JULY 1, 1938 PHYSICAL REVIEW VOLUME 54

Range and Specific Ionization of Alpha-Particles

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(Received April 6, 1938)

A study of the specific ionization of polonium alpha-particles near the end of their range with a shallow ionization chamber and linear pulse amplifier has resulted in an accurate determination of the single particle specific ionization curve and the range energy relation in this region. The method used is to compare the average single particle ionization in chambers of different depths and from the differences to obtain the ionization of particles of residual range less than the depth of the chambers. The single particle specific ionization curve differs from earlier determinations and shows the necessity of a correction to the accepted values of absolute mean range of all alpha-particle groups. The value suggested for the mean range of Th C′ (at 15°C and 760 mm) is 8.570±0.007 cm; for Po it is 3.842±0.006 cm. It is shown that extrapolated ranges obtained from ionization measurements differ from those obtained from number distance curves; an empirical relation is suggested to convert such values to mean range. A corrected range energy relation for alpha-particles extending down to zero range is obtained by combining the results of the present investigations with earlier determinations.

Introduction

One of the most essential relations for the analysis of nuclear disintegration data is that between alpha-particle range and energy. In the region in which alpha-particle groups from natural radioactive sources are available (between 5.3 and 10.5 Mev) precision measurements of the energies have been made with a magnetic deflection technique.[1, 2] The extrapolated ranges from ionization measurements near the end of the range have been accurately observed for a few natural alpha-groups,[3, 4] while the relative values of mean range have been obtained for most of them.[5] Lewis and Wynn-Williams[5] correlated these measurements and established an absolute mean range scale (Th C′ alphas = 8.533 cm at 15°C and 760 mm). In the region below 5.3 Mev (Po = 3.8 cm) several attempts have been made to measure reduced ranges and energies by slowing down the particles in absorbers.[6, 7] These have resulted in extending the relation down to about 3 Mev with reasonable accuracy. A few points on the curve at still lower energies can be computed from atomic mass values and disintegration data, with somewhat less accuracy. The ranges and energies of extremely low energy alphas have been obtained from studies of alpha-particle recoils in He gas.[8] An analysis of the existing experimental evidence, supplemented by a theoretical calculation (valid for the higher energies) for which the constants were determined in the well-known region of natural alphas, has been made by Bethe,[9] and has resulted in a range energy relation which seems to be in reasonable agreement with most disintegration results. The accuracy claimed for this relation below 0.5 Mev is 10 percent, between 0.5 and 2.0 Mev 50 kv (~5 percent), and between 2 and 5 Mev 30 kv (~1 percent).

In the analysis of those nuclear disintegrations from which very short range alpha-particles are observed,[10] the poor accuracy of the range energy relation has been a serious handicap. It has affected the accuracy of the values obtained for neutron energies when measured through the ranges of recoil He atoms. Studies of the energy distribution of neutrons emitted from excited heavy nuclei, which are expected to be of the order of 1 or 2 Mev, also will require a more exact knowledge of the low range region.

This paper reports a method for the accurate determination of the specific ionization of a single alpha-particle and the range energy relation for alpha-particles in the low energy region, extending down to essentially zero range. The results require a new analysis of the Cavendish

[1] Rutherford, Wynn-Williams, Lewis and Bowden, Proc. Roy. Soc. **139**, 617 (1933).
[2] Briggs, Proc. Roy. Soc. **157**, 183 (1936).
[3] Henderson, Phil. Mag. **42**, 538 (1921).
[4] I. Curie, Ann. de physique **3**, 299 (1925).
[5] Lewis and Wynn-Williams, Proc. Roy. Soc. **136**, 349 (1932).
[6] Mano, Ann. de physique **1**, 407 (1934).
[7] Briggs, Proc. Roy. Soc. **114**, 341 (1927).
[8] Blackett and Lees, Proc. Roy. Soc. **134**, 658 (1932).
[9] Livingston and Bethe, Rev. Mod. Phys. **9**, 261 (1937).
[10] Livingston and Hoffman, Phys. Rev. **53**, 227 (1938).

results and give a more exact determination of the absolute range. The work was started with the intention of establishing the position of the mean range of an alpha-particle group with relation to the physical face of a shallow ionization chamber. It was found that this position was a decided function of the electrical characteristics of the amplifier and counter circuits. In order to specify the range as measured by the shallow ionization chamber technique to a better accuracy than the depth of the chamber (usually about 2 mm), it is necessary to know the effective "electrical face" or "electrical center" of the chamber, the relative size of the noise level of the amplifier to alpha-particle pulse heights, the amplitude response characteristic of the amplifier, the grid bias on the recording thyratron, the penetration of the particle into the chamber to produce a count at this bias, etc. In the discussion to follow these various factors are analyzed and methods suggested for determining their effects on the observed values of range.

Discussion

In order to discuss the features pertaining to range measurements it is necessary to define the terms to be used. A very helpful analysis has been given by King and Rayton.[11] Since the loss of energy of alpha-particles through the processes of ionization and excitation occurs in discrete amounts, the energy loss will show statistical fluctuations and the particle ranges will be distributed about a *mean* or most probable value. This distribution may be closely approximated by a Gaussian function:

$$f(x)dx=(1/(\pi^{\frac{1}{2}}\alpha))e^{-(R_0-x)^2/\alpha^2}dx;$$

where $f(x)dx$ is the fraction (of the total number of particles) having a range ending between x and $x+dx$, R_0 is the mean range, α/R_0 is the dimensionless range straggling coefficient, usually indicated by the symbol ρ, and α, the *range straggling parameter*, is the half-width of the distribution curve at $1/e$ of the maximum. Such a distribution is illustrated in Fig. 1 (A), where α has the value 0.062 cm (for Po). Integration of $f(x)dx$ results in a *number distance* curve, similar to those obtained experimentally, in which the mean range is the distance at half-maximum (the point of inflection of the curve). It follows from this that the mean range of a collimated group of alpha-particles is that for which half the particles have a greater, and half a smaller range. The intercept of the essentially straight line portion of the number distance curve is an *extrapolated number distance range* of the particle group, and is the most readily determined feature of experimental number distance curves. It can readily be shown that the difference between these mean and extrapolated ranges, s, is a linear function of the straggling parameter: $s=\frac{1}{2}\pi^{\frac{1}{2}}\alpha$. Such a curve is also indicated in Fig. 1 (B), extrapolating to a value $s=\frac{1}{2}\pi^{\frac{1}{2}}(0.062)=0.055$ cm.

As an alpha-particle moves through a gas it loses energy by ionization and excitation of the gas atoms. The ionization produced per cm length of path (by a single alpha or by a group of alphas) is called the specific ionization (of the single alpha or group of alphas). We will call the specific ionization as measured in a chamber of finite depth (1 to 2 mm) the *specific ionization*, and the value which this approaches as the chamber is made infinitely thin the *differential specific ionization*. The specific ionization is a function of the distance from the end of the

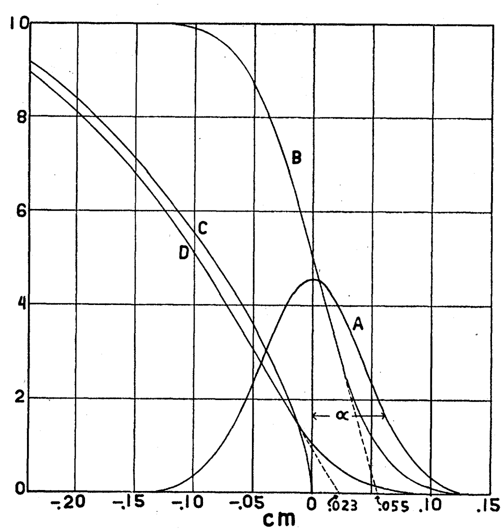

FIG. 1. Characteristics of range curves for polonium alphas. *A*, Range straggling distribution about the mean range; *B*, number distance curve showing extrapolated range; *C*, differential specific ionization of a single alpha-particle of mean range; *D*, average ionization (Bragg) curve for a group of alpha-particles with the range distribution of curve *A*, showing the ionization extrapolated range.

[11] King and Rayton, Phys. Rev. **51**, 826 (1937).

range. The curve of specific ionization against distance from the end of the range for a collimated group of alphas is called a *Bragg curve.* This curve rises asymptotically from zero at the end of the range to a maximum at about 5 or 6 mm from the end, and then decreases slowly to 0.5 of the maximum for particles of about 2.5 cm residual range. The specific ionization curve for a *single* alpha was first obtained experimentally by a study of the photographic density of alpha-particle tracks in a cloud chamber.[12] The chief result of the present investigation is to obtain this curve with accuracy up to the extreme end of the range. As the Bragg curve is the specific ionization of a group of alphas, it is then the resultant of the ionization curve of the single alpha and the straggling distribution of particles in the group. It shows a pronounced tail due to straggling. An extrapolation of the essentially linear front of this curve to zero ionization (first suggested by Marsden and Perkins[13]) results in an *extrapolated ionization range* which is not the same as the extrapolated number distance range obtained from the number distance curve. The ranges measured and reported by Henderson,[3] Curie[4] and others are extrapolated ionization ranges obtained from Bragg curves.

In order to illustrate these features we have plotted in Fig. 1 (C) the single particle differential specific ionization curve obtained from these experiments. The method of taking the data, which will be described in detail later, eliminates the effects of straggling, and so the end of this curve is exactly the mean range. We have also computed the expected shape of the Bragg curve from this single particle curve and the range straggling indicated in the distribution curve ($\alpha=0.062$ cm). This is plotted in Fig. 1 (D), and has an extrapolated intercept 0.023 cm greater than the mean range, but 0.032 cm less than the range extrapolated from the number distance curve. In a latter section of this paper we amplify this analysis to show how the absolute mean range may be determined from the observed Bragg curves of certain standard alpha-particle groups.

The present practice in nuclear physics is to observe the number distance curve and obtain the extrapolated number distance range therefrom. This method has two advantages; that it leads directly to the mean range through subtraction of the quantity $s=\frac{1}{2}\pi^{\frac{1}{2}}\alpha$, and for which α can be determined from the observed straggling of the front of the number distance curve; and that it represents a range value independent of the shape of the Bragg curve. The mean range may be determined from the extrapolated ionization range only through an empirical relation such as that deduced by Lewis and Wynn-Williams:[5] $x=0.8\alpha-0.06$ mm, in which x is the difference between mean range and extrapolated ionization range. It is our impression that the distinction between these two extrapolated ranges is not generally recognized. With the new evidence which we present we find the difference between mean and extrapolated range in the two cases to differ by more than a factor 2.

In the measurement of alpha-particle ranges three essentially different techniques have been used; photographic observation of particle track lengths in a cloud chamber, measurements with sensitive charge measuring devices of the ionization produced in a short length of path, and counting techniques usually involving shallow ionization chambers and pulse amplifiers.

Histographic plots of the lengths of tracks observed in a cloud chamber show a most probable value which is interpreted as the mean range of the group. The absolute range measurements are handicapped by the difficulties of determining gas density at the instant of formation of the track, and the uncertainty of determining the exact end of the range in the region where the specific ionization is rapidly decreasing. The range distribution, however, leads directly to a good estimate of straggling, if errors of measurement are considered.[4, 14]

Ionization measurements, such as those of Henderson,[3] Curie,[4] and Naidu,[15] have resulted in the most consistent data, and from the curves obtained the extrapolated intercepts can be determined with precision. The conversion of such values to mean range requires a knowledge of the specific ionization curve of the single alpha, and of the straggling, including the effects

[12] Feather and Nimmo, Proc. Camb. Phil. Soc. **24**, 139 (1938).
[13] Marsden and Perkins, Phil. Mag. **27**, 690 (1914).
[14] Rayton and Wilkins, Phys. Rev. **51**, 818 (1937).
[15] Naidu, Ann. de physique **1**, 72 (1934).

of the depth of chamber. If properly analyzed, however, it offers possibilities of giving the most exact determinations of range since measurements come from large numbers of alpha-particles, and may be carried out under reduced pressures so that the errors in distance measurements are negligible and the effective chamber depth small.

The shallow ionization chamber technique may be used either to count the number of particles arriving at a certain point (the number distance curve), or, by measuring the size of the pulses, to determine the energy loss by ionization in the chamber. In the first method, the total number of pulses is counted, regardless of size. The most readable quantity from such experimental data is the extrapolated number distance range. Determination of the exact mean range of the particles is handicapped, however, by the finite resolution of the chamber due to its depth. Each experimental apparatus has its own noise level, or background, and only those pulses are counted which are larger than a certain minimum determined by the noise level. This result is usually obtained experimentally by applying the output pulses to a thyratron (grid-controlled gaseous discharge tube) on which the grid bias is adjusted so that the tube flashes and records counts only for pulses greater than a set value. This means that very small pulses, such as those due to particles just entering the chamber, are not counted, and so a minimum depth of penetration corresponding to this value must be added to the distance to the face of the chamber. In order to determine this penetration it is necessary to know the variation of ionization with distance from the end of the range.

This variation can be determined if measurements are taken of the size of pulses obtained from a chamber of given depth, in which case the pulse heights are proportional to the ionization produced in the chamber. This can be obtained by determining the thyratron bias just sufficient to count such pulses or by measuring the height of oscillographic records of the pulses. A spread in pulse heights will be observed due to the range straggling; the average pulse height will be proportional to the ionization of the average alpha-particle in the range straggling distribution. Such measurements give at the same time an alternative direct determination of the end of the range of the average alpha.

In both of these applications of the shallow chamber and counter the resolution can be improved by decreasing the depth of the chamber either physically or by reducing the pressure. With number distance data this results in a smaller correction for penetration and a more accurate result; in number bias curves it makes the error due to the curvature of the specific ionization relation smaller. A most valuable modification of the counter method came from the development of the differential ionization chamber[16] in which the ionization from two adjacent shallow chambers with opposite applied potentials is collected upon a common grid. For particles crossing both chambers the ionization essentially cancels, giving no pulse; only those particles are recorded which stop within the chamber. The resolution of the instrument was further improved by recording only those pulses within set limits of magnitude, effectually measuring particles stopping in a shallow region near the common grid. A peaked curve is obtained broadened by range straggling; the position of this peak represents the point of maximum rate of change of ionization along the path of the alpha-particle. This represents a characteristic range differing from the mean range only by a constant difference, and this difference depends upon the measuring apparatus and not upon the straggling of the alpha-group. The differential chamber technique was used by Lewis and Wynn-Williams[5] and others in the Cavendish Laboratory in their measurements of the relative mean ranges of the natural alpha-groups. All the methods described except that yielding mean pulse heights have the inherent disadvantage of including the range straggling in all observations.

Bohr[17] first suggested that the tail at the end of the Bragg curve was due to the statistical nature of the energy loss and calculated the expected straggling. For Po alphas the straggling parameter, α, computed from the Bohr expression is 0.042 cm; that obtained[14] from the wave mechanical treatment of Bethe[18] is 0.046

[16] Rutherford, Ward and Wynn-Williams, Proc. Roy. Soc. **129**, 211 (1930).
[17] Bohr, Phil. Mag. **30**, 581 (1915).
[18] Bethe, Ann. d. Physik **5**, 325 (1930).

cm. Since all experimental determinations have led to larger values, it has been suggested that such theoretical values are too low because the energy loss by excitation of the inner electrons is neglected.[19] The observed distribution in range nearly always includes other sources of straggling, due to such factors as thickness of the alpha-particle source, finite depth of the measuring chamber, noise or background fluctuations in the amplifiers or electrometers, statistical errors in measurement, etc. The value obtained for the range straggling parameter depends in most instances on the success in identifying and measuring the straggling due to these other factors. The α for Po has been variously reported as: 0.064,[4] 0.052,[14] 0.067[20] cm.

Briggs[21] has made direct measurements of the straggling in mica by magnetic analysis of the energy distribution of Ra C′ alphas after traversing different thicknesses of mica foils. His results, when reduced to air, showed a nearly linear relation between α^2 and distance in the first 4.5 cm where the results had a reasonable consistency. Rutherford, Ward and Lewis[22] extrapolated this relation to obtain an α for the longer range Th C′ alphas. Lewis and Wynn-Williams[5] used Briggs' results to obtain the straggling parameters for all natural alpha-particles. Their absolute values for α may be subject to question, and we will discuss the possible errors in a later section. There are good indications, however, that the variation of α with range is essentially correct and that the relative values are significant. We have chosen to use the Lewis and Wynn-Williams values where needed in this analysis, recognizing the possibility of errors in the absolute values. These values refer to the range straggling parameter only, and so do not necessarily apply to the straggling observed in a particular experiment. We will expect, therefore, that each experiment will have a straggling including types other than range straggling and so will usually be greater than the Cavendish value for the group.

[19] Reference 9, p. 283.
[20] Schulze, Zeits. f. Physik **94**, 104 (1935).
[21] Briggs, Proc. Roy. Soc. **114**, 313 (1927).
[22] Rutherford, Ward and Lewis, Proc. Roy. Soc. **131**, 684 (1931).

Experimental Arrangement and Calibrations

The alpha-particle source used in these investigations was a relatively weak source of polonium (0.5 mC) deposited on palladium from a $0.1N$ HCl solution saturated with hydrogen.[23] The source was about one year old but had not become appreciably thicker than its original condition. The number-range curves obtained show a straggling of only about 1.4 that of the best obtained in experimental measurements. The method of taking data was such as to record only mean ranges, and so eliminate the effects of straggling. In the supplementary experiments on measurements of the absolute mean range a fresh source was prepared and compared with the above to determine the necessary correction for the thickness of the source. This fresh source was prepared in the manner described by Rutherford by placing a drop of purified Po solution of $0.1N$ HNO_3 on nickel, and washing thoroughly before tarnish appeared. It was found to have a straggling only slightly greater than the accepted value for range straggling alone, but the intensity of the source was much weaker.

The ionization chamber was designed to allow the depth to be varied and determined to better than 0.05 mm, by mounting the back plate on a screw of 1 mm pitch. The front face was a closely woven nickel screen from a radio tube, rolled flat and mounted to form the high potential electrode. Microscope measurements showed the grid apertures to be rectangular, of 0.20 by 0.40 mm size; the flattened grid was 0.18 mm thick. The movable back plate was a brass disk $\frac{3}{4}$ inch in diameter and surrounded by a guard ring. It was insulated with amber rings and connected to the grid of the first amplifier tube with a 5-inch lead. The computed electrical capacity of the 1 mm chamber and shielded lead was 5.5 $\mu\mu f$. The source support was arranged to be moved between machined guides perpendicular to the chamber face by means of a 1 mm pitch steel screw equipped with a divided drum and index, and calibrated with a traveling microscope. The alpha-particles from the source

[23] Kanne, Phys. Rev. **52**, 380 (1937). The authors wish to thank Dr. Kanne for supplying the polonium source.

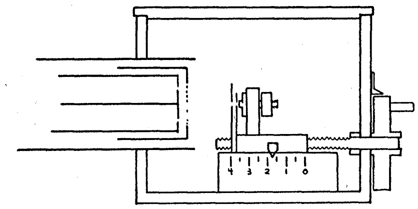

FIG. 2. Diagrammatic sketch of shallow ionization chamber and movable source holder with calibrated screw.

were collimated by two small holes having an average diameter of 0.0435 cm in thin brass sheets 0.365 cm apart immediately in front of the source. The physical distance between source and the inside surface of the grid forming the chamber face was obtained with a traveling microscope; it was found by trial that the distance could be determined to better than 0.005 mm. This was obtained at one setting of the scale and relative distances read off the drum and scale. A scale drawing of the physical arrangement is shown in Fig. 2.

All observations were made at room temperature and pressure and reduced to 15°C and 760 mm of Hg. The barometric pressure was read at intervals of $\frac{1}{2}$ hour during a run and the pressure at the time of the individual readings interpolated from a time chart of the variations. The pressure readings were corrected for the temperature of the mercury column through standard conversion tables, to the 0°C at which the barometer was calibrated. A 1/10°C thermometer near the chamber was read at 5 min. intervals. The total temperature and pressure correction varied between 7.6 and 8.1 percent during one run, and between 5.8 and 6.2 percent during the others.

The pulse amplifier and the counting circuit require special attention, since upon their performance depends much of the accuracy of the results. The amplifier is the resistance capacity coupled type employed by Wynn-Williams,[24] Dunning[25] and others. The usual care was taken in design to eliminate interstage coupling by having for each stage two shielded compartments, one for the tube and the other for the decoupling resistors and condensers. The input stage used a 38 tube operating at low plate and screen potentials. The second, third and fourth stages were voltage amplifiers (6C6 tubes). In the last two stages a 56 tube was used to drive a 210 tube, which had 475 volts plate potential in order to maintain linearity for large plate swings. The output of the amplifier was controlled by means of a potentiometer before the fifth stage. The gain of the amplifier was such that with a 2 mm chamber placed at the peak of the Bragg curve output pulses of a mean height of 250 volts could be obtained. The six stages of amplification were used in order to reduce the gain required of each stage. This insured that each tube would operate on only the linear part of its characteristic. The entire amplifier and thyratron circuit was enclosed in a copper shielding case which allowed operation undisturbed by the cyclotron oscillator in the same room. Five separate power supplies furnished the d.c. potentials for the entire amplifier. The high potential for the ionization chamber came from one supply. The first amplifier stage, with plate and screen voltages of the order of 10 volts, had a separate supply with very good filtering. The second, third and fourth stages had a common supply, as did the fifth and sixth. The latter two, being power stages, required very good regulation of their plate potentials. Another supply was necessary for the thyratron circuit as the flashing of this tube would seriously affect any other tube using the same power supply. The 60-cycle 110-volt supply for the amplifier was a 12 kw synchronous motor generator set which after a half-hour's running gave a very constant voltage.

The thyratron used for measuring output pulse heights was an RCA 885, chosen from several for its reproducibility. Operating at 130 volts plate potential it had a "flash-point" of -14.5 volts grid potential, varying slightly with plate potential and the ambient temperature. The calibration runs to be described below show that the thyratron has a linear characteristic between pulse height and negative grid potential above the flash point. The grid bias supply and meter limited the range of operation to -100 volts, so the gain of the amplifier was adjusted to supply pulse heights of a maximum of 85

[24] Wynn-Williams and Ward, Proc. Roy. Soc. **131**, 391 (1931).
[25] Dunning, Rev. Sci. Inst. **5**, 387 (1934).

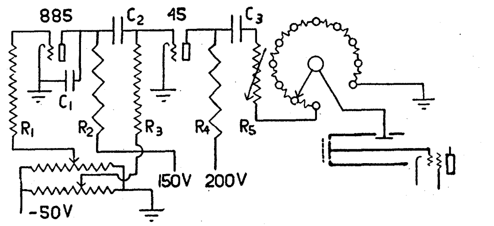

FIG. 3. Diagram of thyratron oscillator circuit for producing pulses of varying amplitudes and time constants; used to simulate alpha-particle pulses in calibrating the amplifier.

volts (when the chamber was set at the position of the peak of the Bragg curve).

In determining the amplitude response characteristic of the amplifier a circuit was used which gave uniform voltage pulses very similar in shape to those produced by an alpha-particle (Fig. 3). In this way the amplifier could be tested under conditions comparable to the counting of alpha-particles. It consisted of an 885 "thyratron oscillator" with condenser C_1 and grid potential adjusted to give a flash frequency of about 1000 per minute. The 45 tube was used to control the size of the pulses and to obtain a convenient inversion. The output was applied to a potential divider composed of ten 200 ohm metallized resistors and a variable resistor R_5. The taps of the voltage divider were connected to a small plate near the grid of the first stage amplifier tube, allowing potential pulses to be induced on the grid of known relative sizes and of a common shape. The time constant controls C_2, C_3 and R_4 were adjusted to make the pulse shapes indistinguishable from those due to alpha-particles, as determined by the use of a cathode-ray oscilloscope with a sweep frequency of 800/sec. The amplitude control R_5 was then set to give an output pulse from the amplifier of the order of 85 volts when the potential divider was set for the maximum. The uniform height pulses were superimposed upon the natural noise of the amplifier; a plot of counts against bias of the amplifier thyratron resulted in a curve similar to that obtained for alpha-particles except for the smaller straggling, which in this case is only that due to noise. The bias for half-maximum counting rate was determined for the various taps of the voltage divider by obtaining points just above and just below half-maximum. These mean bias values are plotted against the fraction of the maximum pulse height in Fig. 4. The plot is seen to be linear over the entire range and to extrapolate to the thyratron flash point of -14.5 volts for zero pulse height. The result makes it reasonably certain that the amplifier is linear in its amplitude response to voltage pulses over its working range. Since the size of a voltage pulse is proportional to the charge collected in the ionization chamber, it may be assumed that the mean pulse height will be proportional to the ionization produced in the chamber. Here and throughout this paper by *mean pulse height* we mean the difference between the mean thyratron bias and the flash point bias.

A noise background is always present in the amplifier, consisting of pulses, both positive and negative, with shapes determined by the time constant of the amplifier, and of such number that data cannot be taken at low bias voltages. Amplifier noise is due to the statistical fluctuations in electron current in the tubes and resistances of the first stage, and the ionization of residual gas in the tube. The use of low plate and screen voltages on the first tube (about 10 volts) reduces the gas noise. Microphonic response to acoustical noise or vibration may be minimized by mounting chamber and tubes on sponge rubber supports. It was found that the noise depended strongly upon the condition of charge or discharge of the filament battery; best results were obtained by discharging to about 0.9 of the capacity of the battery after full charge. The noise level sets a lower limit to the thyratron bias at which the counter is reliable. This was at -33 volts in the calibration run plotted in Fig. 4, or about 18 volts less than the -14.5 volt flash point. In the experiments to be described the noise amounted to about $\frac{1}{4}$ the maximum alpha-particle pulse height in the 1 mm chamber and $\frac{1}{6}$ that for the 2 mm chamber. In deeper chambers the noise is relatively smaller, although nearly the same in absolute magnitude. In general, there is nothing to be gained by distorting the response of the amplifier in order to suppress the noise, as the lower limit of observations exists whether the noise is visible in the oscilloscope or not, and produces a proportionate bias straggling.

Some confusion exists in the literature as to what is meant by the noise background of a pulse amplifier. The r.m.s. value, usually obtained from observations of the noise pattern on the oscilloscope screen, has no significance in counting experiments. The effective noise level is that below which reliable data cannot be taken because of the number of noise counts. This may be several times greater than the r.m.s. value. In describing the operation of a pulse amplifier the usual method is to give a signal to noise ratio, reported by several experimenters to be as large as 50/1. This is probably due either to the use of an r.m.s. value of noise, or to a nonlinearity in the amplifier response, discriminating against the small noise pulses. This last feature is valuable in simple counting experiments, and is readily obtained by adjusting the grid bias of any one of the amplifier tubes. The signal noise ratio has no meaning, however, unless the amplifier has been adjusted to be strictly linear and calibration runs made. An indication of the true signal noise ratio may be obtained from the straggling of number bias curves with alphas near the beginning of their range (for which the range straggling is small). The curves of Fig. 8 (to be described later) show a definite straggling, chiefly due to noise for chamber settings near the source; this straggling spread would have to be less than 1/50 of the average bias value to indicate a ratio of 50/1. No such sharp bias cut-off curves have been reported, and the indication is that such high signal-noise ratios are due to imperfect adjustment of the amplifier and nonlinear response characteristics. However, the signal to noise ratio of 6/1 obtained with the amplifier used in these experiments probably could be improved by a more careful selection of the tube for the first stage.

The instrumental straggling due to noise was determined from the slopes of the curves from which the half-maximum bias values plotted in Fig. 4 were obtained. An average slope was found to extrapolate from the half-maximum to zero in 4.6 volts of bias; this is the "s" for the straggling due to noise, in terms of volts of bias. With the uniform pulses there is no other source of straggling included, such as range or thick source effects. In the low bias region in which the data to be described later were taken it is found that the variation of bias voltage with distance is about 23 volts/mm, which corresponds to a noise straggling parameter α_2 of 0.023 cm. This figure will be used in the analysis of the observed straggling.

Observations and Analysis of the Method

When the collimated beam of alpha-particles was allowed to enter the chamber, and the counting rate observed as a function of distance from the source with a fixed thyratron bias, a number distance curve was obtained. This curve was similar to the analytic curve illustrated in Fig. 1, except for a slight unsymmetrical flattening near the top, due to the particles coming from the deeper layers of the source. It was found that number distance curves obtained with different bias settings led to curves which were displaced relative to each other. In Fig. 5 is

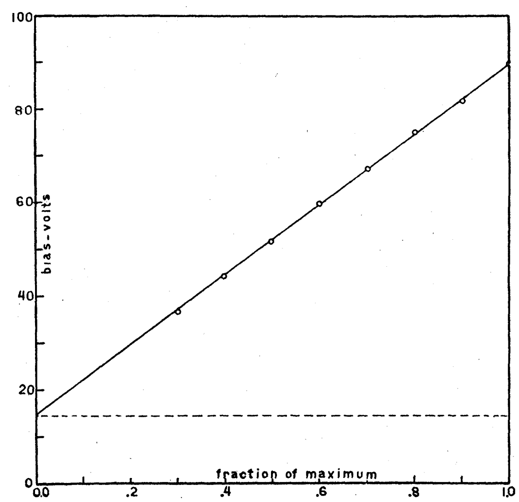

Fig. 4. Amplitude response characteristic of the pulse amplifier showing a linear response above the thyratron flash point.

shown a set of such curves obtained with a 1 mm chamber for bias settings varying from the minimum (just above the noise level of the amplifier) to a maximum (where a few counts are obtained for particles of maximum ionization). For the four lower bias values the curves have a common shape, showing parallel slopes and having the flat top characteristic of true number distance curves. Within this region the mean range and the extrapolated number distance

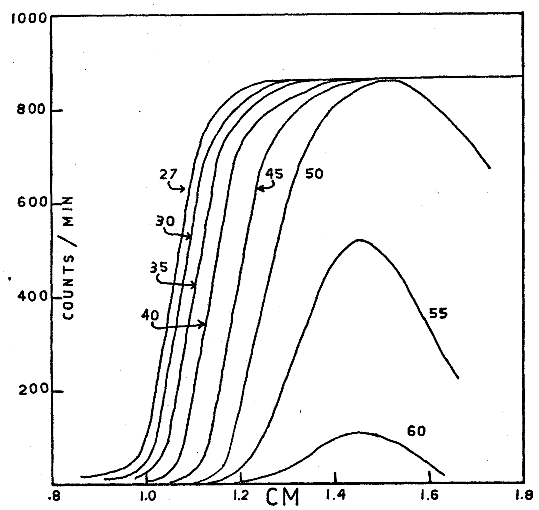

FIG. 5. Number distance curves obtained with the 1 mm depth chamber for a series of thyratron bias voltages, showing the manner in which observed ranges depend upon bias.

range could be obtained by determining the depth of penetration specified by the bias settings. The four higher bias curves all show maxima and changing slopes. The maximum is sharpest at about 55 volts and represents the position of the peak of the Bragg curve at the center of the chamber.[26] This is seen to be about 4 mm from the mean range (half-maximum) indicated by the lowest bias curve. In this region of changing slopes we are measuring the number of particles having a specific ionization greater than that specified by the bias setting.

The difficulty of analyzing such number distance curves is twofold. First, since the curves were taken only for bias values above the noise level, they do not give directly a measurement of the mean range or extrapolated number distance range, which are defined for zero pulse height (zero ionization). A plot of the extrapolated intercepts against pulse height does not result in a straight line and so cannot be extrapolated to zero pulse height with any precision; the same is true of the position of half-maximum representing the mean range. Furthermore, the ionization collected in the chamber does not start sharply at the grid face; there is an effective "electrical face" of the chamber at some distance in front of the grid face itself. Fields of 0.1 the value in the chamber may exist at distances of the order of the grid aperture on the outside of the grid.[27] So the electrical face of the chamber may be several tenths of a mm in front of the grid, and the chamber may have a correspondingly greater "electrical depth."[28]

A method has been devised to determine the position of the electrical face of the chamber, and at the same time to obtain the shape of the extrapolation to zero pulse height. This is accomplished by taking, for a series of biases, number distance curves for two depths of chamber, "1" mm and "2" mm, near the end of the range. The figures are in quotes to indicate that the true electrical depths as determined from the experiments will be different in each case. The physical conditions for the two cham-

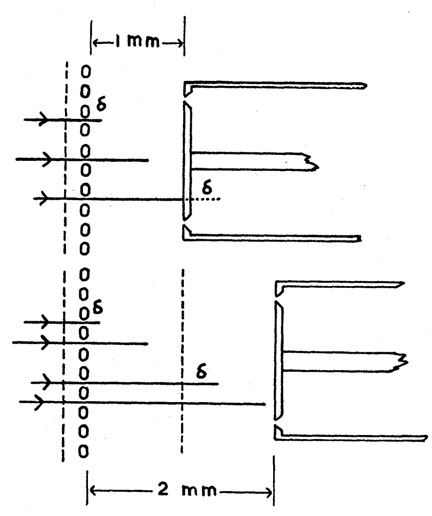

FIG. 6. Schematic diagram of chambers of 1 and 2 mm physical depth, with the external "electrical face" indicated by a dotted line. Particles with residual range δ at the electrical face produce equal ionizations in the two chambers; those with range δ at the back of the 1 mm chamber give different ionizations, the difference being the ionization of a particle of range δ.

[26] This high bias condition is valuable in separating groups of particles of different ranges; the distance between corresponding parts of the curves for different ranges at the same bias will be the difference in range. It can also be used to distinguish between particles of different specific ionizations, such as alpha-particles and protons. The counts for extremely high bias (60 volts) are probably due to a favorable superposition of noise fluctuations with the particle pulses.

[27] Brüche and Scherzer, *Geometrische Elektronenoptik* (Springer, 1934).

[28] The use of a grounded grid in front of the high potential grid might eliminate the extension of the electrical field. However, there is no advantage in this, since for any arrangement an experimental test must be made to determine the effective face.

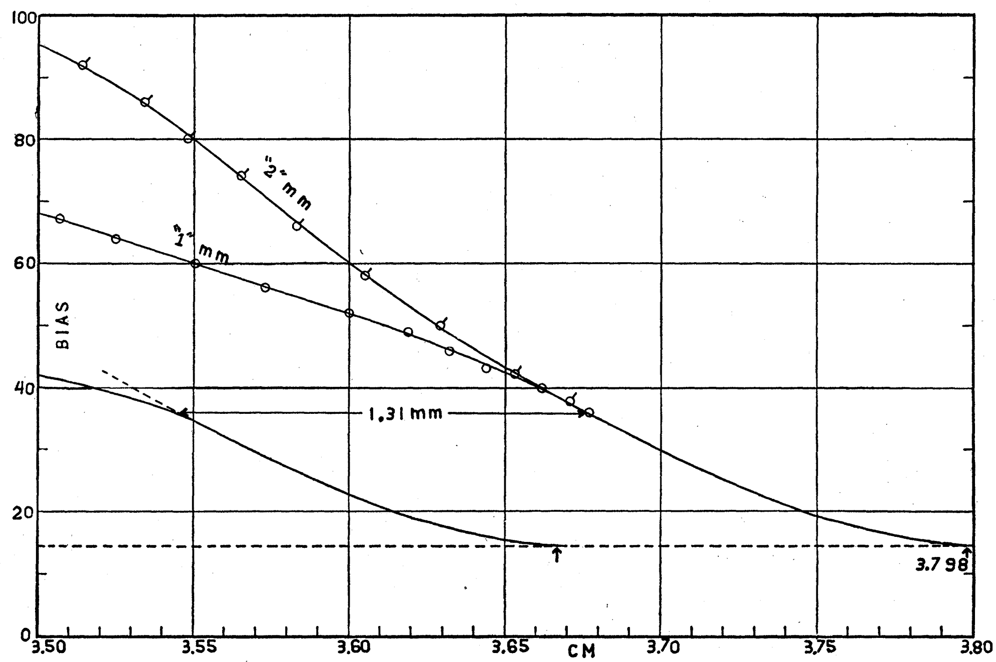

FIG. 7. Plot of observed distance for half-maximum counting rate as a function of bias voltage, for chambers with depths of 1 and 2 mm. The difference curve shown below, and also shifted to the right to join smoothly to the experimental curves, gives the ionization of particles of residual range less than 1 mm. The required shift of 1.31 mm represents the electrical depth of the 1 mm chamber.

bers are represented schematically in Fig. 6. Particles having residual ranges δ at the face of the chamber less than the depth of the "1" mm chamber will produce identical ionizations in the two chambers.[29] Particles having residual range δ at the *back* of the "1" mm chamber will produce different ionizations in the two chambers. This difference will be uniquely the ionization of particles of residual range δ, where δ is less than the 1 mm difference between the two chamber depths. It should be noted that for an alpha which does not completely cross the chamber (i.e., has residual range δ) it is not the specific ionization (averaged over the chamber depth) which is measured, but the

$$I_t(x)=\int_0^\delta I(x)dx,$$

where $I_t(x)$ is the total ionization from the end of range, and $I(x)$ is the differential specific ionization. The "2" mm chamber then gives us the total ionization of particles having a residual range up to about 2 mm.

For a number of different bias voltages and with the 1 mm chamber a series of observations were made of the positions for which the counting rate was half maximum. The maximum rate was measured with the chamber set at the peak of the Bragg curve (about 4500 counts/5 minutes). The distance to the source was increased until the counting rate was just above half-maximum (2250 to 3000 in 5 minutes) and then just below (1500 to 2250). The exact distance corresponding to half-maximum was then obtained by a linear interpolation. The bias voltages are plotted against the corresponding distances for half-maximum in Fig. 7. An entirely similar set of readings made with a physical depth of 2 mm and the same collecting field is also plotted in Fig. 7. These two curves are found to merge at their low bias ends, but to diverge for higher bias values. In order to analyze such data it is necessary that the

[29] Since it was possible that the change in electrical field due to doubling the depth of the chamber might result in a different collection time for the ions and so a different pulse height, observations were taken with full and half-voltage on each chamber. The results were identical, indicating that the collection time was very short compared to the time constant of the amplifier, and that there was no appreciable shift in the position of the electrical face.

two curves overlap, and the relative chamber depths to accomplish this result had been determined in previous preliminary observations.

Smooth curves are drawn through the experimental points and the differences between these curves plotted separately in Fig. 7. We have assumed a power relation between total ionization I_t and residual range R, *viz.* $I_t = R^n$, n being in general a function of R. Such a relation is found to hold with good accuracy over the last mm of the difference curve, with $n = 1.64$. Using this relation we find the end of the difference curve to be at 3.667 cm. If the effective depths of the two chambers had been exactly 1 and 2 mm, shifting of the difference curve by 1 mm should have made it coincide with the "2" mm curve. It is found, however, that the difference curve must be shifted 1.31 cm (at 15°, 760 mm) to make a smooth fit. This represents the electrical depth of the "1" mm chamber. All distance measurements were taken to the inside face of the grid, and corrected to 15°C, 760 mm. The 1 mm physical depth of the chamber has a reduced value of 0.92 mm, so the "electrical face" is at a position $1.31 - 0.92 = 0.39$ mm in *front* of the inner face of the grid to which measurements were made. The "2" mm chamber has then a depth of $0.92 + 1.31 = 2.33$ mm. The abscissae of Fig. 7 represent the distance to the electrical face of the chamber at 15°, 760 mm; the indicated end of range is at $3.667 + 0.131 = 3.798$ cm. From the magnitude of the experimental fluctuations of the data plotted in Fig. 7, and from the assumption made in extrapolating the difference curve, we estimate the probable error in the determination of the end of range to be ± 0.008 cm. *The end of the range is now defined by the extrapolation to zero total ionization of the total ionization distance curve of the single particle of mean range by use of the range exponent found to hold over the last mm of path.*

When counts are taken as a function of bias voltage with the chamber set at a fixed position, a number bias curve is obtained. Such a curve shows the distribution of pulse heights about a half-maximum or mean value. Half of the alpha-particles produce more ionization than this value, and half produce less. Therefore this mean bias value represents the specific ionization of the alpha-particle having a range equal to the mean range of the group. This follows from the fact that in a group of alphas half have their individual specific ionization curves shifted to one side of the specific ionization curve of the alpha of mean

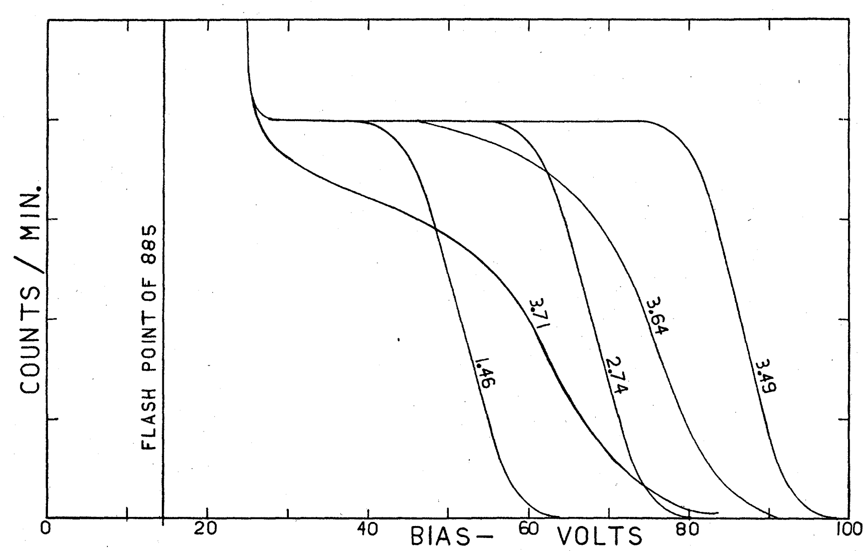

FIG. 8. Number bias curves obtained with the 1 mm depth chamber for settings at various distances from the source. The mean bias (for half-maximum counting rate) represents the specific ionization of the average alpha; the slopes of the curves are determined by the instrumental straggling, increased near the end of the range by range straggling.

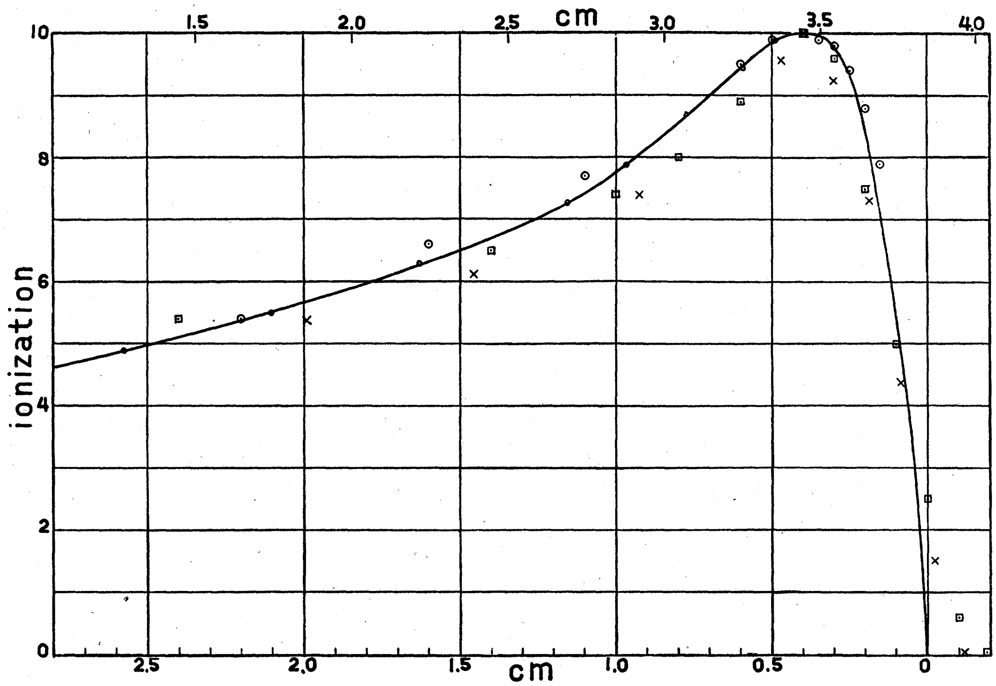

FIG. 9. The specific ionization curve of a single alpha-particle corrected for the effect of the finite chamber depth. The experimental points (small solid circles) are the values of mean bias plotted against the position of the electrical center of the chamber. The scattered points are taken from other reported single particle ionization curves: Feather and Nimmo (large circles), Stetter and Jentschke (squares), and Schulze (crosses).

range, and half to the other side. Due to the finite slope of the specific ionization curve those alphas with curves shifted to one side will produce greater (or less) ionization than those with curves shifted to the other side.

A family of number bias curves obtained for several settings of the chamber position is shown in Fig. 8. The figures on the curves represent the distance between source and chamber. In each curve a straggling of pulse heights about the mean value is observed, and a linear portion which can be extrapolated to the axis. Where the range straggling is small (close to source), the slopes are nearly uniform and can be used to determine the straggling in pulse heights due to the instrument alone. The slopes increase with increasing distance, due to the effects of the range straggling of the individual particles on their ionization in the region of rapidly decreasing specific ionization. The most significant feature of one of these curves is the half-maximum or mean bias. As the distance between source and chamber is increased this mean bias increases, reaching a maximum when the chamber is set at the peak of the Bragg curve. With further increase the mean bias drops rapidly. A plot of pulse heights obtained from such mean bias values, as a function of distance, is the specific ionization of the alpha-particle of mean range.

Using a "1" mm chamber the bias for half-maximum counting rate was determined at different points along the path. For each setting of position three bias values were used, one to determine the maximum counting rate (35 volts) and the others at biases giving counting rates just above and just below half-maximum. The mean bias value is obtained by linear interpolation between the latter two points. The corresponding mean pulse height is proportional to the ionization per 1.31 mm length of path when plotted against the distance to the "electrical center" of the chamber ($\frac{1}{2}(1.31)-0.39=0.27$ mm *behind* the physical face). It is desirable to have the differential specific ionization rather than that in any finite depth of chamber. This requires a correction for the variation of the specific ionization in the chamber depth, which can be determined from the curvature of the observed curve.[30] The corrected curve is shown in Fig. 9; it shows $I(x)$, the differential specific ionization (relative to that at the peak), for a single alpha-particle as a function of distance from the end of the range. The small heavy circles are experimental points. For comparison with the results of other observers points are indicated taken from the reported single particle ionization curves of Feather and Nimmo,[12] Schulze,[20] and Stetter and Jentschke,[31] matched at the peak. The method described above is neither valid nor practicable within the last 1.31 mm of range, but points can be obtained in this region from the slope of the bias distance curve of Fig. 7. The curve in Fig. 7 represents $I_t(x)$, the total ionization up to 2.23 mm residual range. But since

$$I_t(x)=\int_0^x I(x)dx, \quad I(x)=dI_t(x)/dx.$$

In this manner $I(x)$ can be obtained for the last 2.23 mm of range.

A larger scale plot of the end of the curve beyond the peak is shown in Fig. 10. Here the dotted curve is drawn through the observed points (circles) with the 1.31 mm chamber. The solid curve is corrected to zero chamber depth (the differential specific ionization) and extended to zero residual range by means of the points obtained from the slope of the bias distance curve (crosses). The resulting curve fits the experimental points accurately and shows an essentially vertical intercept at the end of the range.[32]

A graphical integration of the areas under the curves of Figs. 9 and 10 was next performed to obtain a relation between range and total ionization. If we assume that the energy loss per ion pair remains constant throughout this region, the integration gives a relative range energy relation for alpha-particles having a range equal to the mean range of the group. This requires that the ratio between energy loss by ionization and by excitation remain constant. We consider this assumption to be the chief cause of uncertainty in the range energy relation. The curve obtained was adjusted to match the natural alpha relation in the region between 2.5 and 2.8 cm residual

[30] Let $f(x)$ be the ordinate representing the observed ionization in a chamber of depth l, whose center is at x. Then:

$$f(x)=E(x+\tfrac{1}{2}l)-E(x-\tfrac{1}{2}l), \tag{1}$$

where $E(x)$ is the energy of a particle of residual range x. By the expansion of the right-hand side of (1) in a Taylor's series about x:

$$f(x)=l\left(\frac{dE}{dx}+\frac{1}{6}\left(\frac{l}{2}\right)^2\frac{d^3E}{dx^3}+\cdots\right). \tag{2}$$

Similarly, if we expand $f(x+\tfrac{1}{2}l)+f(x-\tfrac{1}{2}l)$ about the point x:

$$f(x+\tfrac{1}{2}l)+f(x-\tfrac{1}{2}l)=2f(x)+l(\tfrac{1}{2}l)^2d^3E/dx^3+\cdots. \tag{3}$$

Since the ionization $I(x)=ldE/dx$, by combining (2) and (3) we obtain:

$$I(x)=(4/3)f(x)-(1/6)f(x+\tfrac{1}{2}l)-(1/6)f(x-\tfrac{1}{2}l),$$

where $I(x)$ is the differential specific ionization at x.

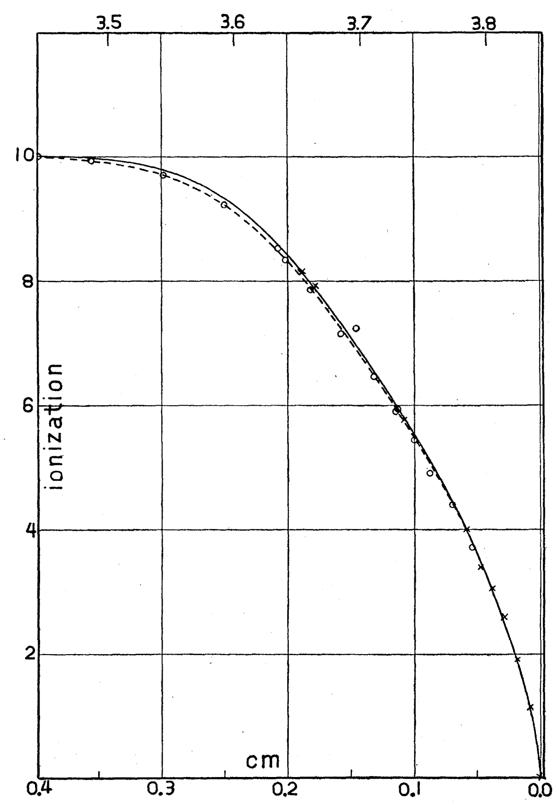

FIG. 10. Enlarged scale plot of the end of the single particle specific ionization curve, showing observed points from number bias data (circles) and from the slope of the range bias curve of Fig. 7 (crosses). The solid curve is the differential specific ionization, corrected for the effect of finite chamber depth.

[31] Stetter and Jentschke, Physik. Zeits. **36**, 441 (1935).

[32] The assumption of a constant range exponent used in obtaining the extreme end of range makes the shape of this extrapolation uncertain within the last 0.1 mm. The known phenomena of capture and loss of charge in this region would suggest that the true ionization curve has a slight extension at the foot. However, in order to provide a satisfactory standard on which to base range determinations we have used the assumption of constant range exponent resulting in the vertical intercept.

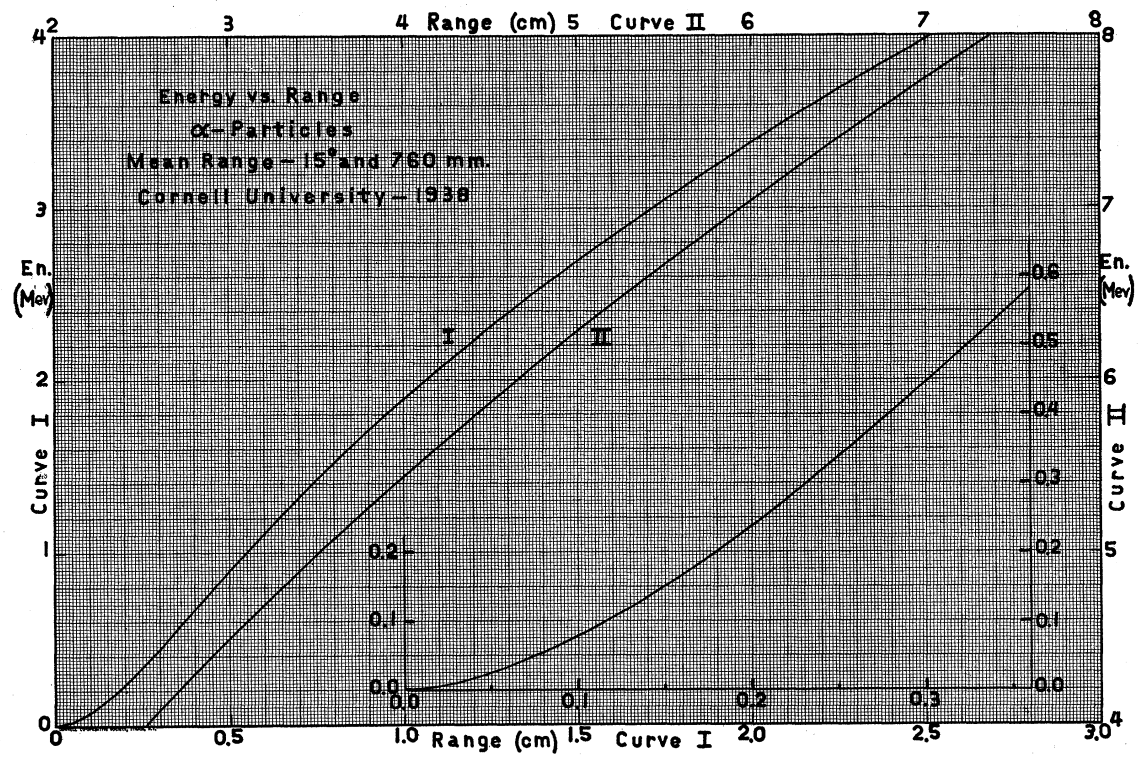

FIG. 11. Energy *vs.* range for alpha-particles. Obtained by graphical integration of the specific ionization curve of Fig. 9 and matched to the known relation in the region of natural alphas. Accuracy: 5 kv from 0 to 0.5 Mev, 1 percent from 0.5 to 2.0 Mev, 20 kv from 2.0 to 4.0 Mev, 0.5 to 0.3 percent from 4.0 to 5.0 Mev, and 0.2 percent in range and 0.1 percent in energy from 5.0 to 8.0 Mev.

range where it is known to a good accuracy. The result, in the customary Mev units is shown in Fig. 11 on a sufficiently large scale to be used directly. The method used was to plot the ranges and energies of the natural alphas, corrected as indicated in the discussion to follow and given in Table II, and to extend it to energies below 5 Mev by the use of Mano's[6] data on the energies of retarded alphas (also corrected to the new standard range). When matched in absolute value the experimental curve also matched the natural alpha-curve accurately in slope over a considerable region, a partial justification of the assumption above. For if there had been any appreciable variation in the energy loss per ion pair at low energies the computed range ionization curve would not have had the same slope as the range energy curve.

Straggling

In Fig. 5 the low bias number distance curves are observed to have a common slope which is a measure of the total straggling. The difference between mean range (half-maximum) and the extrapolated range, under standard temperature and pressure conditions, shows an average value, s, of 0.070 cm. Another estimate of this total straggling is obtained from the slopes of the straight lines used in interpolating the values of half-maximum count which are plotted in Fig. 7. From an average value of slope in the low bias region we obtain an s of 0.079 cm. The first method has a statistical error due to smaller numbers of counts on each point (1 minute); the second has an error due to the long extrapolation through the two points near the half-value. They are probably of equivalent accuracy and the variation in s obtained is an indication of the errors. We use an average of 0.074 cm as the best estimate of the distance s. This results in a total straggling parameter, $\alpha_t = 0.074/(\frac{1}{2}\pi^{\frac{1}{2}}) = 0.084$ cm.

The observed straggling is the resultant of many separate straggling effects. We will list and discuss these in turn:

(*a*) *Range straggling.*—The experimental value of the range straggling parameter which we have chosen to use is that given by Lewis and Wynn-Williams[5] for Po ($\alpha_1=0.062$ cm), obtained from Briggs'[21] measurements of range straggling in mica and reduced to air. In the discussion to follow the experimental Bragg curves of Curie and Naidu are found to be in good agreement with this value.

(*b*) *Noise straggling.*—Analysis of the slopes of the number-bias curves of the amplifier calibration run gave an "*s*" of 0.020 cm; from this $\alpha_2=0.023$ cm.

(*c*) *Ionization straggling.*—The chamber used in these measurements has an electrical depth at standard conditions of only 1.31 mm. The number of ionization collisions occurring in this length of path can be estimated to be less than 5000 for particles with 1 mm or less residual range, allowing a statistical variation of bias in the various readings. Furthermore the statistical variation of energy loss per collision about the mean value (32 volts/ion pair) results in additional straggling. An approximate calculation of these two effects leads to a range equivalent of this ionization straggling, α_3 of 0.011 cm.

(*d*) *Angular straggling.*—The geometry of the collimation slits leads to a correction due to the fact that the average alpha-particle observed makes a slight angle with the normal (see below). The variation in angle about this average angle within the collimation limits introduces a straggling which is nearly symmetrical about the mean angle. The total correction for the average angle was only 0.0043 cm, so the angular straggling, α_4, will be only a small fraction of this, and may be neglected.

(*e*) *Chamber depth straggling.*—The method of taking the data results in bias readings determined by the chamber depth and specific ionization; the smooth curves plotted through the points and the methods of subtracting bias curves have essentially reduced the depth of the chamber to zero. However, the electrical face of the chamber as determined is an average value, and it will vary for particles entering the chamber adjacent to a grid wire or in the center of a grid aperture. This depends on the shapes of the equipotential lines near the face of the grid, which are not known. An estimate of the straggling can be obtained from the difference between the outer surface of the grid wires and the average electrical face, which is about 0.021 cm. Assuming this to represent the over-all straggling ($2s$) we obtain a $\alpha_5=0.012$ cm.

(*f*) *Source straggling.*—The straggling effects discussed above are essentially symmetrical, and would result in a distribution centered about the mean range. As such, they can have no effect on the measured value of mean range. However, there is at least one other effect which is not symmetrical, the straggling due to depth of source. This will not only increase the total straggling observed, but it will result in a smaller value of mean range. The best estimate of the source straggling comes from the observed difference in range of particles from the relatively thick source used in the range and ionization measurements, and the freshly prepared source which had no appreciable thickness. The observed difference in range of these two sources under identical conditions was 0.042 cm (see below). This is directly the correction to be applied to the mean range value obtained and from it we can deduce a $\alpha_6=0.048$ cm.

If our analysis of the various straggling factors is correct we should be able to deduce the total straggling parameter from the individual effects. We find for the collimated group of polonium alphas $\alpha_t=(0.062^2+0.023^2+0.011^2+0.012^2+0.048^2)^{\frac{1}{2}}=0.083$, in agreement with the observed $\alpha_t=0.084$ cm.

Determination of the Absolute Mean Range of Po Alphas

The mean range of 3.798 cm determined above and illustrated in Fig. 7 must be corrected for certain known factors, such as the angle of collimation, the thickness of source and the composition of the air (absolute ranges are for dry air at 15°C and 760 mm of Hg pressure).

The largest error in absolute range was that due to the finite thickness of source, and indicated by the straggling. The correction was obtained directly by observing the difference between the source used above and a freshly prepared source having no appreciable thickness. The fresh source was first checked for straggling, and was found to give essentially that expected from range and instrumental straggling alone. With a common bias voltage of 50 volts, corresponding to 0.418 of the peak of the specific ionization curve, measurements were made of the positions for half-maximum counting rates for the two sources. The absolute measurements of distance from source to chamber face were taken and reduced to standard conditions; the distance for the new source was found to be 0.042 cm greater than for the old source. This is exactly the correction to be added to the end of range observed; it is the "s" of the source straggling and gives a α_6 of 0.048 cm.

The imperfect collimation of the beam would result in an apparent range less than the true range for perfect collimation. The collimation used allowed a maximum deviation from the direction normal to the chamber face of 6.8 degrees. The average particle observed corresponds to an average angle, and hence to a distance slightly less than the true range along the normal. This average angle, θ, can be determined following the procedure outlined by Bethe.[33] The distribution of particles with angle may be approximated by a Gaussian function: $g(\theta) \backsimeq e^{-(\sin^2 \theta/\beta^2)}$; where $\beta^2 = (1.27a^2/b^2) = 0.0045$ and where a is the average radius of the collimating holes and b the distance between them. By integration we find the average $\langle\theta^2\rangle_{Av} = \frac{1}{2}\beta^2$. The correction to the observed range, for a particle of 3.84 cm range is $3.84((1/\cos\theta)-1) = 3.84(\frac{1}{4}\beta^2) = 0.0043$ cm. On adding these two corrections to the observed value of the end of range of 3.798 ± 0.008 cm, we obtain a value for the absolute mean range of Po alphas of 3.840 ± 0.010 cm. This probable error includes an arbitrary error of ± 0.006 cm which, because of the uncertainty in the corrections, we have added to the purely experimental errors.

The relative humidity of the air was about 20 percent at 25°C during the experiments, indicating a water vapor content of 0.6 percent of the observed pressure. Taking the stopping power of water vapor to be $\frac{3}{4}$ relative to air we find a correction in range of -0.007 cm. The CO_2 content of the air in the closed room was estimated to be 0.5 percent greater than normal, resulting in a positive correction of about the same magnitude. Since the resultant correction could not be determined accurately and would be smaller than the experimental error, no correction was applied. The standardization of the gravitational constant to 45° latitude was estimated and found to be trivial.

[33] Reference 9, p. 279.

Comparison with Results of Other Experiments

The best determinations of the ionization extrapolated range for alpha-particle groups are by Henderson.[3] He obtains for Th C′ and Ra C′, respectively, the values: 8.616 ± 0.004, and 6.953 ± 0.004 cm in dry air at 15°C and 760 mm of Hg pressure. Harper and Salaman[34] check the Th C′ result, with a larger error. Curie and Behounek[35] and Harper and Salaman[34] are in sufficient agreement for Ra C′, but Naidu[15] reports a low value. For Po three observers are in excellent agreement; Curie[4] gives $3.870(\pm 0.006)$, Naidu[15] 3.868 ± 0.006 and Harper and Salaman[34] 3.870 ± 0.008 cm. Since no other method of measurement has resulted in such precise determinations, ionization measurements should be used in establishing the absolute range scale. We have chosen to use Henderson's values of Th C′ and Ra C′ and the average of Curie's and Naidu's results for Po as standards of ionization extrapolated range.

The most complete survey of the relative mean ranges of natural alpha-particle groups has been made in the Cavendish Laboratory with the differential ionization chamber.[1,5] The measurements were all relative to Th C′ as a standard, and this standard was established by comparisons with Henderson's results. Lewis and Wynn-Williams[5] used the only single particle ionization curve available at that time, obtained by Feather and Nimmo[12] from the photographic density of alpha-particle tracks in a cloud chamber, and the data of Briggs[21] on the straggling parameters for the various groups, to construct an average ionization (Bragg) curve. With several assumed values of α they computed average ionization curves, and from the extrapolated intercepts they obtained an empirical relation for the difference between such extrapolated values and the mean range: $x = 0.8\alpha - 0.006$ cm. They then used this relation to determine the absolute scale for their relative mean range determinations, namely, that for which Th C′

[34] Harper and Salaman, Proc. Roy. Soc. **127**, 175 (1930).
[35] Curie and Behounek, J. de phys. et rad. **7**, 125 (1926).

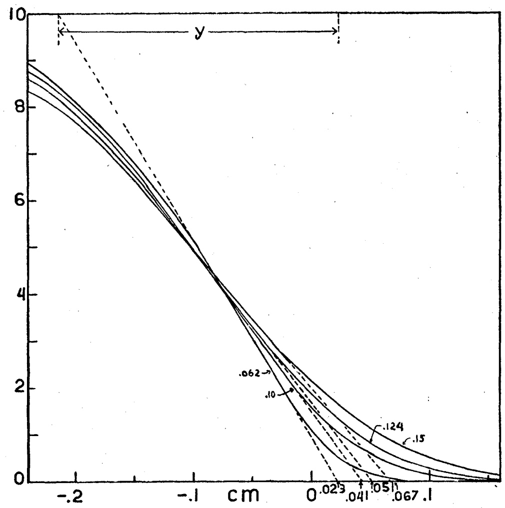

FIG. 12. Average ionization (Bragg) curves computed from the single particle ionization curve and the range distribution functions characterized by straggling parameters of 0.062, 0.100, 0.124 and 0.150 cm. The extrapolated intercepts are used to obtain an empirical relation between mean range and ionization extrapolated range. The slopes, expressed in terms of the intercept distance, y, provide a method of determining the straggling from experimental ionization curves.

alphas had a mean range of 8.533 cm. It is "the value such that the extrapolated ranges calculated from the mean ranges determined for Th C′, Ra C′ and (with less weight) Th C and Po, differ by a minimum amount from the most accurate absolute determinations of these extrapolated ranges." These minimum differences are +0.007, −0.008 and −0.021 cm for Th C′, Ra C′ and Po, greater than the stated errors in the measurements. This value for Th C′ has been accepted and used as a range standard. It should be pointed out that it is a derived value, based upon an incorrect specific ionization curve for a single alpha-particle.

The first single particle ionization curve reported was deduced by Curie[4] from an observed average ionization curve and a measured range straggling parameter for Po. Following the early work of Feather and Nimmo[12] others have attempted to measure the single particle curve, but their results were handicapped by instrumental straggling. Schulze[20] used a proportional amplifier and measured the average size of pulses due to the ionization produced in a short section of path; his results show a decided "tail" similar to the average ionization curve. Stetter and Jentschke[31] used two shallow chambers and linear pulse amplifiers and measured average heights of oscillographically recorded pulses. They obtain a steeper curve, in rough agreement with that obtained in these investigations.

With our more precise determination of the single particle ionization curve it is possible to make a better analysis of the Bragg curves. Using assumed Gaussian distributions with range straggling parameters of 0.062 (Po, illustrated in Fig. 1), 0.100, 0.124 and 0.150 cm we have computed the shapes of the average ionization curves.[36] They are illustrated in Fig. 12. From the indicated extrapolated intercepts we find that the empirical relation between x (the difference between mean and ionization extrapolated range) and α is essentially linear and can be expressed as: $x=0.47\alpha-0.006$ cm. Extreme lines drawn through the points suggest a probable error of 8 percent in the region of the natural alpha-particles, exclusive of the error in α. It is now possible to use this empirical relation in the manner of Lewis and Wynn-Williams to relate extrapolated ionization ranges and relative mean ranges. However, before doing this it is essential to consider that each experimental observation of extrapolated range includes possible straggling factors other than range straggling, and so may have an α differing from the range straggling parameter used by Lewis and Wynn-Williams.

The most definite quantity connected with the shape of a computed average ionization curve of Fig. 12 is the slope of the straight line used in the extrapolation. When the slopes are plotted against the corresponding values of α a relation is observed which is linear to a better accuracy than the empirical relation between x and α described above. If the slope is expressed in terms of the distance y between the upper and lower intercepts of this straight line (through the ordinates representing the peak of the curve and

[36] This was done following the procedure outlined by Lewis and Wynn-Williams in obtaining the average ionization, $I: I=\sum_{x=z}^{x=\infty} f(x)I(x-z)$, where $I(x-z)$ is the ionization produced by a single alpha-particle in a short interval at an average distance $x-z$ from the end of its track, and $f(x)$ is the distribution function characterized by the chosen value of α. The summation was performed with intervals of 0.02 cm, and the result normalized to fit the maximum to the peak of the Bragg curve.

zero ionization) we obtain a relation: $\alpha = 0.917y - 0.160$ cm$\pm$5 percent. A value for y may be obtained from any observed ionization curve, and this relation used to determine the α characteristic of the experiment.

Henderson's published curve for Th C′ has a value y of 0.304 cm, indicating an α of 0.119 cm, in good agreement with the Cavendish value of 0.120 cm. For his Ra C′ curve $y = 0.292$ and $\alpha = 0.108$, somewhat higher than the range straggling value 0.102 cm and indicating additional straggling in the experiment. For Po the curve by Curie shows $y = 0.242$ and $\alpha = 0.062$ while that of Naidu gives $y = 0.246$ and $\alpha = 0.066$ cm. The experimental values of α determined in this way are in general slightly higher than the Cavendish range straggling values, which is to be expected. The reasonably close checks indicate that the Cavendish tabulation of α is essentially correct and, furthermore, that our computed ionization curves are in satisfactory agreement with the experimentally observed curves in this essential feature of slope of the extrapolation line. When the experimental curves are compared point-by-point with the computed curves for the same α the agreement is not as satisfactory, the experimental curves in general falling below in the region of 0.7 to 0.9 of the peak, and also differing in the shape of the tail past the extrapolated limit. We assume that the slope of the front of the curve is less dependent upon variable experimental factors than the extreme regions.

We are now able to compute the mean ranges indicated by the ionization extrapolation measurements of the standards chosen. Using the relation $x = 0.47\alpha - 0.006$ cm, and values of α obtained above from the observed curves we obtain the values given in Table I. When the Lewis and Wynn-Williams values for relative mean ranges are increased by 0.037 cm we obtain for the standards 8.570, 6.907 and 3.842 cm, which differ by minimum amounts from the computed mean range values. These differences are now 0.004, -0.001 and -0.003 cm, less than the experimental errors in the ionization measurements and also less than the relative errors of Lewis and Wynn-Williams. In order to preserve the relative accuracy inherent in the differential ionization chamber data we have chosen to use as mean range standards the Lewis and Wynn-Williams values for Th C′, Ra C′ and Po, increased by 0.037 cm.

The validity of the various methods for determination of mean range is indicated in the case of Po. That computed from extrapolated ionization range (Bragg curves) is 3.845 cm; that obtained by correcting the Lewis and Wynn-Williams relative mean range is 3.842; the value obtained from the number distance bias method reported in this paper is 3.840 cm; and a direct cloud chamber study by Curie gives 3.85 cm. These four measurements, obtained by four different techniques, are in excellent agreement, well within the limits of error. A weighted average of these values gives a range of 3.843 $\pm$0.005 cm for Po. It is interesting to note that the cloud chamber determination of mean range checks the corrected mean ranges obtained from electrical measurements. Hence the apparent disagreement between ranges of nuclear disintegration particles observed in the cloud chamber and with ionization chamber and counter techniques was largely a result of the low range standards.

In Table II the corrected values for mean range, the probable error in the mean range, the straggling parameter, α, the ionization extrapolated range and the number distance extrapolated range are tabulated with the magnetically determined energies. The values in bold faced type in Table II are those chosen as standards, the starred values are those for which measurements were made relative to a standard. Unstarred values for the other types of range are derived. The computed extrapolated ranges are based upon the tabulated values of range straggling parameter, α. The set of values A are the corrected relative mean ranges of Lewis and Wynn-Williams;[5] the set B for the long range groups are from Rutherford, Wynn-Williams,

TABLE I. *Determination of mean range standards.*

	Ion. extr. range R_i:	Conv. factor ($x = .47\alpha - .006$) $R_i - R_m$:	Mean range std. (computed): R_m:	Rel. mean range std.: (L. & W. W. +0.037)
Th C′ (Hend.)	8.616±0.004	0.050±0.005	8.536±0.007	8.570
Ra C′ (Hend.)	6.953±0.004	$0.045\pm0.004_5$	6.908±0.006	6.907
Po				
(Curie)	3.870±(0.006)	$0.023\pm0.002_3$	3.847±0.007	
(Naidu)	3.868±0.006	$0.025\pm0.002_5$	3.843±0.007	
Po (Av.)			3.845±0.006	3.842

Lewis and Bowden;[1] the U values, C, are those reported by Rayton and Wilkins[14] relative to Po. The errors in mean ranges are obtained from the stated experimental errors plus the error involved in the conversion to mean range from ionization range. The values for α were taken directly from the tabulation by Lewis and Wynn-Williams[5] with the exception of the bracketed items which were extrapolated from the Cavendish relation of α and mean range.

The energies of the alphas in the various groups have been measured by the magnetic deflection method. The energy values in Table II are from tabulations by Lewis and Bowden,[37] Briggs,[2] and Rosenblum and Dupouy.[38] Lewis and Bowden base their energy scale upon a velocity of $(1.922_{00}\pm 0.002)\times 10^9$ cm/sec. for the alphas of the 9.907 cm group from Ra C′. Briggs, using improved apparatus and a better value of e/m for He, has redetermined this velocity and finds it to be $(1.92148\pm 0.00009)\times 10^9$ cm/sec. This latter velocity corresponds to an energy of $(7.6802\pm 0.0006)\times 10^9$ electron volts. On this basis Briggs has recomputed the energies of the alphas of several groups (Rn, Ra A, Ra C′, Tn, Th X, Th A, and Th C′) whose relative velocities he had measured. His energy values are the ones given in Table II for these groups. To get the energies of the alphas in the other groups the values of Lewis and Bowden and of Rosenblum and Dupouy are multiplied by the ratio, 7.6802/7.6830, between Briggs' new Ra C′ energy value and that of Lewis and Bowden. Of these latter groups the An and Ac C (short and long) were measured by Rosenblum,[38, 39] and the rest by the Cavendish group. The probable errors in the table are taken from the original papers.

TABLE II. *Ranges and energies of natural alpha-particles.*

		Mean range:	Acc:	Strag. par. α:	Extrap. ranges Ioniz:	Extrap. ranges N-dist:	Energy (Mev):	Energy Acc:
A:	An (short) (mean)	5.240*	±0.015	0.081	5.272	5.312		
	An (long)	5.692*	±0.015	0.087	5.727	5.769	6.8235	±0.014
	Ac A	6.457*	0.008	0.096	6.496	6.542		
	Ac C (short)	4.984*	0.015	0.078	5.015	5.053	6.272_7	0.012
	Ac C (long)	5.429*	0.015	0.083	5.462	5.503	6.618_6	0.013
	Ac C′	6.555*	0.015	0.097	6.595	6.641		
	Tn	5.004*	0.008	0.078	5.035	5.073	6.2818	0.0008
	Th A	5.638*	0.008	0.086	5.672	5.714	6.7744	0.0008
	Th C (mean)	4.730*	0.008	0.074	4.778	4.796	6.0537	0.0007
	Th C′	**8.570**	0.007	0.120	**8.616**	8.676	8.7759	0.0009
	Rn	4.051*	0.008	0.065	4.076	4.109	5.4860	0.0005
	Ra A	4.657*	0.008	0.073	4.685	4.722	5.9981	0.0005
	Ra C′	**6.907**	0.006	0.102	**6.953**	6.997	**7.6802**	0.0006
	Ra F (Po)	**3.842**	0.006	0.062	**3.870**	3.897	5.2984	0.0021
B:	Ra C′ (long)	7.792*	0.015	(0.112)	7.839	7.891	8.2769	0.0033
	Ra C′ (long)	9.04*	0.02	(0.125)	9.09	9.15	9.0655	0.0037
	Ra C′ (long)	11.51*	0.02	(0.15)	11.57	11.64	10.5052	0.0043
	Th C′ (long)	9.724*	0.008	(0.132)	9.780	9.841	9.4877	0.0038
	Th C′ (long)	11.580*	0.008	(0.15)	11.64_3	11.71_3	10.5379	0.0043
C:	U I	2.653*	0.007	(0.045)	2.669	2.693		
	U II	3.211*	0.007	(0.053)	3.230	3.258		

Bold face=standards.
*=Measured relative to a standard.

Geiger[40] has reported ionization measurements of some 20 alpha-particle groups, giving extrapolated ranges differing so widely from the Cavendish results that they are difficult to evaluate. For example, he reports extrapolated ranges for Th C′, Ra C′, and Po of 8.617±0.007, 6.971 ±0.004 and 3.925±0.004 cm, respectively, differing from the accepted values above by +0.001, +0.018 and +0.055 cm in the three cases. In the case of Ac A his value is 0.088 cm greater than that computed from the Cavendish results (Table II), yet he quotes an error of only ±0.010 cm. In attempting to find a reason for these discrepancies we have determined the straggling parameter from the slope of the extrapolation line of his published curve for Ra C′ through the method described above, and obtain a value of nearly 0.20 cm. This is essentially double the range straggling for Ra C′ and would result in a much larger extrapolated range than that obtained by Henderson. We must conclude that Geiger's high values are due to excessive straggling, seemingly differing widely from one group to another. The curves reported by Geiger contain groups and hence the straggling cannot be determined at all accurately. For this reason Geiger's curves are not subject to the analysis used above and so are not included in the tabulation.

The Range Energy Relation

The new values for mean range of the natural alphas require a small but significant correction

[37] Lewis and Bowden, Proc. Roy. Soc. **145**, 235 (1934).
[38] Rosenblum and Dupouy, J. de phys. et rad. **4**, 262 (1933).
[39] Curie and Rosenblum, Comptes rendus **196**, 1663 (1933).
[40] Geiger, Zeits. f. Physik **8**, 45 (1922).

to the range-energy relation, essentially increasing the range for all values of energy in this region by 0.037 cm. In Fig. 11 is plotted the complete corrected relation, based upon the natural alpha-particle ranges given in Table II. In this region the errors given for mean range and energy specify the accuracy of the curve, about 0.2 percent in range and 0.1 percent in energy. The extension through Mano's reduced range values in the region in which the experimental curve was matched (about 4 Mev) is of somewhat less accuracy, estimated to be about 0.5 percent at the point of matching. Errors in the single particle ionization curve are thought to be of the order of ± 1 volt in bias values; the range bias curve obtained by graphical integration will be somewhat more accurate due to the smoothing inherent in the graphical integration process. When matched at zero and at 4 Mev the maximum percentage error will be in the median region (about 2 Mev). We estimate ± 5 kv from zero to 0.5 Mev, 1 percent from 0.5 to 2 Mev, and 20 kv from 2 to 4 Mev. In addition to the experimental errors, a possible variation of the energy loss per ion pair may further increase the error in the low energy region.

The range energy relation described above[41] supercedes the Cornell relation of 1937.[9] The theoretically determined curve is in close agreement with the new experimental relation, however, agreeing to within its limits of error even in the low energy region. The theoretical relation for energies above 8 Mev must be corrected for the change in the alpha-particle standards. This requires the addition of 0.037 cm to the range values of the published curves.[9] In the extreme low energy region the experimental points of Blackett and Lees[8] match the curve with reasonable accuracy when corrected by 9 percent for the range energy relation used.[9] The unpublished data of Blewett and Blewett on the ranges and energies of retarded alphas are in reasonable agreement between 3.5 and 5 Mev when corrected to the revised Po range standard,

[41] A limited number of large scale blue prints of this relation are available and may be obtained by addressing the senior author (M. S. L.).

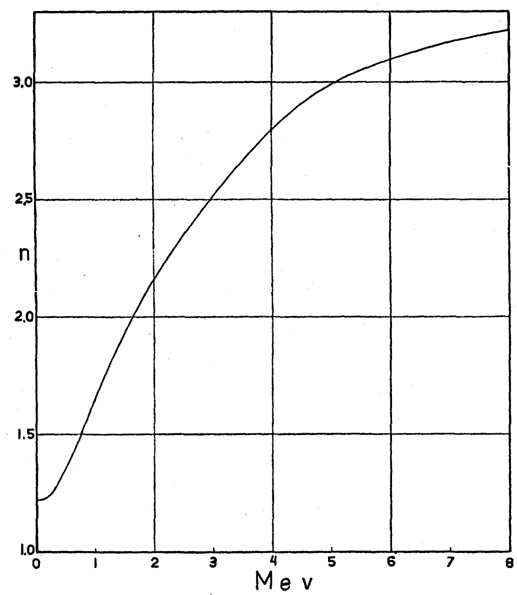

FIG. 13. Range exponent, n, for alpha-particles as a function of energy. n is defined in terms of the logarithmic derivative, $n=2d\log R/d\log E$. Important for thick target and angular straggling corrections.

but lie definitely above the curve at lower energies.

In computing the corrections to experimental range data due to thickness of target, angular straggling, etc., it is valuable to know the variation of range exponent with energy. Bethe[42] uses the logarithmic derivative of the range with respect to the energy: $n=2d(\log R)/d(\log E)$ $=2(E/R)(dR/dE)$, and plots a curve of n *vs.* E obtained from the range energy relation available at that time. From our range energy and specific ionization curves we have determined this range exponent; the result is plotted in Fig. 13. The curve differs from the previously published curve considerably in shape and absolute value in the low energy region. The assumption of constant range exponent used in extrapolating to the end of range shows in the horizontal intercept at zero energy, partially justified by the trend of the curve within the last 3 mm.

The authors wish to express their appreciation to Professor H. A. Bethe for many helpful suggestions in the analysis and interpretation of the results. The work was partially supported by a grant from the Research Corporation.

[42] Reference 9, p. 271.