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Atomic Electron Velocities in Hydrogen

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When conditions are favorable to single scattering of fast electrons, those scattered through a suitable angle by gases of low atomic number fall into two distinct classes, (1) those scattered elastically by the nuclei and (2) those scattered inelastically by the atomic electrons. The inelastically scattered electrons have a distribution of energies about the most probable value, which is $V = V_0 \cos^2 \theta$, where V_0 and V are the energies of the incident electron before and after collision with an atomic electron, and θ is the angle of deflection. The distribution of energies among the inelastically scattered electrons is determined by the distribution of energies among the atomic electrons in such a way that the curve representing the *distribution of energies* among the scattered electrons, above or below the most probable value, is identical in shape with the *distribution of component velocities* among the atomic electrons. A beam of electrons with energies between 1737 and 4040 volts was directed into hydrogen at low pressure and the distribution of energies among those electrons which had been scattered at 34.2° measured. From this the distribution of component velocities among the atomic electrons follows immediately. The experimental curve is wider than the theoretical curve computed by Hicks by about 11 percent at the "half-width." In view of the exact agreement obtained previously with the same method in the case of helium, it is suggested that the assumptions underlying the theoretical calculation for the hydrogen molecule may have to be revised.

IN a previous paper[1] it was shown how, under certain conditions, measurements on the scattering of electrons by matter could be interpreted so as to give information on the distribution of velocities among the atomic electrons. The principle underlying the method is as follows. If a beam of sufficiently fast electrons passes through matter when the conditions are such that "single scattering" prevails, then an electron in the beam may be deflected *either* by a nucleus *or* by an atomic electron. (For electrons of sufficiently high speed, the probability of two or more scattering centers contributing deflections of the same order of magnitude to any one electron is negligibly small.) The electrons deflected by the nuclei suffer no loss of energy, while those deflected by atomic electrons lose an amount $V = V_0 \cos^2 \theta$, giving $V_1 = V_0 \sin^2 \theta$ to the atomic electron. Here V_0 and V are the energies of the incident electron before and after scattering, V_1 the energy acquired by the atomic electron, and θ the angle of deflection of the incident electron. Thus we should expect to find that the electrons scattered through an appreciable angle by matter will be separated into two distinct groups, those scattered elastically by the nuclei and those scattered inelastically by the atomic electrons. Those scattered by the atomic electrons, however, will have a range of energies about a mean value, if the atomic electrons are in random motion instead of being at rest. The distribution of energies among the atomic electrons determines the distribution of energies among the scattered electrons, a relation which permits the former to be calculated from an experimental determination of the latter. This relation takes a particularly simple form if, in place of the energies of the atomic electrons, we consider their component velocities.[2] If $f(u)$ is the distribution of *component velocities* among the atomic electrons, $F(V'')$, the resulting distribution of *energies* among the electrons scattered at a suitably selected angle, has exactly the same shape as $f(u)$. V'' is the excess energy (positive or negative) which the scattered electron acquires as a result of a collision with an atomic electron having a component velocity u in a certain direction.[3] Thus an experimental study of electron scattering at a suitable angle, giving us $F(V'')$, the distribution of energies among the scattered electrons about the mean value, immediately tells us the shape of $f(u)$ for the component velocities of the atomic electrons.

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[1] A. L. Hughes and Marvin M. Mann, Jr., Phys. Rev. **53**, 50 (1938).

[2] G. E. M. Jauncey, Phys. Rev. **50**, 326 (1936).

[3] The particular component of velocity which is significant in determining V'' is the one which the atomic electron had, before collision, in a direction at right angles to that of the scattered electron after collision. If the velocities of the atomic electrons are distributed isotropically in space then all the component velocities in any direction are on the average identical, and related in the same way to the resultant velocity.

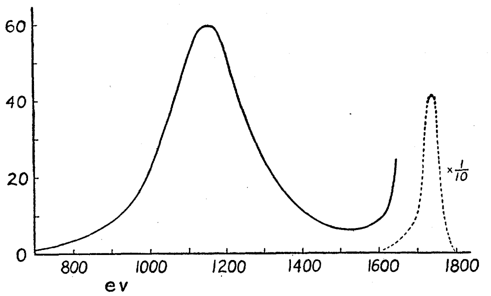

FIG. 1. Energy distribution of 1737-volt electrons scattered through 34.2° by hydrogen.

$f(\lambda'')$, the profile of the Compton modified band for x-ray scattering by the same atoms turns out to be identical in shape with $f(u)$. A full discussion of the relationship between $F(V'')$, $f(\lambda'')$, and $f(u)$ will be found in the first paper[1] on this topic.

This paper is concerned with the scattering of electrons by hydrogen and the conclusions which one can draw as to $f(u)$ for hydrogen. No description of the apparatus and of the method of taking and reducing observations will be given as they were identical with those described in the previous paper on helium.[1] Hydrogen was obtained from a commercial cylinder and was said to be 99.9 percent pure. No further purification was attempted beyond passing it into the reservoir through two charcoal traps immersed in "dry-ice." The pressure of the hydrogen in the collision chamber was held at a constant value, about 0.004 mm during the experiments.

RESULTS

Measurements were made with electrons of energy 1737, 2985, 3588 and 4030 electron volts.

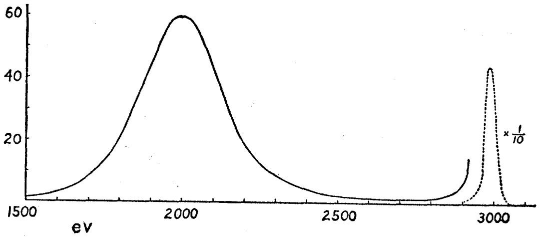

FIG. 2. Energy distribution of 2985-volt electrons scattered through 34.2° by hydrogen.

The results are presented in the form of curves (Figs. 1, 2, 3, and 4) similar to those published for helium in the previous paper.[1] Each of the curves represents the mean of over 250 separate readings. The spread of the individual points about each curve is so small that, on the scale on which the drawings are reproduced, the dots representing the individual measurements would be so close to the curve that their separation from the curve would hardly be visible. It is evident that the higher the incident energy, the narrower, in relation to the value of the abscissa at which the maximum occurs, is the band representing the inelastically scattered electrons, and the more complete is the separation between it and the elastic peak.

The band representing the inelastically scattered 1737-volt electrons is not quite symmetrical with respect to the maximum. This is no doubt to be attributed to the fact that, as the energy of the incident electrons is reduced, the conditions for "single center scattering," as defined in the previous paper[1] are less and less strictly fulfilled. The separations, S_{exp}, between the elastic peak and the inelastic maximum are given in Table I. The theoretical separation is

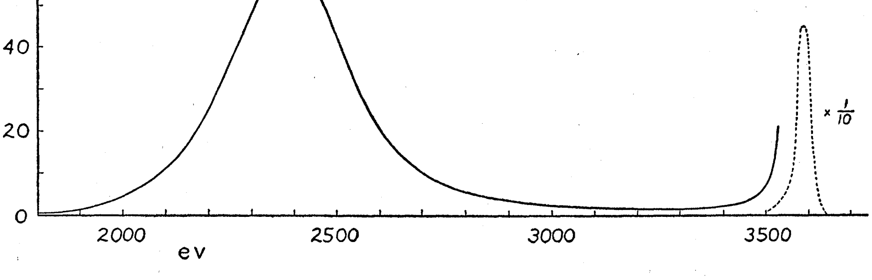

FIG. 3. Energy distribution of 3588-volt electrons scattered through 34.2° by hydrogen.

S_{theo} which is equal to $V_0 \sin^2 \theta$. In our apparatus, $\theta = 34.2°$.

It will be seen that the difference $S_{\text{exp}} - S_{\text{theo}}$ varies in such a way that no definite conclusions can be drawn. However, the value is always positive, which is the result that would follow from the very simple considerations presented in the previous paper on the assumption that the atomic electrons are not perfectly free. No quantitative conclusion can be drawn. It may be pointed out that the variations of the kind found in the last column of the table could be the result of an uncertainty in the exact value of the angle of scattering. The final adjustment of the electron beam to the middle of the apertures between the gun and the collision chamber was made by means of a small weak magnet near the anodes of the gun. This may alter θ by a few tenths of a degree. To obtain more precise information as to the meaning and reality of the difference, $S_{\text{exp}} - S_{\text{theo}}$, would require an apparatus in which the value of the angle could be set and held to within 0.1°.

TABLE I. *Comparison of experimental and theoretical separations between the elastic and inelastic maxima for various electron energies.*

INCIDENT ENERGY	S_{exp}	S_{theo}	$S_{\text{exp}} - S_{\text{theo}}$
1737 volts	587 volts	549 volts	38 volts
2985	988	943	45
3588	1200	1134	66
4030	1350	1274	76

The main purpose of this investigation is to determine the value of $f(u)$ for the atomic electrons in hydrogen. We shall use only the experimental results obtained with incident electron energies of 2985, 3588, and 4030 volts, since for these the distribution of energies among the inelastically scattered electrons is symmetrical about the middle. To obtain $f(u)$ for the atomic electrons we proceed as follows. In the notation of the earlier paper,[1] the abscissas both to the right and left of the maximum of the inelastic band may be labeled V''. Each experimental curve, as plotted in Figs. 2 to 4, therefore gives us two $F(V'')$ curves which may be averaged together. To get the $f(u)$ curves we merely change the abscissas in V'' into abscissas in u (or β) by the substitution

$$\beta(=u/c)\times 10^3 = V'' \div (0.5782\, V_0^{\frac{1}{2}}).$$

The $f(u)$ curves so obtained from Figs. 2 to 4 were almost exactly superposable. They were drawn on a large scale on accurate cross-section paper and the best possible mean curve constructed. This mean curve is made up from a total of 890 individual observations on the scattered electrons. From the way in which the individual points are scattered about the curve, the values of the points on the curve may be considered accurate to 2 percent over the range $\beta = 0$ to $\beta = 5\times 10^{-3}$ cm/sec. and to 5 percent over the range $\beta = 5\times 10^{-3}$ to $\beta = 9\times 10^{-3}$ cm/sec. The mean curve, adjusted to a maximum of 60.0, is plotted against β in Fig. 5, and the values of the ordinates, $f(u)$ are tabulated in the last column of Table II.

DISCUSSION

In order to compare our experimental results with theory we have tabulated in Table II, and plotted in Fig. 5, the values for $f(u)$ which have been obtained theoretically. The column and curve marked "K.R.R." are due to Kirkpatrick, Ross and Ritland[4] who computed $f(\lambda'')$, the profile of the modified band in the Compton effect for x-ray scattering by hydrogen atoms.

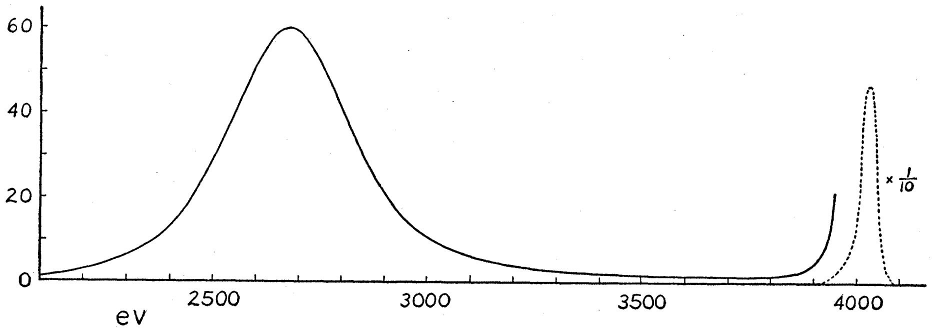

FIG. 4. Energy distribution of 4030-volt electrons scattered through 34.2° by hydrogen.

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[4] P. Kirkpatrick, P. A. Ross, and H. O. Ritland, Phys. Rev. 50, 928 (1936).

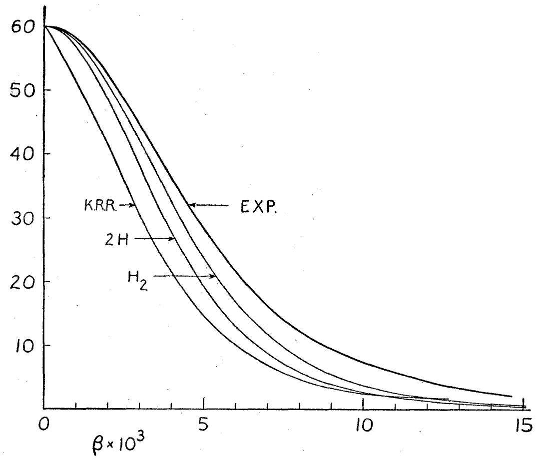

FIG. 5. Theoretical and experimental distributions of component velocities of atomic electrons in hydrogen, EXP., experimental values obtained in this investigation. K.R.R., theoretical value for the hydrogen atom due to Kirkpatrick, Ross and Ritland. 2H and H_2, theoretical curves due to Hicks for atomic and molecular hydrogen, respectively. Ordinates: values of $f(u)$. Abscissas: each unit $=\beta\times10^3=u/(3\times10^7)=\lambda''$ in X.U. (for $\lambda=695$ X.U. and $\theta=90°$).

Since $f(\lambda'')$ and $f(u)$ have the same shape, the one automatically gives the other. The columns in Table II and the curves in Fig. 5 marked "2H" and "H_2" are due to Hicks.[5] The one marked "2H" is that computed for the hydrogen atom. It is puzzling that the theoretical values computed in these two papers for the hydrogen atom should differ as much as they do, in view of the fact that the wave mechanical picture of the hydrogen atom is presumably accurately known. One certainly cannot expect $f(u)$ for the hydrogen molecule to be the same as $f(u)$ for the atom; it is however instructive to have both of them available for comparison with our experimental $f(u)$ values. Various wave mechanical descriptions of the hydrogen molecule have been suggested. The one proposed by James and Coolidge[6] has given results in excellent accord with certain experimental data. But because of certain technical mathematical difficulties in adapting the James-Coolidge treatment of the hydrogen molecule to the problem of deriving a theoretical $f(u)$ curve, Hicks was unable to make use of their ideas. He therefore assumed another wave mechanical description of the hydrogen molecule, that due to Weinbaum. With this as a starting point he computed the values listed under "H_2" in Table II, and plotted as the curve marked "H_2" in Fig. 5. It will be seen that our experimental values for $f(u)$ differ appreciably from Hicks' "H_2" curve, and still more from the curves for atomic hydrogen. In view of our estimate of the degree of accuracy of our values, discussed earlier in the paper, it must be concluded that the difference is real. This is particularly significant in view of the fact that, for helium, the agreement with the "K.R.R." curve is very good, and with the Hicks "H-4" curve excellent. Since the experimental procedures for hydrogen and helium were identical, we must conclude that the experimental values for hydrogen are fully as accurate as those for helium. If one assumes that the agreement in the case of helium is not merely an accidental one, it follows that the theoretical calculations for hydrogen are probably in error.

TABLE II. *Theoretical and experimental velocity distributions for the atomic electrons in hydrogen, and the profile of the associated Compton modified band for $\lambda=695$ X.U. and $\theta=90°$.*

		$f(u)$ (and $f(\lambda'')$)			
			HICKS		
λ'' (X.U.)	u (CM/SEC.)	K.R.R.	(2H)	(H_2)	$f(u)$ EXPERIMENTAL
0	0×10^7	60.0	60.0	60.0	60.0
1	3	51.4	56.8	57.6	58.1
2	6	41.5	48.4	50.9	52.4
3	9	30.4	37.8	41.7	44.4
4	12	21.7	27.5	32.2	36.0
5	15	14.8	19.19	23.67	28.3
6	18	9.9	12.97	16.77	21.4
7	21	6.9	8.67	11.60	16.1
8	24	4.7	5.76	7.90	12.2
9	27	3.2	3.84	5.36	9.4
10	30	2.47	2.59	3.61	7.3
11	33		1.78	2.46	
12	36	1.73	1.21	1.67	4.4
13	39		.86	1.15	
14	42	.99	.60	.83	2.5
15	45		.43	.60	
20	60		.11	.16	

There is one point which should be mentioned in that it may possibly have some bearing on the discrepancy. Helium is monatomic and hydrogen diatomic. The theory whereby $f(u)$ is shown to be identical in shape with the experimental $F(V'')$ is based on a strictly particle

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[5] B. Hicks, Phys. Rev. **52**, 436 (1937).

[6] H. M. James and A. S. Coolidge, J. Chem. Phys. **1**, 825 (1933).

point of view of the electrons and their interactions, and completely disregards their wave aspect. The agreement between theory and experiment may justify this in the case of helium. We have plenty of evidence, however, from electron diffraction effects that the elastic electron scattering does require us to use the wave aspect for a complete description of what happens. This raises the question as to whether or not it is necessary to modify the method of using the experimental data described in the earlier paper so as, in some way, to take account of the wave nature of the electrons in the inelastic electron scattering. We are inclined to think, however, that the simple particle view can be retained in the interpretation of this kind of experiment, and that the discrepancy will be removed by a more accurate method of handling the wave mechanical description of the hydrogen molecule.

We wish to thank Professor Kirkpatrick and Dr. Hicks for letting us have the numerical values from which their published curves were drawn.

AUGUST 1, 1938 PHYSICAL REVIEW VOLUME 54

A New Precision Method for the Determination of e/m for Electrons

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The excellent focusing properties of crossed electric and magnetic fields have been utilized in the development of a new, precision method for the determination of e/m for electrons. This method differs from previous methods in that the final equation for e/m does not involve the velocity explicitly. Moreover, focusing criteria have been worked out which effectively eliminate any possible influence of electron energy upon the value of e/m. That this is a great source of error and uncertainty in other methods is shown by the great difference between the energy of the electrons before and after emergence from a slit. This effect is too great to arise from a contact potential difference but it can be attributed to direct electron bombardment of the slit and the subsequent formation of a surface charge on it. The magnitude of this charge is not constant but varies between 9 volts and 24 volts, depending upon the applied accelerating potential. The value of e/m obtained with the present apparatus is, $e/m_0=(1.7571\pm 0.0013)\times 10^7$ e.m.u., where 0.0013 is the probable error derived from a least squares solution of a set of observations for various electric and magnetic field intensities. Other sets differed from this by less than 1 : 5000. The mechanical accuracy of the present cylindrical condenser sets the limit on the precision attainable with the present apparatus. However, the error due to this cause is less than the probable error stated above. The limitations of the present condenser can be reduced considerably through the use of a new condenser designed in accordance with kinematic principles. The method presented here for the production of magnetic fields of great uniformity and a new, precise cylindrical condenser would permit a determination of e/m to be made with the method of crossed fields to within an accuracy of 1 : 3000.

I. Introduction

The focusing properties of crossed electric and magnetic fields for electrons, in the case of circular orbits, were discussed in a recent[1] paper in this journal. It was found that this combination of fields is capable of extremely sharp focusing of electron beams that vary both in direction and in velocity.

This same field combination was investigated experimentally to determine its suitability for the accurate measurement of the specific charge of electrons. In practically all deflection measurements of e/m, the precision is limited by uncertainties in the velocity of the electrons. In the method of crossed fields, the final equation does not contain the velocity explicitly, hence uncertainties in the accelerating potential do not enter. Although the accelerating potential does not appear explicitly in the equation for e/m, provision must be made by focusing to adjust the velocity for any given ratio of intensities of the electric and magnetic fields.

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[1] A. E. Shaw, Phys. Rev. **44**, 1006 (1933).