Note on Internal Conversion of Arbitrary Multipole Order

In the activity coming down with gallium chemically separated from a zinc target bombarded with deuterons, Alvarez[1] found two homogeneous groups of electrons near 100 kev energy. The sharpness and spacing of these groups, the presence of zinc x-rays, and the relative intensities indicated that the electrons were the secondaries of an internally-converted γ-ray of about 100 kev energy. Few γ-rays of this energy were observed, though radiative transitions cannot be completely forbidden in an odd nucleus like Zn^{67}. The study of the internal conversion in natural radioactive elements has revealed[2] no cases of a conversion as high as that indicated here in zinc. The approximate formulae given by Rasetti[3] and Bethe[4] give values of a few percent for this case, and the Z^3 dependence of the conversion coefficient suggests that conversion becomes unimportant for the lighter elements. The formulae cited, however, were for the conversion of radiation from an electric dipole. One expects conversion to vary with multipole order, 2^l, roughly as the parameter $(\lambda^2/\lambda^2e)^l$, where λe is the de Broglie wave-length of the emitted electron, λ the γ-ray wave-length. Thus for low energy γ-rays, the conversion could increase greatly with multipole order.

Making a nonrelativistic calculation, neglecting spin, and neglecting the ratio $n=Ze^2/\hbar v$ of the velocity of the electron in its bound state to its velocity v after emission, one obtains a simple formula. We take the internal conversion coefficient a_K as

$$\frac{\text{number of electrons emerging from the atom}}{\text{number of } \gamma\text{-rays emerging from the atom}}.$$

For conversion by one electron in the K shell, we have for an electric 2^l pole:

$$a_K=\frac{l}{l+1}\cdot\alpha^4\cdot Z^3\left(\frac{2mc^2}{E}\right)^{l+5/2}. \quad (1)$$

Z = effective nuclear charge; $\alpha=e^2/\hbar c$; E = energy of γ-ray; m = electron rest mass. If the effective nuclear charge were the same for both shells, the lowered electron density for the L shell would introduce a factor $\frac{1}{8}$. Screening will still further reduce the L conversion. In this limit, where we neglect the orbital motion of the bound electron, magnetic multipole radiation is not converted. This corresponds physically to the absence of radial forces on a stationary charge in a magnetic multipole field.

For Zn, $n\sim 0.3$, and neglect of the binding is hardly justified. We have therefore made calculations using the nonrelativistic wave functions for the Coulomb field; for the electric dipole we find:

$$a_K=\alpha^4Z^3\left(\frac{2mc^2}{E}\right)^{7/2}\frac{\pi n}{1-e^{-2\pi n}}\left|\left[1+\frac{8}{1+n^2}\left(f+\frac{k}{2p}g\right)\right]\right|^2 \quad (2)$$

where $f=[x(1-x)(1-u)+(in/2)(1-2x)+n^2/2]$;
$g=[(1-2x)(1-u)-in(1+u)]$;
$u=(-x/(1-x))^{in}$; $|arg\,(-u)|<\pi$;
$x=\rho/2ip$; $n=a/p$.
Here: $\rho=a+i(p-k)$; $a=Ze^2m/\hbar^2$; $p=mv/\hbar$; $k=E/\hbar c$.

The other multipole orders lead to analogous but longer formulae. Numerical results for this case of a 100 kev γ-ray and atomic number 30 are:

	a_k	a_k from (1)
electric dipole	0.03	0.1
electric quadripole	0.7	1.4
electric octupole	5.	16.

The same trend with multipole order appears here. It is clear that the large conversion observed by Alvarez can be understood if the radiation is not dipole radiation.[5]

These nonrelativistic formulae fail if the γ-ray energy becomes of the order mc^2. Since $\lambda/\lambda e\rightarrow 1$ for high γ-ray energies, one would expect the change with multipole order to become much less marked as the γ-ray energy increased, and relativistic calculations confirm this. Casimir has published[6] a formula for the electric dipole valid in the limit of very high γ-ray energies. Since the multipole order is unimportant here, Casimir's formula will give the order of magnitude of the conversion for all multipole orders, if only the γ-ray energy is $\gg mc^2$.[7]

All results are here stated for one electron. There are ordinarily two electrons in the K shell, but in cases like Alvarez', where K electron capture occurs, it is possible that the nuclear transition which causes electron emission precedes the atomic transition which refills the depleted K shell. Further work on internal conversion in the artificially radioactive nuclei may help to settle this question, and to determine the angular momenta of the states involved.

We are happy to acknowledge the continual kind guidance of Professor J. R. Oppenheimer and Dr. R. Serber.

S. M. Dancoff
P. Morrison

Department of Physics,
University of California,
Berkeley, California,
June 17, 1938.

[1] Alvarez, Phys. Rev. **53**, 606 (1938).
[2] Hulme, Mott, F. Oppenheimer and Taylor, Proc. Roy. Soc. **155A**, 315 (1936).
[3] Rasetti, *Elements of Nuclear Physics* (Prentice-Hall 1936), p. 139 et seq.
[4] Bethe, Rev. Mod. Phys. **9**, 228 (1937). The formulae given by Bethe and Rasetti do not agree as published. In the limit of $n\rightarrow 0$ they agree with each other, but differ from our formula (1) by a factor 2. The consistency of the approximations employed in their derivation is not clear.
[5] We have to thank Dr. Alvarez for informing us that there are at least as many secondaries as 100 kev quanta emerging.
[6] Casimir, Physik. Zeits. **32**, 665 (1931).
[7] A more detailed discussion of the relativistic formulae and the derivation of those given above will be published later .

Radioactive As Isotopes

Germanium samples activated last year at Berkeley with 5.5 Mev deuteron beam from the cyclotron were brought back by one of the authors to Tokyo. The study of the activity shows the existence of two long periods. According to the data so far obtained, the periods are estimated to be about 17 days and 50 days, respectively. The decay curves are still being followed in order to determine the periods more accurately. Chemical separations show that both activities are due to arsenic isotopes.

The shorter period was also obtained by bombarding arsenic with fast neutrons from Li+D in the Tokyo cyclotron. This neutron loss process was first reported by

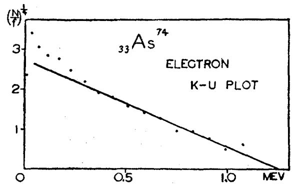

FIG. 1(a). K-U plot for electrons.

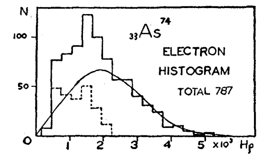

FIG. 1(b). Energy histogram for electrons. The part drawn by a broken line is considered to be due to annihilation rays.

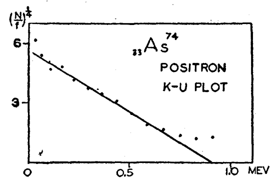

FIG. 1(c). K-U plot for positrons.

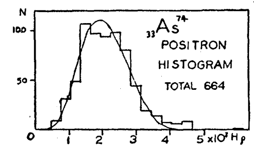

FIG. 1(d). Energy histogram for positrons.

Pool, Cork and Thornton,[1] and Curtis and Cork[2] who reported that it has the half-life of 13.5 days and emits both positive and negative electrons. The upper limit of the positron is said to be 0.65 Mev but for negative electrons, relative numbers emitted and chemical separations have not been reported yet.

The study of the β-ray histogram of the germanium samples with a hydrogen-filled Wilson chamber gave results as shown in Fig. 1. The upper limit for positive and negative electrons reduced from the K–U plots are 0.9 Mev and 1.25 Mev, respectively. Although the value for positrons is a little higher than that obtained by the Michigan group, we are certain that the process should be written as follows:

$$Ge^{73}+D^2\rightarrow As^{74}+n'$$
$$As^{75}+n'\rightarrow As^{74}+2n'.$$

To check the fact that As^{74} emits positive and negative electrons three series of photographs were taken on the same sample on three different dates about two weeks apart. The ratios of the number of positive and negative electrons are nearly constant as shown in Table I. The branching ratio is thus estimated to be 0.9. As for the longer period it is most probable that it is due to the following process:

$$Ge^{76}+D^2\rightarrow As^{77}+n'.$$

TABLE I. *Relative number of positive and negative electrons.*

Date	pos.	neg.	ratio
March 23	255	233	0.92
April 6	146	113	0.78
April 24	263	212	0.81
Total	664	558	0.84

This isotope emits very soft β-rays which can be absorbed completely by 4/1000″ Al sheet. A study with a low pressure chamber is now in progress.

It is a pleasure to express our thanks to Professor E. O. Lawrence for the privilege of using his cyclotron. Thanks are also due to Dr. Y. Nishina and Professor K. Kimura for their valuable help and encouragement. We wish to acknowledge the assistance given to us by Mr. K. Sinma, Mr. F. Yamasaki and Mr. N. Mori. The experiment has been aided by grants from the Research Corporation, the Chemical Foundation, the Josiah Macy, Jr. Foundation, the Japan Society for the Promotion of Scientific Research, Oji Paper Manufacturing Company, Mitsui Ho-onkwai Foundation, Tokyo Electric Light Company and Japan Wireless Telegraph Company.

R. SAGANE
S. KOJIMA
M. IKAWA

Department of Physics (R. S. and S. K.),
Department of Chemistry (M. I.),
Tokyo Imperial University,
Tokyo, Japan,
May 28, 1938.

[1] Pool, Cork and Thornton, Phys. Rev. **52**, 239 (1937).
[2] Curtis and Cork, Phys. Rev. **53**, 681 (1938).