matter is approximately the same as that of the showers observed at sea level. The general features of this conclusion are in agreement with the results of an earlier investigation[5] on shower production under thick layers of various materials. We observed transition effects which indicated that the showers were electronic in character. We suggested that such showers might have their origin in the generation of shower producing radiation (possibly secondary electrons and photons) by the penetrating component. It also follows from these transition curves that some of such secondary particles must have considerable energy.

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The Formation of Deuterons by Proton Combination

H. A. Bethe, *Cornell University, Ithaca, N. Y.*

AND

C. L. Critchfield, *George Washington University, Washington, D. C.*

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The probability of the astrophysically important reaction $H+H=D+\epsilon^+$ is calculated. For the probability of positron emission, Fermi's theory is used. The penetration of the protons through their mutual potential barrier, and the transition probability to the deuteron state, can be calculated exactly, using the known interaction between two protons. The energy evolution due to the reaction is about 2 ergs per gram per second under the conditions prevailing at the center of the sun (density 80, hydrogen content 35 percent by weight, temperature $2\cdot 10^7$ degrees). This is almost but not quite sufficient to explain the observed average energy evolution of the sun (2 ergs/g sec.) because only a small part of the sun has high temperature and density. The reaction rate depends on the temperature approximately as $T^{3.5}$ for temperatures around $2\cdot 10^7$ degrees.

§1. Introduction

It seems now generally accepted that the energy production in most stars is due to nuclear reactions involving light elements. Of all the elements, hydrogen is favored by its large abundance, by its large internal energy which makes a considerable energy evolution possible, and by its small charge and mass which enable it to penetrate easily through nuclear potential barriers. Again, of all reactions involving hydrogen, the most primitive is the combination of two protons to form a deuteron, with positron emission:

$$H+H=D+\epsilon^+. \qquad (1)$$

In fact, this reaction must stand in the beginning of any building up of chemical elements; it has already been discussed in this connection by v. Weizsäcker.[1] However, there seems to be a general belief that reaction (1) is too rare to account for any appreciable fraction of the energy production in stars and that it can serve only to *start* the evolution of elements in a star which will then be carried on by other, more probable, processes. It is the purpose of this paper to show that this belief is unfounded but that reaction (1) gives an energy evolution of the correct order of magnitude for the sun.

On the other hand, we do not want to imply that reaction (1) is the only important source of energy. An analysis of all possible nuclear reactions with light elements[2] shows that the capture of protons by carbon and nitrogen will also play an important role. It is more important

[1] v. Weizsäcker, Physik. Zeits. **38**, 176 (1937).
[2] Bethe, to appear shortly in the Physical Review.

than (1) for heavy stars, less important for light ones and about equally important for the sun.

In calculating the energy evolution from reaction (1), it must be considered that (1) is followed by a number of other reactions which are all "fast" in comparison with (1) because they involve the emission of radiation or of heavy particles rather than of β-rays.[3] The deuterons formed in reaction (1) will first capture another proton, with γ-ray emission:

$$D+H=He^3+\gamma. \tag{2}$$

The ratio of the rates of reactions (2) and (1) is about 10^{18} which means that, in equilibrium, there will be about one deuteron for 10^{18} protons. This makes collisions between two deuterons very improbable and thus excludes any appreciable production of neutrons by such collisions.

The fate of the He^3 formed in (2) depends to some extent on the question whether this nucleus or H^3 is more stable. The most recent determination[4] of the energy evolution in the reaction $H^2+H^2=He^3+n^1$ seems to show that He^3 is more stable while older determinations[5] gave the opposite result. The processes which occur are:

If He^3 is more stable:

$$\begin{aligned} He^3+He^4&=Be^7+\gamma,\\ Be^7&=Li^7+\epsilon^+,\\ Li^7+H&=2He^4. \end{aligned} \tag{3}$$

If H^3 is more stable:

$$\begin{aligned} He^3+\epsilon^-&=H^3,\\ H^3+H&=He^4+\gamma. \end{aligned} \tag{4}$$

The net effect is in both cases the same, *viz.* the combination of four protons and two electrons to form an α-particle. (The emission of a positron is equivalent to the consumption of an electron since the positron will ultimately annihilate an electron.) One α-particle is formed for each process (1). The energy produced per process (1) is therefore equal to the difference in weight between four hydrogen atoms and one helium atom, *viz.* (cf. reference 5, 6) $Q=4\cdot 1.008\,13-4.003\,86=0.028\,66$ mass units, i.e.,

$$Q=4.3\cdot 10^{-5}\text{ erg}. \tag{5}$$

While the energy evolution in (3) and (4) is exactly the same, the processes lead to the formation of different intermediate products which could be of influence on the building up of heavier elements. However, it can be shown[2] that the building up of elements heavier than He^4 is negligible under any circumstances so that the question of stability of He^3 or H^3 is actually not important.

In order to calculate the probability of the proton combination, the following factors have to be considered:

(1) The probability of collision of two protons which involves the penetrability of their mutual potential barrier. This can be calculated very accurately since the force between two protons is very well known from scattering experiments.[7]

(2) The probability of emission of a positron during the collision. This involves a theory of the β-decay. All available experimental evidence points to the conclusion that the original Fermi theory gives good agreement with the experimental dependence of lifetime on energy.

(3) The energy distribution of the protons in the star which is given by the Boltzmann law.

§2. The Probability of Positron Emission

According to Fermi's theory, the probability of β-emission (per second) is

$$\beta=gf(W)|G|^2. \tag{6}$$

Here G is the matrix element of the nuclear transition,

$$G=\int\psi_i\psi_f d\tau, \tag{7}$$

ψ_i and ψ_f the initial and final state of the nucleus, except for the substitution of one neutron by a proton. f is a function of the energy W of the emitted β-particle,

$$f(W)=(W^2-1)^{\frac{1}{2}}\left(\frac{1}{30}W^4-\frac{3}{20}W^2-\frac{2}{15}\right)+\frac{1}{4}W\log\left[W+(W^2-1)^{\frac{1}{2}}\right], \tag{8}$$

[3] Cf. Bethe, Rev. Mod. Phys. **9**, 188, § 76C (1937).

[4] Bonner, Phys. Rev. **53**, 711 (1938).

[5] See Livingston and Bethe, Rev. Mod. Phys. **9**, 373 (1937).

[6] Bainbridge, Phys. Rev. **53**, 922A (1938).

[7] Tuve, Heydenburg and Hafstad, Phys. Rev. **50**, 806 (1936).

where W is expressed in units of mc^2 and includes the rest mass. A lower limit for the Fermi constant g can be deduced by assuming that $G=1$ for transformations such as $N^{13}\rightarrow C^{13}+\epsilon^+$. This reaction has a half-life of 11.0 minutes and an upper limit of the β-spectrum (observed) of about 1.25 Mev, therefore $\beta=\log 2/660=1.05\cdot 10^{-3}$ sec.$^{-1}$, $W=3.5$, $f(W)=11.86$ and

$$g=0.9\cdot 10^{-4}\ \text{sec.}^{-1}. \tag{9}$$

The (maximum) energy of the positron emitted in the combination of two protons follows from the very accurately known masses of proton and deuteron,[5] *viz.:*

Two protons	$=2\cdot(1.008\,13-0.000\,55)$	$=2.015\,16$
Deuteron	$=\quad 2.014\,73-0.000\,55$	$=2.014\,18$
Positron energy, including mc^2		$0.000\,98$

(mass units), equivalent to $1830\cdot 0.000\,98\ mc^2$
$=1.80\ mc^2$.

For $W=1.8$, we have $f(W)=0.132$ and therefore, with (6) and (9),

$$\beta=1.2\cdot 10^{-5}|G|^2\ \text{sec.}^{-1}. \tag{10}$$

A similar value is obtained by extrapolating the empirical Sargent curves to lower energy. β would be larger, if the matrix element G for the transformation $N^{13}\rightarrow C^{13}$ is less than one. On the other hand, the Konopinski-Uhlenbeck theory would give about 10 times less for the decay constant. However, that theory seems to give too strong a dependence of decay constant on energy throughout, and will therefore not be used.

In calculating the matrix element G for the proton combination (1), it must be considered that the wave function of the deuteron is symmetric with respect to interchange of space and spin coordinates of proton and neutron while the wave function of two protons is antisymmetric.[8] In particular, when the two protons are in an S state (which is most favorable for their coming close together), their spins will be antiparallel (singlet) whereas the ground state of the deuteron is a triplet S state. The transition is therefore allowed only if the Gamow-Teller form[9] of the β-theory is used. This formulation permits a change of direction of the spin of the proton which transforms into a neutron, and therefore a change of the total spin by one unit. Strong evidence for this theory is found[10] in the short life of He^6; the β-transformation $He^6\rightarrow Li^6$ leads very probably from a ^1S to a ^3S state and is closely analogous to our process $2H\rightarrow D$. We shall therefore accept the Gamow-Teller theory in this paper. Moreover, we shall assume that the matrix element G in (10) can be calculated simply by integrating over the spatial coordinates of the two particles, i.e., that the summation over spins which is also implied in G gives unity. This assumption does not seem serious in view of the uncertainty in the numerical factor in (10).

If the original Fermi theory were taken instead of the Gamow-Teller theory, the transition $H+H=D+\epsilon^+$ could occur only by virtue of the "small" components of the Fermi interaction, *viz.* $\boldsymbol{\alpha}_{\text{heavy particle}}\cdot\boldsymbol{\alpha}_{\text{electron-neutrino}}$. Since the Dirac operator $\boldsymbol{\alpha}_{\text{heavy}}$ (velocity) is "odd," its application on the ^3S wave function of the deuteron ground state will give a function of ^3P character. This can combine with the ^3P part of the wave function of the incident protons. Due to the smallness of the latter and of the α-operator itself, the transition probability will be greatly reduced, *viz.*, approximately by a factor

$$(W/I)^2(E/Mc^2)\approx 10^{-5},$$

where $W=1.8\ mc^2$ is the energy evolution in the process, $I=4.4\ mc^2$ the binding energy of the deuteron, $E\approx 50$ kev the kinetic energy of the protons and $Mc^2=931$ Mev. In this case, then, the energy evolution in the sun due to proton combination would be negligibly small.

§3. The Collision Cross Section

According to the general principles of quantum theory, the cross section σ for the combination of two protons of relative velocity v is given by (6), (7) if we insert for ψ_i the wave function of two protons normalized to unit incident current. Thus we have

$$\sigma=gf(W)v^{-1}\left|\int\psi_p\psi_d d\tau\right|^2, \tag{11}$$

where ψ_d is the wave function of the ground state of the deuteron and ψ_p is normalized per unit

[8] We are indebted to Professor Oppenheimer for drawing our attention to this point.

[9] Gamow and Teller, Phys. Rev. **49**, 895 (1936).

[10] Goldhaber, Phys. Rev. (to be published).

density at infinity. The integration goes over the space of the relative coordinates of the two protons (or proton and neutron, in the deuteron).

If we assume a square potential well of radius r_0 and depth V_0, the normalized wave function of the deuteron is[11]

$$\psi_d = \begin{matrix} (B/r)\exp(-(x-x_0)) & \text{for} \quad r>r_0 \\ (B/r)\sin\mu x/\sin\mu x_0 & \text{for} \quad r<r_0 \end{matrix} \quad (12)$$

with the abbreviations

$$x=r/b, \quad x_0=r_0/b, \quad (12a)$$

$$b=\hbar(M\epsilon)^{-\frac{1}{2}}=4.37\cdot 10^{-13}\text{ cm}, \quad (12b)$$

$$\mu=(V_0-\epsilon/\epsilon)^{\frac{1}{2}}, \quad (12c)$$

$$B=(2\pi b)^{-\frac{1}{2}}(1+x_0)^{-\frac{1}{2}}(1+\mu^{-2})^{-\frac{1}{2}}. \quad (12d)$$

M is the proton mass, ϵ the binding energy of the deuteron, i.e., 2.17 Mev. Depth and width of the well are related by the condition

$$\mu\cot\mu x_0=-1. \quad (12e)$$

The wave function of two protons, normalized to unit density, has the form e^{ikz} at large distances. (For considerations of symmetry, see §4.) It can be expanded in spherical harmonics and, at the small velocities prevailing in stars (~ 10 kilovolts), only the zero order term will be important. It has, at large distances, the form

$$\psi_p=\sin(\rho-\varphi)/\rho, \quad (13)$$

where $\rho=kr, \quad k=Mv/2\hbar \quad (13a)$

and φ is a phase (depending logarithmically on ρ). At small distances, i.e., inside the nuclear potential well, we have

$$\psi_p=\frac{w}{r}=\frac{A}{r}\sin\frac{(MD)^{\frac{1}{2}}}{\hbar}r, \quad (14)$$

where D is the depth of the potential well between two protons, assumed "square" and of radius r_0, A is a normalization factor. Outside of the well, ψ_p is a solution of the Schroedinger equation in the Coulomb field which may be written[12]

$$\psi_p=e^{iK}\cos K(F+G\tan K)/kr. \quad (15)$$

Here F is the regular and G the irregular solution of the Coulomb equation which behave asymptotically as $\sin(\rho-\varphi_0)$ and $\cos(\rho-\varphi_0)$, respectively. As can be seen easily, (15) goes over asymptotically into (13). K is the phase shift due to the nuclear field, fixed by the condition of continuity of (14), (15) at $r=r_0$; we have[12]

$$\tan K=F^2\delta/[1-FG\delta] \quad (16)$$

with $$\delta=\left(\frac{d\log F}{d\rho}-\frac{d\log w}{d\rho}\right)_{r0}. \quad (16a)$$

For the small proton energies concerned, $F(r_0)$ is very small (see below), both because of the Coulomb barrier and the small value of k. Therefore K will be very small ($K=0.0017$ for $v=e^2/\hbar$, i.e. for a relative kinetic energy of 12.5 kev which gives the largest contribution to the cross section at a temperature of $5\cdot 10^7$ degrees) and $e^{iK}\cos K$ (cf. (15)) can be replaced by unity. Then the factor A in the internal wave function (14) is

$$A=\frac{F(r_0)+G(r_0)\tan K}{k\sin(MD)^{\frac{1}{2}}r_0/\hbar}. \quad (17)$$

According to Yost, Wheeler and Breit,[12] the Coulomb wave functions may be written (for orbital momentum $L=0$)

$$F=C\rho\Phi, \quad G=C^{-1}\Theta, \quad (18)$$

where

$$C=(2\pi\eta)^{\frac{1}{2}}e^{-\pi\eta}, \quad \eta=e^2/\hbar v \quad (19)$$

contains the effect of the Coulomb barrier, and Φ and Θ are slowly varying functions of r which have the value 1 for $r=0$. Inserting into (16), (15) we get

$$\tan K=C^2kr_0\lambda, \quad (20)$$

$$\lambda=\Phi^2\zeta/(1-\Phi\Theta\zeta), \quad (20a)$$

$$\rho=\left(\frac{d\log F/w}{d\log\rho}\right)_{r0} =\left(1+r\frac{d\log\Phi}{dr}-r\frac{d\log w}{dr}\right)_{r0}, \quad (20b)$$

$$\psi_p=C[\Phi(r)+\lambda(r_0/r)\Theta(r)] \quad (r>r_0), \quad (21)$$

$$\psi_p=C\frac{r_0}{r}[\Phi(r_0)+\lambda\Theta(r_0)] \times\frac{\sin\nu r/b}{\sin\nu r_0/b} \quad (r<r_0). \quad (22)$$

[11] Bethe and Bacher, Rev. Mod. Phys. **8**, 110 (1936).
[12] Yost, Wheeler and Breit, Phys. Rev. **49**, 174 (1936); J. Terr. Mag. **40**, 443 (1935).

In the last expression, we have introduced the abbreviation

$$\nu=(MD)^{\frac{1}{2}}\hbar^{-1}b=(D/\epsilon)^{\frac{1}{2}}, \qquad (22a)$$

which will be useful for the integration. λ represents the effect of the resonance due to the nuclear potential well.

For slow protons, the wave functions at small r are almost independent of the proton energy and have the form[12]

$$\Phi(y)=y^{-\frac{1}{2}}I_1(2y^{\frac{1}{2}})$$

$$=1+\frac{y}{1!2!}+\frac{y^2}{2!3!}+\frac{y^3}{3!4!}+\cdots, \qquad (23)$$

$$\Theta(y)=-2y^{\frac{1}{2}}K_1(2y^{\frac{1}{2}})=1+y(\log y+2\gamma-1)\Phi$$

$$-\sum_{s=1}^{\infty}\frac{y^{s+1}}{s!s+1!}\sum_{t=1}^{s}\left(\frac{1}{t}+\frac{1}{t+1}\right) \qquad (23a)$$

with

$$y=2r/a, \quad a=2\hbar^2/Me^2=5.75\cdot 10^{-12}\text{ cm}, \qquad (23b)$$
$$\gamma=0.577\cdots \quad \text{(Euler's constant)}.$$

The deviations from (23), (23a) for $\eta=1$ are of the order of a percent.

Collecting our formulae, we may now write for the cross section (11)

$$\sigma=\frac{gf(W)}{v}C^2\frac{(4\pi b^2)^2}{2\pi b}\Lambda^2, \qquad (24)$$

where

$$(1+x_0)^{\frac{1}{2}}(1+\mu^{-2})^{\frac{1}{2}}\Lambda=\Lambda_1+\Lambda_2+\Lambda_3, \qquad (25)$$

$$\Lambda_1=\frac{x_0[\Phi(r_0)+\lambda\Theta(r_0)]}{\sin\mu x_0\sin\nu x_0}\int_0^{x_0}\sin\mu x\sin\nu x dx, \qquad (25a)$$

$$\Lambda_2=\int_{x_0}^{\infty}xdx\exp\left(-(x-x_0)\right)\Phi(2bx/a), \qquad (25b)$$

$$\Lambda_3=\lambda x_0\int_{x_0}^{\infty}dx\exp\left(-(x-x_0)\right)\Theta(2bx/a). \qquad (25c)$$

The integrations are elementary and give

$$\Lambda_1=\frac{\frac{1}{2}x_0^2[\Phi(r_0)+\lambda\Theta(r_0)]}{\sin\mu x_0\sin\nu x_0}$$

$$\times\left[\frac{\sin(\mu-\nu)x_0}{(\mu-\nu)x_0}-\frac{\sin(\mu+\nu)x_0}{(\mu+\nu)x_0}\right], \qquad (26a)$$

$$\Lambda_2=1+x_0+(2b/a)(1+x_0+\tfrac{1}{2}x_0^2)$$
$$+\tfrac{1}{2}(2b/a)^2(1+x_0+\tfrac{1}{2}x_0^2+\tfrac{1}{6}x_0^3)+\cdots, \qquad (26b)$$

$$\Lambda_3=\lambda x_0\{1+(2b/a)[\log(2b/a)+2\gamma-1]\Lambda_2$$
$$-[(2b/a)^2\tfrac{3}{2}(1+x_0+\tfrac{1}{2}x_0^2)$$
$$+\tfrac{1}{2}(2b/a)^3(7/3)(1+x_0+\tfrac{1}{2}x_0^2+\tfrac{1}{6}x_0^3)$$
$$+\cdots]+\sum_{s=1}^{\infty}(2b/a)^s f_s(x_0)/(s-1)! \qquad (26c)$$

with

$$f_s(x_0)=s!^{-1}\int_{x_0}^{\infty}\exp\left(-(x-x_0)\right)x^s\log x dx, \qquad (26d)$$

$$f_1=1+(1+x_0)\log x_0-E_i(-x_0)\exp(x_0), \qquad (26e)$$

$$f_2=\tfrac{3}{2}+\tfrac{1}{2}x_0+(1+x_0+\tfrac{1}{2}x_0^2)\log x_0$$
$$-E_i(-x_0)\exp(x_0), \qquad (26f)$$

$$f_3=(11/6)+(5/6)x_0+\tfrac{1}{6}x_0^2$$
$$+(1+x_0+\tfrac{1}{2}x_0^2+\tfrac{1}{6}x_0^3)\log x_0$$
$$-E_i(-x_0)\exp(x_0). \qquad (26g)$$

Table I gives the numerical results for two values of the radius r_0 of the potential well, *viz.* e^2/mc^2 and $e^2/2\,mc^2$. The depth D of the proton-proton well was taken from Breit, Condon and Present.[13] All other quantities were calculated from the formulae in the text. It is seen that the contribution of the inside of the potential well, Λ_1, is rather small even for the larger r_0, which shows that the result will not depend sensitively on the shape of the potential well.

TABLE I. *Numerical results for two values of the radius.*

	$r_0=e^2/mc^2$	$r_0=e^2/2\,mc^2$
x_0	0.645	0.322
V_0 (Mev)	20.9	66.5
D (Mev)	10.3	47.0
μ	2.94	5.45
ν	2.18	4.65
$(rd\log w/dr)$	0.236	0.110
$\Phi(r_0)$	1.050	1.025
$\Theta(r_0)$	0.769	0.854
$(rd\log\Phi/dr)$	0.050	0.025
ζ	0.814	0.915
λ	2.63	4.80
Λ_1	0.689	0.277
Λ_2	1.949	1.547
Λ_3	1.205	1.030
$(1+x_0)(1+\mu^{-2})$	1.835	1.367
Λ	2.84	2.44
Λ^2	8.08	5.93

[13] Breit, Condon and Present, Phys. Rev. **50**, 825 (1936).

Furthermore, the contribution Λ_3 which is due to resonance is smaller than Λ_2 which would be present even if there were only the Coulomb field between the two protons. The final result for Λ^2 increases somewhat with increasing radius of the well. For $\eta=1$, a numerical calculation gave $\Lambda^2=8.8$ instead of 8.1.

§4. Symmetry, Statistics, etc.

If N is the number of protons per cm^3, the probability of finding one proton with velocity $\mathbf{v}_1$ and one with $\mathbf{v}_2$, is

$$N^2\varphi(\mathbf{v}_1)\varphi(\mathbf{v}_2)d\mathbf{v}_1 d\mathbf{v}_2, \tag{27}$$

where φ is the Boltzmann distribution function. In calculating the total probability of our process, we must integrate over each pair of volume elements $d\mathbf{v}_1 d\mathbf{v}_2$ only once; if we want to integrate over $\mathbf{v}_1$ and $\mathbf{v}_2$ independently, (27) must be divided by 2 (this corresponds to the fact that the total number of proton pairs is $\frac{1}{2}N^2$). We can transform to relative velocity $\mathbf{v}=\mathbf{v}_1-\mathbf{v}_2$ and center-of-gravity velocity $\mathbf{V}=\frac{1}{2}(\mathbf{v}_1+\mathbf{v}_2)$ and integrate over the latter, then we obtain (including the factor $\frac{1}{2}$ mentioned)

$$\tfrac{1}{2}N^2\varphi(\mathbf{v})d\mathbf{v} \tag{27a}$$

$$\varphi(\mathbf{v})d\mathbf{v}=(M/4\pi kT)^{\frac{3}{2}} \times\exp(-Mv^2/4kT)4\pi v^2 dv. \tag{28}$$

Of the proton pairs with given velocities $\mathbf{v}_1\mathbf{v}_2$, one in four will have opposite spins, three in four parallel spins. The latter have an antisymmetrical spatial wave function and therefore do not contribute to the cross section of our nuclear process. The former have a symmetrical wave function which is, normalized to unit density:

$$[(\exp(i\mathbf{k}_1\cdot\mathbf{r}_1+i\mathbf{k}_2\cdot\mathbf{r}_2)) +(\exp(i\mathbf{k}_2\cdot\mathbf{r}_1+i\mathbf{k}_1\cdot\mathbf{r}_2))]/\sqrt{2}. \tag{29}$$

Separating off the motion of the center of gravity and expanding into spherical harmonics, we obtain

$$(\exp(ikz)+\exp(-ikz))/\sqrt{2} =\sqrt{2}\sin kr/kr+\cdots, \tag{29a}$$

which differs from (13) by a factor of $\sqrt{2}$. This gives a factor 2 in the cross section and, with the

Table II. *Energy evolution due to proton combination, for* $\rho c_{\mathrm{H}}^2=10\ g/cm^3$.

t(million degrees)	5	10	15	20	30	40	50	100
τ	19.8	15.70	13.72	12.56	10.89	9.90	9.18	7.28
ϵ(ergs/g sec.)	0.0040	0.15	0.76	2.2	9.1	20	36	150

factor $\frac{1}{4}$ for the *a priori* probability of opposite spin, there remains a factor $\frac{1}{2}$.

The total number of processes per cm^3 per sec. becomes thus

$$p'=\tfrac{1}{4}N^2\int\varphi(\mathbf{v})d\mathbf{v}v\sigma. \tag{30}$$

By the insertion of (24), (28), (19), this becomes

$$p'=\pi^{\frac{1}{2}}gf(W)N^2b^3\Lambda^2\left(\frac{M}{kT}\right)^{\frac{3}{2}}\int_0^\infty v^2dv \times\frac{2\pi e^2}{\hbar v}\exp\left(-\frac{2\pi e^2}{\hbar v}-\frac{Mv^2}{4kT}\right). \tag{31}$$

The integrand has a strong maximum very close to

$$v=(4\pi e^2kT/\hbar M)^{\frac{1}{3}}. \tag{31a}$$

Approximating it, in the usual way, by a Gaussian around the maximum, we find

$$p'=16\pi\cdot 3^{-5/2}\cdot gf(W)N^2b^3\Lambda^2\tau^2e^{-\tau} \tag{32}$$

with

$$\tau=3(\pi^2Me^4/4\hbar^2kT)^{\frac{1}{3}}. \tag{33}$$

If t is the temperature in millions of degrees, we have

$$\tau=33.8t^{-\frac{1}{3}}. \tag{33a}$$

§5. Result

We insert into (32) the values of the constants b (cf. (12b)), $gf(W)$ from (10), Λ^2 from Table I (for $r_0=e^2/mc^2$), and assume that the concentration of hydrogen (by weight) is c_{H} and the density (in g/cm^3) is ρ. Then the number of processes per *gram* per second is

$$p=p'/\rho=16\pi\cdot 3^{-5/2}\cdot 1.2\cdot 10^{-5}\cdot(6.02\cdot 10^{23})^2 \cdot(4.37\cdot 10^{-13})^3\cdot 8.10\cdot\rho c_{\mathrm{H}}^2\tau^2e^{-\tau} =0.95\cdot 10^7\rho c_{\mathrm{H}}^2\tau^2e^{-\tau}. \tag{34}$$

Each combination of two protons leads (§1) ultimately to the formation of an α-particle out of four protons and two electrons, with an energy liberation of $4.3\cdot 10^{-5}$ erg. Therefore the energy production is

$$\epsilon = 410\rho c_H^2 \tau^2 e^{-\tau} \text{ ergs/gram sec.} \quad (35)$$

The density at the center of the sun[14] is about 80, the hydrogen content $c_H \approx 0.35$, therefore $\rho c_H^2 \sim 10$. Table II gives the energy evolution due to the proton-proton reaction at various temperatures. With a hydrogen content of 35 percent, the central temperature of the sun is about $20 \cdot 10^6$ degrees. At this temperature, the energy production is about 2.2 ergs/g sec. This is of the same order as the observed average energy production of the sun (2.0 ergs/g sec.). Thus we come to the conclusion that the *proton-proton combination gives an energy evolution of the right order of magnitude for the sun.*

For a quantitative comparison, it must be remembered that both the temperature and the density of the sun decrease fairly rapidly from the center outwards, and that the rate of reaction decreases with both of these quantities. The average energy production per gram will thus be considerably smaller than that at the center, perhaps by a factor of 10. Since our calculations are rather accurate, it seems that there must be another process contributing somewhat more to the energy evolution in the sun. This is probably the capture of protons by carbon (reference 2).

For many problems, it is necessary to know the temperature dependence of the energy production ϵ. It is convenient to express this dependence as a power law, $\epsilon \sim T^n$. Then (cf. (35))

$$n = d \log \epsilon / d \log T = \tfrac{1}{3}(\tau - 2). \quad (36)$$

At 20 million degrees, this gives in our case $n = 3.5$. This is large enough to make the point source model of stars a rather good approximation. On the other hand, it is too slow a dependence on temperature to explain, with temperatures of the order $2\text{–}4 \cdot 10^7$ degrees, the very high rates of energy production found in very heavy stars. However, we believe that our process is the principal source of energy in stars lighter than the sun.

[14] Stromgren, Ergebn. d. exakt. Naturwiss. **16**, (1937).

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On the Production of Heavy Electrons*

L. W. Nordheim and G. Nordheim
Duke University, Durham, North Carolina

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The barytron theory of nuclear forces shows a close correspondence to the theory of the electromagnetic field. Estimates based on this analogy are given for the probabilities of the processes leading to actual production of barytrons. A comparison with the experimental evidence regarding the occurrence of barytrons in the cosmic radiation shows that the theoretical cross sections are too small to explain the properties of the hard component. The difficulty is increased by the short lifetime of the barytrons which is estimated to be of the order $\sim 10^{-8}$ sec. from radiative β-decay. Therefore no simple picture of barytron production in terms similar to radiation theory can be given. This failure, however, indicates only the inapplicability of perturbation calculations, but does not constitute an actual disproof of the link between nuclear forces and cosmic radiation.

Introduction

At present there are two different lines of evidence for the existence of a new particle, the barytron,[1] of electronic charge (positive and negative) and of a mass μ about 200 times the electronic mass m.

From cosmic-ray investigations there can be little doubt that a large percentage of the hard component, which itself constitutes the greater part of all radiation from sea level downwards, can be neither electrons nor protons[2] but must

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[1] This name was proposed at the Washington Meeting of the American Physical Society, April 1938.

[2] Anderson and Neddermeyer, Phys. Rev. **51**, 884 (1937); Street and Stevenson, Phys. Rev. **51**, 1005 (1937).