probability $\phi_B$ for the production of a barytron by a proton over the one $\phi_{h\nu}$ for the production of a light quantum by an electron of the same energy as the proton, the order of magnitude

$$\frac{\phi_B}{\phi_{h\nu}} \sim \frac{X}{Z^2}\frac{AR_0^2}{\alpha r_0^2} \sim 0.75\left(\frac{\mu}{M}\right)^3 \frac{X}{Z^2} < 10^{-3}. \quad (3)$$

The factor $X$ is introduced to take care of the composite structure of the nucleus and should be approximately equal to its mass[13] number. The ratio of these probabilities therefore is of the order of $10^{-3}$.

To show the significance of this result we remark that an electron has about an even chance to create a quantum of comparable energy in one unit length of the radiation theory of showers,[14] which is for air $\sim 1/30$ of the whole atmosphere. Therefore a proton will generate approximately only $30\times 10^{-3} \sim 1/30$ barytrons when traversing the whole atmosphere. As the number of protons in the cosmic radiation is certainly much smaller than the number of barytrons, the above emission process cannot explain the occurrence of the latter near sea level so long as the analogy formula has any meaning.

### Section 3. Production of barytrons by light quanta or electrons[15]

A production of barytrons by light quanta or electrons can only be effected by a simultaneous action of the electromagnetic and nuclear-force field. The simplest mechanism of this kind is the direct photoeffect, i.e. a photon is absorbed by a proton and a barytron is reemitted. The process is due to the combined effects of the interactions photon-barytron and barytron-proton (compare Section 6). It follows then from dimensional arguments that the cross section must be of the order $\rho_0 \times R_0 = e^2/\mu \times G^2/M \times F(E/M)$, where $F$ is a homogeneous function of the primary energy $E$ and the rest energies of the particles concerned. If this function is assumed to be of the order unity for high energies[16] the ratio of the cross sections for producing a barytron by a photon over the cross section for the production of a normal electron-positron pair would be of the order of magnitude

$$\frac{\phi_B^{h\nu}}{\phi_{\mathrm{el}}^{h\nu}} \cong \frac{X}{Z^2}\frac{\rho_0 R_0}{\alpha r_0^2} \sim \frac{X}{Z^2}\left(\frac{137m}{M}\right)^2 < 10^{-2}. \quad (4)$$

This is larger than (3) by roughly a factor 10 and as we can expect the presence of many more quanta than protons in the high atmosphere it might offer a more promising picture.

On the assumption that the hard component of the cosmic radiation is produced as a secondary radiation by photons, one can now estimate from observational results how large the ratio (4) has to be in order that enough hard rays are produced. If, in the production process, the whole energy of a quantum is given to the barytron, the total number of barytrons of energy $E$ produced in the atmosphere will be

$$N(E) = \int \phi_B^{h\nu}(E) n(E, x) Y(x) dx, \quad (5)$$

[13] As the range of the nuclear forces is of the same order as the average distance between its constituents one should expect an independent superposition of their action with regard to our emission effect.

[14] Bhabha and Heitler, Proc. Roy. Soc. **A159**, 432 (1937); Carlson and Oppenheimer, Phys. Rev. **51**, 220 (1937).

[15] The hypothesis that the hard component is produced by the soft component in the atmosphere itself has been first put forward by Bowen, Millikan and Neher, Phys. Rev. **53**, 217 (1938).

[16] As we shall see in the later discussion this assumption is subject to grave doubts since, with the present form of the theory, the function $F$ generally decreases with increasing energy.

where $\phi_B{}^{h\nu}(E)$ is the cross section as a function of energy, $n(E, x)$ the number of quanta of energy $E$ as a function of depth $x$ below the top of the atmosphere, and $Y$ the number of atoms per unit volume. If we measure the cross section in units of the cross section of normal pair production and the depth in the unit length $l_0$ of multiplication theory[14] ($l_0 \sim 0.35$ m water equivalent for air) $dl = \phi_{\mathrm{el}}{}^{h\nu} Y dx$, and (5) reduces to

$$N(E) = \int \phi_B(E) n(E, l) dl. \tag{6}$$

The distribution $n(E, x)$ of quanta in the atmosphere can be obtained in sufficient approximation from the analysis of one of the authors.[17] According to the multiplication theory the number of quanta will be slightly larger (by a factor of $\sim 1.5$) than the number of electrons of the same energy and shows approximately the same energy distribution, which is

$$n(E)dE = f(x)E^{-s}dE, \quad \text{for } E > E_0, \tag{7}$$

where $f(x)$ gives the dependence on depth. The exponent $s$ lies between 2 and 3, and $E_0$ is the energy down to which the multiplication of rays extends, i.e. $\sim 1.5 \times 10^8$ ev for air. The form of the distribution is nearly independent of altitude and therefore $f(x)$ is approximately proportional to the total soft intensity in the atmosphere which, in turn, gives the greater part of the intensity in the atmosphere itself. The number of slow electrons below $E_0$ should, furthermore, be of the same order as those above $E_0$ (i.e. those obeying the distribution (7)). Therefore one can say that the number of quanta of energy above $E_0$ at a certain depth will be roughly equal to the total number of soft particles, i.e. to nearly the total measured intensity, and that their energies will roughly follow the same distribution (7). Integrating the intensity distribution (Fig. 3 of reference 17) at the equator[18] over depth one obtains in the units used there 600 particle meters or, as 1 m water equivalent $\sim 3$ unit lengths, about 2000 photon lengths for photons above an energy of $1.5 \times 10^8$ ev. The number $Q(E)$ of photon lengths for quanta of an energy larger than $E$ will be therefore

$$Q(E) = 2000(E_0/E)^{s-1}; \quad E > E_0 = 1.5 \times 10^8 \text{ ev}. \tag{7a}$$

The number of hard particles at sea level[19] is around 7 (in the units used above) and certainly more than half of these, i.e. around 4, have energies larger than $10^9$ ev.

We make now the most favorable assumption that no barytrons get lost in the atmosphere itself by spontaneous disintegration. (This would mean, of course, that the theory of $\beta$-decay, outlined in Section 1, could not be correct.) Then most of the barytrons will come from the upper part of the atmosphere where the intensity of quanta is largest, and will have already lost an energy of 1 to $2 \times 10^9$ ev by ionization. Therefore, only quanta above about 2 to $3 \times 10^9$ ev should be counted. With $s \sim 2.5$ we obtain from (7a) $Q \sim 40$ for $E = 2 \times 10^9$ ev.

This means that such a quantum must have a chance of about 1 in 10 to create a barytron of comparative energy, before it is absorbed through the normal production of an electron pair. This estimate should hold for any mechanism. In case the production cross section depends on energy, the ratio 1/10 would mean an average over all energies, and if the production occurs in multiple processes, the cross section would be smaller in proportion. But it has to be noted, that the actual energy distribution of the hard rays as measured by Blackett[20] demands that

[17] Nordheim, Phys. Rev. **51**, 1110 (1937); **53**, 694 (1938).

[18] Only few of the photons produced by the field sensitive part of the soft primaries would be energetic enough to produce barytrons which could travel down to sea level. This would be consistent with the smallness of the geomagnetic effect at sea level (15 to 20 percent), and it would seem therefore quite possible to account for the features of this effect on the photon hypothesis.

[19] Compare Table III, reference 17. These figures were estimated from experimental data and not deduced from the tentative analysis given there.

[20] Blackett, Proc. Roy. Soc. **A159**, 1 (1937). It is interesting to remark that the energy distribution for the hard rays would be in agreement with the one observed and compatible with the absorption curve of the hard rays underground if the cross section for barytron production is independent of the quantum energy like the cross section of electron pair production. In this case the energy distribution would be the same as the energy distribution of the producing quanta, i.e., of the form (7) which is at least in qualitative agreement with observation as shown in reference 17. According to this hypothesis of the secondary nature of the hard component it is not necessary to assume that the hard rays produce hard secondaries in great amounts. The main cause for absorption of the hard rays will then be again energy loss through ionization and the distribution law (7) will give also in this case an absorption law $E \sim x^{s-1}$ ($x$ = depth) as observed.

also barytrons of very high energies are produced.[21]

The correspondence formula (4) gives for the ratio of barytron production to electron pair production by photons for air a value of about $2\times 10^{-3}$ ($m/M=1850$; $Z^2=50$, $X\sim 20$), which is thus too small by a factor $\sim 50$. It seems, therefore, not to be possible to give a simple account of the barytron production on lines similar to radiation theory.

The discrepancy would still be larger, if the lifetime of the barytron is actually as short as estimated in sections 1 and 4. The connection between the barytrons and radioactive $\beta$-decay is, however, not cogent, and it might be changed by a reformulation of the $\beta$-theory.

## Part II. A Model Theory for Barytron Production

### Section 4. Interaction Hamiltonian and lifetime of barytrons

The considerations of the preceding sections were of a purely qualitative nature and based on simple analogies to the theory of radiation. It remains to be seen whether the characteristic properties of the barytron field (i.e. that it describes the motion of charged particles of finite rest mass having no negative energy states) introduce any features which might invalidate this analogy. For this reason we give in the following sections an actual model theory for the effects in question. For this it is necessary to have an interaction which can be applied to high energy phenomena. This demands that the perturbation developments converge at least in the same way as those in quantum electrodynamics. The only nuclear force field so far proposed which satisfies this requirement is the scalar one,[22] originally introduced by Yukawa. Though this field admittedly does not give satisfactory results for the spin dependence of nuclear forces, it seems to be worth while to study the mechanism of barytron production with a definite model. The result is that the orders of magnitude come near to the estimates of Part I though there are some differences in details.

For the scalar field of Yukawa the interaction with the heavy particles is described by the Hamiltonian

$$\mathcal{H}=2\sqrt{\pi}\,\hbar c\cdot G\int\{\varphi^*(\psi_N{}^*\beta\psi_P)+\varphi(\psi_P{}^*\beta\psi_N)\}dV, \quad (8)$$

where $\psi_N$, $\psi_P$ are the wave functions of a heavy particle in the neutron or proton state. They are assumed to obey Dirac's wave equation so that $(\psi_N{}^*\beta\psi_P)$, etc., are the simplest scalars. $\beta$ is the Dirac operator and $\varphi$ is the scalar wave function of the barytron, which is assumed to obey the Klein-Gordon equation, and is subject to second quantization according to the procedure of Pauli and Weisskopf[23] for Einstein-Bose particles. $G$ is an interaction constant with the dimension of a charge; the other factors are chosen so that the simple form (1) of the potential, without any additional constant, is obtained.

For the quantization of the barytron field following the procedure of Pauli-Weisskopf, a Fourier decomposition can be used.

$$\varphi=\sum q_B\exp\left((i/\hbar c)\mathbf{B}\cdot\mathbf{r}\right);$$
$$q_B=-\frac{i}{(2E_B)^{\frac{1}{2}}}(-a_B+b_B{}^*); \quad (9)$$
$$q_B{}^*=-\frac{i}{(2E_B)^{\frac{1}{2}}}(a_B{}^*-b_B).$$

[21] Similar considerations would apply to the purely electromagnetic production of positive and negative pairs of barytrons by quanta. The cross section for this process is, as well known, for high energies, nearly independent of energy the ratio to normal electron pair production being of the order $(m/\mu)^2$. As two barytrons are created in a single act this ratio had to be of the order 1/20 or $\mu\lesssim 5$ m. For the probable value of $\mu\sim 100$ to 200 m this kind of production can only contribute less than 1 percent of the total. The action of electrons compared to photons will in both cases (mixed electromagnetic-nuclear field and purely electromagnetic production) be smaller by a factor $\sim 1/137$.

[22] An alternative form of the barytron field of vectorial nature and resembling therefore more closely the electromagnetic field, has recently been suggested independently by Yukawa (reference 7), Fröhlich, Heitler, and Kemmer, Proc. Roy. Soc. **A166**, 154 (1938); and Bethe, Phys. Rev. Abstract (1938). The possibilities for the interaction Hamiltonian of the barytron field with heavy particles have been studied in detail by Kemmer, Proc. Roy. Soc. **A166**, 127 (1938). The interaction (8) used here is the same as Kemmer's with his interaction constant $g_a\neq 0$ and all others zero. It seems that a formal perturbation treatment of the emission processes with his generalized interaction would give rise to terms which contain positive powers of the initial energy and which, therefore, would become very large at high energies. On the other hand, the divergences in this theory are far more serious and such formal developments would have no physical meaning. It might be, however, that this lack of convergence gives an indication of qualitatively new phenomena (multiple processes of higher order in the sense of Heisenberg) which might finally bring the solution of the barytron problem, though at present no method exists for treating these questions.

[23] Pauli and Weisskopf, Helv. Phys. Acta **7**, 709 (1934).

Here $E_B$ and $\mathbf{B}$ are the energy and the momentum vector (in energy units i.e. $\mathbf{B}$ = momentum $\times c$) and the quantities $a_B$ etc. stand for the operators

$a_B^*$ = creation of a barytron of positive charge,
$b_B^*$ = creation of a barytron of negative charge,
$a_B$ = annihilation of a barytron of positive charge,
$b_B$ = annihilation of a barytron of negative charge.

From (8) and (9) we obtain the following matrix elements for the change of a proton into a neutron with momentum $\mathbf{N}/c$ and emission of a positively charged barytron

$$\mathcal{H}_{P;\,N+B^+}=\frac{-iG\hbar c(2\pi)^{\frac{1}{2}}}{(E_B)^{\frac{1}{2}}}(u_N{}^*\beta u_P),\qquad \mathbf{P}=\mathbf{N}+\mathbf{B}^+;\quad E_B{}^2=B^2+\mu^2, \tag{10}$$

where $u_P$ and $u_N$ are the four component Dirac amplitudes of the corresponding states (all wave functions are assumed to be normalized per unit volume). The matrix element for the reverse process of absorption of a barytron with negative charge by a neutron is the conjugate complex to (10), and for the process $\mathbf{N}\rightarrow\mathbf{P}+\mathbf{B}^-$, i.e. the emission of a negatively charged barytron by a neutron, the subscripts $P$ and $N$ in (10) have to be exchanged.

From the matrix elements (10) the interaction potential between a proton and a neutron is obtained by the normal second order perturbation formula. For states of motion for which the kinetic energies of the heavy particles are small compared to $\mu$ the $\beta$-brackets in (8) can be replaced by unity, and the energies of the intermediate states are simply the energies of the intermediate barytrons. Positive and negative barytrons give the same contribution, and one obtains therefore for the interaction potential

$$V(r)=2\pi\hbar^2c^2G^2P\cdot 2\int\int\int\frac{\exp\left(-(i/\hbar c)\mathbf{B}\cdot\mathbf{r}\right)d\mathbf{B}}{(2\pi\hbar c)^3(B^2+\mu^2)}=-\frac{G^2}{r}\exp\left(-r\mu/\hbar c\right)\cdot P, \tag{11}$$

where $r$ is the distance between neutron and proton, and the factor $(2\pi\hbar c)^{-3}$ represents the density of barytron states in momentum space. $P$ is an operator which exchanges the proton-neutron character of the heavy particles. Its effect on a function antisymmetrical in the character variables (as must be assumed for an $^3S$ state which is symmetrical in all other variables) is a change of sign, while the sign is preserved for a symmetrical character function ($^1S$ state). Equation (11) gives therefore a Heisenberg force of space dependence (1), but repulsive for the $^3S$ state of the deuteron.[7, 24]

In order to describe the radioactive $\beta$-decay an additional interaction between barytrons and electrons and neutrinos has to be introduced. The simplest possible assumption is, in analogy to (8), that

$$\mathcal{H}_\beta=g\hbar c2\sqrt{\pi}\{\varphi(\psi_n{}^*\beta\psi_{\mathrm{el}})+\text{conjugate complex}\}. \tag{12}$$

which couples the emission of an electron $\psi_{\mathrm{el}}$ and the absorption of a neutrino $\psi_n$ (that is emission of an antineutrino) with the production of a positive or disappearance of a negative barytron and so on. Here $g$ is a new interaction constant also of the dimension of a charge. A second-order perturbation theory over the stages $\mathrm{N}\rightarrow P+B^-$; $B\rightarrow$ electron + antineutrino gives then the formula for the probability of $\beta$-decay as

$$dW=\frac{2}{\pi}\left(\frac{gG}{\hbar c}\right)^2\left(\frac{m}{\mu}\right)^4\frac{(\epsilon_0-\epsilon)^2(\epsilon^2-m^2)^{\frac{1}{2}}\epsilon d\epsilon}{m^5}\cdot\frac{m}{\hbar}, \tag{13}$$

where $\epsilon$ is the energy of an emitted electron and $\epsilon_0$ its maximal possible energy (determined by the total available energy).

This formula is identical with the one derived from a Fermi interaction of the usual form

$$\mathcal{H}_F=G_F\cdot m(\hbar c/m)^3\{(\psi_n{}^*\beta\psi_{\mathrm{el}})(\psi_P{}^*\beta\psi_N)+\text{conj. compl.}\}, \tag{14}$$

where $G_F$ is a dimensionless constant and

$$G_F{}^2=\left(\frac{2\pi gG}{\hbar c}\frac{m^2}{\mu^2}\right)^2.$$

As $G_F$ is of the order[25] $5\times10^{-12}$, we obtain by use of (2)

$$\frac{g^2}{\hbar c}=\frac{1}{4\pi^2}\frac{\hbar c}{G^2}\left(\frac{\mu}{m}\right)^4G_F{}^2\sim10^{-74}.$$

[24] Lamb and Schiff, Phys. Rev. **53**, 651 (1938).
[25] Calculated from the data given by Nordheim and Yost, Phys. Rev. **51**, 992 (1937). For light positron emitters for which the heavy particle matrix element is $\sim1$ one has $(2\pi)^{-3}G_F{}^2\cdot(m/\hbar)=\tau_0{}^{-1}=\tau_f{}^{-1}\sim10^{-4}$.

The decay probability of a free barytron (i.e. its reciprocal lifetime) at rest becomes on the other hand

$$W=\frac{1}{\tau}=\frac{g^2}{\hbar c}\cdot\frac{\mu}{\hbar}=\sim 1.2\left(\frac{\mu}{m}\right)^4; \tag{15}$$

i.e. the lifetime is of the order $10^{-8}$ seconds in agreement with the estimate of Section I.

As another simple effect for barytrons we mention the scattering by a neutron or proton,[26] i.e. the analogy of the Compton effect for light quanta. The total cross section integrated over all scattering angles for this effect is, with the interaction (8), given by

$$\sigma=\pi R_0^2\left\{\frac{(1+2/\gamma)^2}{1+2\gamma}+\frac{2(1+\gamma)}{(1+2\gamma)^2}\right\};\quad \gamma=\frac{E_0}{M}, \tag{16}$$

where $E_0$ is the initial energy of the barytron. For $E_0>M$ the cross section decreases as $M/E_0$ as in the ordinary Compton effect. For $E_0<M$ (analogy to Thomson scattering) one obtains a factor $(M/E_0)^2$ to the geometrical cross section $\pi R_0^2$. This factor is due to the special form (8) of our interaction. In ordinary radiation theory one has instead of the $\beta$-brackets in (8) the Dirac $\alpha$-operators which are of the order $v/c$ where $v$ is the velocity of the charged particles. Expressed in the formalism of perturbation theory the ordinary Compton scattering is due to virtual transitions into negative energy states while with the $\beta$-interaction this is not necessarily so.

As most of the results in this section have already been obtained by Yukawa we have omitted the detailed derivations.

## Section 5. The radiative emission of barytrons by heavy particles

According to the conservation of charge, barytrons can be emitted in collisions of heavy particles according to the schemes given in (17).

$$\begin{aligned}
&\text{I.} && P_0+N_0\rightarrow P_1+P_2+B^-\\
&\text{II.} && N_0+P_0\rightarrow N_1+N_2+B^+\\
&\text{III.} && P_1+P_2\rightarrow P+N+B^+\\
&\text{IV.} && N_1+N_2\rightarrow N+P+B^-.
\end{aligned} \tag{17}$$

The cross sections for these processes will be equal so that we need only to consider the reaction (I). With the matrix elements (10), connecting the change of character of a heavy particle with the emission or absorption of a barytron, the reaction has to go over two successive intermediate states and there exist the six possibilities given in (18).

$$\begin{array}{ccc}
a & b & c\\
P_0\rightarrow N'+B^{+\prime} & P_0\rightarrow N'+B^{+\prime} & P_0\rightarrow N'+B^{+\prime}\\
N_0+B^{+\prime}\rightarrow P_2 & N_0+B^{+\prime}\rightarrow P_1 & N'\rightarrow P_1+B^-\\
N'\rightarrow P_1+B^- & N'\rightarrow P_2+B^- & N_0+B^{+\prime}\rightarrow P_2\\
d & e & f\\
P_0\rightarrow N'+B^{+\prime} & N_0\rightarrow P_2+B^{-\prime} & N_0\rightarrow P_1+B^{-\prime}\\
N'\rightarrow P_2+B^- & P_0+B^{-\prime}\rightarrow N'' & P_0+B^{-\prime}\rightarrow N''\\
N_0+B^{+\prime}\rightarrow P_1 & N''\rightarrow P_1+B^- & N''\rightarrow P_2+B^-
\end{array} \tag{18}$$

So long as the heavy particles can be considered as free the formulae (17) and (18) are directly the equations for the conservation of the respective momenta. The dashes in (18) denote intermediate states. We consider the process from the system of reference in which the neutron is initially at rest, i.e. $\mathbf{N}_0=0$. According to perturbation theory the differential cross section will then be

$$d\phi=\frac{2\pi}{\hbar}\frac{E_{P_0}}{P_0c}|H_{s_0s}|^2\delta(E_s-E_{s_0})d\rho_Bd\rho_{P_1} \tag{19}$$

with $$H_{s_0s}=\sum_{s's''}\frac{H_{s_0s'}H_{s's''}H_{s''s}}{(E_{s_0}-E_{s'})(E_{s'}-E_{s''})}, \tag{19a}$$

where the indices $s_0$, $s'$, $s''$, $s$ denote the initial, two successive intermediate, and the final state of the total system and $d\rho_B$ and $d\rho_{P_1}$ the density functions for the corresponding particles

$$d\rho_B=\frac{\mathbf{B}E_BdE_Bd\Omega_B}{(2\pi\hbar c)^3};\quad d\rho_{P_1}=\frac{\mathbf{P}_1E_{P_1}d\Omega_{P_1}}{(2\pi\hbar c)^3} \tag{20}$$

with $d\Omega_B$ and $d\Omega_{P_1}$ the elements of the solid angles for the barytron and one of the final protons, respectively. The factor $\delta(E_s-E_{s_0})$ denotes the conservation of energy. Only those values of the momenta will contribute to the integral cross section for which the resonance denominators in the sum (19a) are small. We in-

[26] A positive barytron can only be scattered by a neutron and a negative one only by a proton.

vestigate separately the case $A$ of high energy of the oncoming proton, i.e. $E_{P_0} > M$ and case $B$ of small energy i.e. $E_{P_0} < M$.

*A. Relativistic case:* $E_{P_0} > M$.—The smallest resonance denominators are obtained for small values of the momenta $\mathbf{B}^{+\prime}$ and $\mathbf{B}^{-\prime}$ for the intermediate barytron, i.e. for small momenta transfers between the two heavy particles. If we neglect the symmetrization in the final protons, i.e. in $P_1$ and $P_2$ and designate with $P_2$ the particle in which the original neutron $N_0$ goes over, then only the series $a$ and $e$ in (18) contain the corresponding small resonance denominator and need alone to be taken into account.[27] For these terms we have then

$$\begin{aligned} &\mathbf{B}^{+\prime} = \mathbf{P}_0 - \mathbf{P}_1 - \mathbf{B}^- = -\mathbf{B}^{-\prime}; \quad E'_{B^+} = E'_{B^-} \\ &\mathbf{N}' = \mathbf{N}'' = \mathbf{P}_1 + \mathbf{B}^- \sim \mathbf{P}_0. \end{aligned} \tag{21}$$

Since only very small momentum transfers contribute (see the formulae (30) and (31)), $E_{P_2}$ will still be only $\sim M$. Therefore we can neglect differences $E_{P_2} - E_{N_0}$ against $E_B'$ even in the denominators and we have as an expression for the conservation of energy

$$E_{P_0} = E_{P_1} + E_{B^-}. \tag{22}$$

Denoting the energies of the heavy particles from now on by $E_0$, $E_1$, $E'$, $E''$ and those of the barytrons by $E_B = \epsilon$ and $E_B' = \epsilon'$ we obtain from (10), (18), (19a), (21) and (22)

$$\mathcal{H}_{s_0 s} = \frac{i(2\pi)^{\frac{3}{2}} G^3 \hbar^3 c^3}{\epsilon^{\frac{1}{2}} \cdot \epsilon'^2} \left\{ \frac{(u_{N'}{}^* \beta u_{P_0})(u_{P_1}{}^* \beta u_{N'})}{\epsilon' + E' - E_0} - \frac{(u_{N''}{}^* \beta u_{P_0})(u_{P_1}{}^* \beta u_{N''})}{E'' - E_0 - \epsilon'} \right\}. \tag{23}$$

[27] This procedure is allowed as long as $E_{P_1}$ and $E_{P_2}$ are of different order of magnitude, i.e., as long as $E_B - < E_{P_0}$. We have checked the correctness of these and other similar neglections later on by more detailed calculations, which are elementary but tedious and have, therefore, been omitted here.

Summing over positive and negative energy states of the intermediate neutron and the spins of the initial and final states and transforming the resonance denominators

$$\begin{aligned} E'^2 - (E_0 - \epsilon')^2 &= 2(E_0\epsilon' - \mathbf{P}_0 \cdot \mathbf{B} - \tfrac{1}{2}\mu^2) \\ &\cong 2(E_0\epsilon' - \mathbf{P}_0 \cdot \mathbf{B}'), \\ E''^2 - (E_0 + \epsilon')^2 &= 2(E_0\epsilon' + \mathbf{P}_0 \cdot \mathbf{B}' + \tfrac{1}{2}\mu^2) \\ &\cong 2(E_0\epsilon' + \mathbf{P}_0 \cdot \mathbf{B}'), \end{aligned} \tag{24}$$

we obtain for the differential cross section (terms containing $\mathbf{B}'$ in the nominator are neglected)

$$d\phi = \frac{1}{\pi^2} A R_0^2 \frac{P_1}{P_0} \times \frac{M^4 E_0^2 (M^2 + E_0 E_1 - \mathbf{P}_0 \cdot \mathbf{P}_1) \mathbf{B} d\epsilon d\Omega_B d\Omega_{P_1}}{\epsilon'^2 \{E_0^2 E'^2 - (\mathbf{P}_0 \cdot \mathbf{B}')^2\}^2}. \tag{25}$$

This expression contains the dependence on the direction of the final particles through the scalar product $(\mathbf{P}_0 \cdot \mathbf{P}_1)$ in the nominator and through the intermediate momentum $\mathbf{B}'$ and energy $\epsilon'$ in the denominator.

For the integration we introduce the following variables: the vector

$$\mathbf{Q} = \mathbf{P}_1 + \mathbf{B} \tag{26}$$

through its absolute value $Q$ (which determines the opening angle $\theta$ between $\mathbf{P}_1$ and $\mathbf{B}$ with $QdQ = P_1 B \sin\theta d\theta$) and the polar angles $\vartheta$ and $\varphi$ (the latter is cyclic) with respect to the axis $\mathbf{P}_0$, and finally the azimuth $\psi$ of the plane $(P_1 B)$ around $\mathbf{Q}$ ($\psi$ is contained only in $\mathbf{P}_0 \cdot \mathbf{P}_1$). The integration over the azimuths $\varphi$ and $\psi$ leads to

$$\iint (M^2 + E_0 E_1 - \mathbf{P}_0 \cdot \mathbf{P}_1) d\varphi d\psi = 4\pi^2 \{ M^2 + E_0 E_1 - (P_0/2Q)(P_1^2 + Q^2 - B^2) \cos\vartheta \}$$

and we obtain for (25)

$$d\phi = 2AR_0^2 P_0^{-1} d\epsilon \times T,$$

$$T = \iint \frac{E_0^2 M^4 \{2Q(M^2 + E_0 E_1) - P_0(P_1^2 + Q^2 - B^2)\cos\vartheta\} \sin\vartheta d\vartheta dQ}{\epsilon'^2 \{E_0^2 \epsilon'^2 - P_0^2 (P_0 - Q\cos\vartheta)^2\}^2}. \tag{27}$$

With

$$\epsilon'^2 = \mu^2 + B^2 = \mu^2 + P_0^2 + Q^2 - 2P_0 Q \cos\vartheta \tag{28}$$

and the new variables

$$z = 1 - \cos\vartheta \quad \text{and} \quad y = P_0 - Q \tag{29}$$

we obtain (only the lowest powers in $z$ and $y$ which give the chief contribution to the cross section are retained)

$$\begin{aligned} T &\sim \int_\Delta^{y_0} dy \int_0^2 dz \frac{4P_0E_0^2M^6}{(\mu^2+y^2+2P_0^2z)\{E_0^2\mu^2+M^2y^2+2zE_0^2P_0^2\}^2}, \\ &\sim \frac{2M^6}{P_0}\int_\Delta^{y_0}\frac{dy}{y^2}\left\{\frac{1}{E_0^2\mu^2+M^2y^2}+\frac{1}{P_0^2y^2}\lg\frac{\mu^2+y^2}{\mu^2+M^2y^2/E_0^2}\right\}. \end{aligned} \tag{30}$$

The limits for $y$ are

$$\begin{aligned} \Delta &= P_0-(P_1+B) \cong \epsilon\left\{\frac{M^2}{2E_0E_1}+\frac{\epsilon-B}{\epsilon}\right\}, \\ y_0 &= P_0-|P_1-B| \cong \text{smaller of } 2\epsilon \text{ or } 2E_1, \end{aligned} \tag{31}$$

(relativistic approximations and (22) are used). Only the lower limit $\Delta$ is important which signifies the minimum possible momentum transferred to the initial neutron. Carrying out the integration over $y$ we obtain in sufficient approximation as final result

$$d\phi = 16AR_0^2\left(\frac{M}{\mu}\right)^2\left(\frac{M}{E_0}\right)^2 \times \frac{1}{1+(\mu E_0/\epsilon M)^2}\,\frac{E_0-\epsilon}{E_0}\,\frac{d\epsilon}{\epsilon}. \tag{32}$$

This cross section has a maximum as a function of the emitted energy for $\epsilon\sim(\mu/M)E_0$.

It is interesting to compare (32) with the result one would obtain in the limit of vanishing rest mass of the barytron (which would give a pure Coulomb law for the proton-neutron interaction). Putting $\mu\to 0$ in (30) and integrating one obtains

$$d\phi_{\mu\to 0} \sim \frac{32}{3}AR_0^2\frac{E_0}{M}\,\frac{(E_0-\epsilon)^3}{\epsilon^3}\,\frac{d\epsilon}{M}. \tag{33}$$

We see that this would give a much larger effect than the analogous formula of radiation theory and especially a strong divergence for small $\epsilon$. This is apparently due to the used $\beta$-interaction instead of the $\alpha$-operators (see the end of section 4) which increases the cross section enormously. The finite rest mass, however, introduces a very effective limitation for the decrease of the resonance denominators.

By use of the correspondence picture of Weizsaecker-Williams for the description of radiative effects we can explain the factors in (32) as follows: The increase by the factor $(M/\mu)^2$ comes from the choice of the $\beta$-interaction. (For this reason it would perhaps be more logical to introduce $G^2/\mu$ as the critical "barytron radius" in place of $G^2/M$ as in Section 2.) The decrease by the other energy dependent factors is due to the screening expressed by the exponential factor in (1).

Integrating (32) over $\epsilon$ we obtain for the total cross section for the production of a barytron of any energy

$$\phi \sim 16AR_0^2\left(\frac{M}{\mu}\right)^2\left(\frac{M}{E_0}\right)^2\left[\lg\frac{M}{\mu}-1\right] \tag{34}$$

and for the average energy loss of a proton ($N$ = number of atoms per unit volume and $X$ = the effective number of scattering particles in the atom, which is of the order[28] $3Z$,

$$-dE/dx \sim XN8AR_0^2(M/\mu)^2M^2/E_0. \tag{35}$$

For energies from $10^9$ to $10^{10}$ ev (34) is of the same order of magnitude as would follow from the analogy consideration of Section 3.

*B. Nonrelativistic case:* $E_{P_0}<M$.—It is necessary to investigate this case separately because of the large effect of the $\beta$-interaction at small energies. In this case ($E_0<M$) the energies of all heavy particles concerned will be smaller than $M$ and therefore all the $\beta$-brackets in the matrix elements can be replaced by unity. Furthermore all energy differences of the heavy particles in the denominators can be neglected compared to the barytron energies $\epsilon$ and $\epsilon'$, but all six series of the intermediate states (18) have to be taken into account. The evaluation of the cross section is otherwise similar to the relativistic case and leads to the result

$$d\phi \sim 8AR_0^2\left(\frac{M}{\mu}\right)^2\frac{T_0^{\frac{1}{2}}(T_0-\epsilon)^{\frac{3}{2}}}{(2T_0-\epsilon)^2}\,\frac{Bd\epsilon}{\epsilon^2} \tag{36}$$

[28] According to the scheme (17) a proton can emit a barytron in a single way when encountering another proton but in two ways in an encounter with a neutron.

and integrated

$$\phi \sim 2AR_0^2(M/\mu)^2 \lg T_0/\mu, \qquad (37)$$

where $T_0 = P_0^2/2M$ is the kinetic energy of the oncoming particle. We see that the cross section (37) is a smooth continuation of the relativistic case (34) with only a slightly different numerical factor.

To obtain an estimate of the importance of the emission effect investigated in this section we calculate the probability $W$ that a proton creates a barytron when passing through matter before it is stopped entirely by normal ionization. Since the specific energy loss (35) is small compared to the specific energy loss $\gamma$ by ionization, the energy as a function of depth $x$ is

$$E(x) = E_0 - \gamma x,$$

where $E_0$ is the initial energy. We have then with the help of (34)

$$W = XN\int \phi(E(x))dx = XN\int_M^{E_0} \phi(E)\frac{dE}{\gamma} \sim 16XNAR_0^2\left(\frac{M}{\mu}\right)^2 \frac{M}{\gamma}, \qquad (38)$$

if $E_0$ is sufficiently large. Inserting the numerical values for air we obtain

$$W \sim (XM/\mu)\times 10^{-2} \sim 2\times 10^{-2}$$

which as has been discussed in Section 3 is insufficient to explain the occurrence of barytrons in the cosmic radiation.

### Section 6. Barytron production by photons

The simplest reactions of this type are

$$\begin{array}{ll} \text{I.} & k+P \rightarrow N+B^+, \\ \text{II.} & k+N \rightarrow P+B^-, \end{array} \qquad (39)$$

where $k = h\nu$ and $\mathbf{k}$ are energy and momentum (in energy units) of the light quantum. Since both reactions will have the same cross section we consider only (I). The process goes in two different ways over one intermediate state each. The first possibility consists in the conversion of the light quantum into a positive-negative barytron pair and subsequent reabsorption of the negative barytron by the proton. The second consists in the emission of the positive barytron by the proton and subsequent absorption of the light quantum by the barytron. The momentum relations are

$$\text{I.} \begin{array}{l} \mathbf{k} \rightarrow \mathbf{B}^+ + \mathbf{B}^- \\ \mathbf{P} + \mathbf{B}^- \rightarrow \mathbf{N} \end{array} \quad \text{II.} \begin{array}{l} \mathbf{P} \rightarrow \mathbf{B}^+ + \mathbf{N} \\ \mathbf{B}' + \mathbf{k} \rightarrow \mathbf{B}^+ \end{array} \qquad (40)$$
$$\mathbf{B}' = -\mathbf{B}^-$$

The relations between the energy of the emitted barytron and its direction relative to the direction of the initial light quantum (angle $\vartheta$) are very similar to those of the original Compton effect, i.e. in sufficient approximation (if we assume the initial proton to be at rest, i.e. $\mathbf{P}=0$)

$$\begin{array}{ll} \epsilon^+ = \dfrac{k}{1+(k/M)(1-\cos\vartheta)}; & k \gg \mu, \\ \epsilon^+ \sim k; & k < M. \end{array} \qquad (41)$$

The matrix elements for pair production and absorption are according to Pauli and Weisskopf[29]

$$\begin{aligned} \mathcal{H}_{\text{pair}} &= -e\hbar c\left(\frac{\pi}{2k}\right)^{\frac{1}{2}} \frac{\mathbf{e}_k\cdot(\mathbf{B}^+ - \mathbf{B}^-)}{(\epsilon^+\epsilon^-)^{\frac{1}{2}}}; \\ \mathcal{H}_{\text{abs}} &= e\hbar c\left(\frac{\pi}{2k}\right)^{\frac{1}{2}} \frac{\mathbf{e}_k\cdot(\mathbf{B}^+ + \mathbf{B}')}{(\epsilon^+\epsilon')^{\frac{1}{2}}}. \end{aligned} \qquad (42)$$

where $\epsilon^+$ and $\epsilon^- = \epsilon'$ denote the energies of the barytrons and $\mathbf{e}_k$ the unit vector of polarization of the quantum ($\mathbf{e}_k$ is orthogonal to $\mathbf{k}$). The matrix element for the second step is again (10). We have then

$$\mathbf{e}_k\cdot(\mathbf{B}^+ - \mathbf{B}^-) = \mathbf{e}_k\cdot(\mathbf{B}^+ + \mathbf{B}') = 2\mathbf{e}_k\cdot\mathbf{B}^+$$

and the cross section becomes

$$\begin{aligned} d\phi &= \frac{2\pi}{\hbar c}\rho_B\delta(E_s - E_{s_0})\left|\sum \frac{H_{s_0s'}H_{s's}}{E_{s'} - E_{s_0}}\right|^2 \\ &= G^2e^2\delta(E_s - E_{s_0})(\mathbf{e}_k\cdot\mathbf{B}^+)\left|\frac{2\epsilon^-(u_N{}^*\beta u_P)}{(\epsilon^+ - k)^2 - \epsilon^{-2}}\right|^2 \frac{B^+ d\Omega}{k\epsilon^{-2}}. \end{aligned} \qquad (43)$$

[29] Pauli and Weisskopf, reference 23 formulae (49) and (53). It is interesting to note that in (42) the difference $\mathbf{B}^+ - \mathbf{B}^-$ enters (in reference 23 the definition of one of the momenta is reversed). This clearly has to be so as the resultant electric current of the two barytrons is proportional to this difference.

We treat again separately the cases of high and low energy.

*A. Relativistic case:* $k \gg \mu$.—We sum over the spin states of the heavy particle and use the following relations coming from the conservation of energy and momentum (compare (39) and (40))

$$\delta(E_s - E_{s_0})d\Omega$$
$$= 2\pi \frac{d\epsilon^+}{dE}\bigg|_\vartheta \sin\vartheta d\vartheta = 2\pi \frac{E_N B}{Mk}\frac{Md\epsilon^+}{\epsilon^{+2}}, \quad (44)$$
$$\epsilon^{-2} = N^2 + \mu^2 = \mu^2 + (k-\epsilon^+)^2 + 2M(k-\epsilon^+). \quad (45)$$

Then we obtain for the differential cross section with $E_N = M + k - \epsilon^+$

$$d\phi = 2\pi G^2 e^2 \frac{M+E_N}{Mk^2} \cdot \frac{2M^2(k-\epsilon^+)B^4 d\epsilon^+}{k\epsilon^{+3}[\mu^2 + 2M(k-\epsilon^+)]^2}. \quad (46)$$

The total cross section will be then (putting $\epsilon^+ = \epsilon$)

$$\phi = 4\pi R_0\rho_0 \frac{M^2\mu}{k^3}\int_{\epsilon_{nm}}^{k} \frac{[2M+(k-\epsilon)](k-\epsilon)\epsilon d\epsilon}{[\mu^2+2M(k-\epsilon)]^2}. \quad (47)$$

The main contribution to this integral comes from the region $k-\mu<\epsilon<k$ which means that practically always the whole energy is given to the barytron. Carrying out the integration over $\epsilon$ we have with sufficient approximation

$$\phi = \pi R_0\rho_0 F(k),$$
$$F(k) = \frac{\mu M}{k^2}\left\{2 \lg\left(1+\frac{2Mk}{\mu^2}\right) - \frac{9}{4} + \frac{k}{2M}\right\}. \quad (48)$$

This expression contains the factor $R_0\rho_0$ as discussed in Section 3 but the dimensionless factor $F(k) = F(h\nu)$ decreases strongly for high energies similar to the ordinary Compton effect, which again means that the effect cannot account for the number of observed barytrons.

*B. Nonrelativistic case:* $k \sim \mu$.—Here the $\beta$-brackets in (10) are unity,

$$\epsilon^+ \sim k, \quad \frac{d\epsilon^+}{dE} = 1,$$

and the integral cross section becomes after some calculation

$$\phi = 2\pi R_0\rho_0 \frac{M\mu B}{k^3}\left[\frac{k}{B}\lg\frac{k+B}{k-B} - 2\right];$$
$$B^2 = k^2 - \mu^2. \quad (49)$$

This is of the same order as one would expect from the extrapolation of (48) to low energies. For $k\sim\mu$, (49) contains a factor $M/\mu$ in addition to the dimensional expression $R_0\rho_0$. This cross section would actually approach the magnitude required by observation (comp. Section 3) if it were independent of energy.

The authors wish to express their thanks to Professor E. Teller for stimulating discussions on the subject.

*Note added in proof:* A more exact evaluation of the fundamental constant $G$ of the theory has recently been given by Sachs and Goeppert-Mayer (Phys. Rev. **53**, 991 (1938)). They obtain a value $\sim$0.3 for $G^2/\hbar c$ instead of 0.1 as assumed in this paper. The cross sections of the production processes are thereby increased very effectively (especially for the process discussed in Section 3) so that they approach already those required by observation. Furthermore, the convergence of the perturbation calculations becomes doubtful, even for the scalar barytron field. This means that the energy dependence of the cross sections cannot be determined any more. Even if the possibility for a better estimate of the production probabilities seems thus to be still more remote, the prospect for establishing the discussed connection between nuclear forces and the hard component of the cosmic radiation is certainly much better now.