The Theory of Excitation Functions on the Basis of the Many-Body Model

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The theory of transmutation functions for charged particles is examined in the light of recent developments in nuclear theory. It is found that ignoring all factors besides the penetrability of the incident particle is justifiable only for processes without effective competition. For these, the usual applications of the Gamow-Condon-Gurney theory must be modified to include the influence of incident particles with nonvanishing orbital momentum which causes a continued rise of the yield for energies greater than the Coulomb barrier. For processes subject to effective competition, the comparative density of residual states available to each alternative process is important in determining relative yields. The densities of the states are calculated on the basis of the liquid drop model for the nucleus and a table is obtained giving the relative probabilities of neutron and charged particle emissions. The overwhelming factor in favor of neutron emission by heavy nuclei seems to be confirmed by experiment. The shape of an excitation curve expected for the less probable of two processes is shown. Finally the effect of selection rules in the yield curves of reactions involving very light nuclei is considered. Special cases are discussed.

§1. INTRODUCTION

HERETOFORE, attempts to explain the variation of the yield from charged particle disintegrations as a function of the bombarding energy were largely limited to applications of the simple Gamow-Condon-Gurney (G-C-G) theory to the penetration of nuclear potential barriers by incoming particles. This was done with the hope that other factors, lumped together under the title "internal disintegration probability," would be comparatively unimportant in determining the variation. The development of the Bohr-Breit-Wigner theory of nuclear processes has made it clearer what some of the other factors are and encourages an attempt to evaluate their effects. The quantitative results of such an attempt must for the present remain very uncertain. However, they will show to what extent the neglect of influences other than that of the simple penetrability are justified.

When the penetration probability of the incoming particle is alone considered to vary with the energy in the evaluation of the excitation function for a given process, it is being assumed: first, that the probability of the nucleus' and particle's sticking together after the penetration does not depend importantly upon the energy, and secondly, that the formation of the compound nucleus itself guarantees the completion of the process. No improvement can be made on the first assumption until a more workable model of the nucleus presents itself. The second assumption is justified only if the completion of the process in question involves a type of emission which is much more probable than any other way in which the compound nucleus may disintegrate. Our evaluation of the excitation functions for such cases, presented in §4, differs from the usual procedure in one respect: The contributions of incident particles having orbital momenta other than zero are taken into account. When the process being considered meets with effective competition from other possible modes of disintegration of the same compound nucleus, the relative densities of the final states available to each process have an important effect on the excitation functions. This point is discussed in §5, for heavy nuclei. In §6, we treat the effect of selection rules, which have practical importance for very light nuclei.

The consideration of other influences besides the penetrability by particles making direct hits has a particularly important bearing on the deduction of nuclear radii from transmutation functions. It has been the practice to assume that the height of the Coulomb barrier of a nucleus is equal to the energy at which the yield reaches a maximum; such a behavior is expected on the basis of the "simple" ($l=0$) G-C-G theory. The practice is not justified, since it is now clear that the yield continues to rise rapidly beyond the barrier height energy because of the increasing contributions from particles having high orbital momenta and in some cases also because of the

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greater density of available final states at higher excitations. (Fig. 3.)

§2. General Formulation

Most of the effects considered in this paper will be discussed in terms of a formulation which conveniently represents the Bohr picture and which may be derived either by the simple statistical method used in papers of Weisskopf[1] and Bethe[2] or by averaging the Breit-Wigner "dispersion" formula over the resonances in the neighborhood of the energy of the compound nucleus formed in the process. One obtains by the latter method (see I, Eq. (405)) for the cross section of a process in which a nucleus A is bombarded by particles P and from which there results the emission of a particle Q and the formation of a residual nucleus B:

$$(\sigma^{Ppi}{}_{Qqi'})_{\mathrm{Av}}=\frac{2\pi^2\lambda\!\!\!^{-}{}_P{}^2}{(2i+1)(2s+1)}\times\sum_{Jljl'j'}(2J+1)\left(\frac{\Gamma^J{}_{Pplj}\Gamma^J{}_{Qql'j'}}{\Gamma_J D_J}\right)_{\mathrm{Av}}. \quad (1)$$

$2\pi\lambda\!\!\!^{-}{}_P$ is the de Broglie wave-length of P. $\Gamma^J{}_{Pplj}$ is a partial width of the compound nuclear level, corresponding to the emission of particles P having an energy denoted by p and orbital and total angular momenta l and j. $\Gamma^J{}_{Qql'j'}$ is a corresponding quantity for particles of kind Q. Γ_J is the *total* width of the level, i.e.,

$$\Gamma_J=\sum_{Q'q'}\Gamma^J{}_{Q'q'}=\sum_{Q'q'l''j''}\Gamma^J{}_{Q'q'l''j''}. \quad (2)$$

D_J is the average spacing of the levels of angular momentum J in the neighborhood of the compound level formed. i, s, J, i' and s' are the respective spins of A, P, C, B and Q in the states concerned.

When (1) is summed over all possible emission processes Qq, one obtains

$$\sigma^{Ppi}{}_C=\frac{2\pi^2\lambda\!\!\!^{-}{}^2{}_P}{(2i+1)(2s+1)}\sum_{Jlj}(2J+1)\frac{\Gamma^J{}_{Pplj}}{D_J}. \quad (3)$$

This must be the cross section for the formation of the compound nucleus since it must disintegrate in *some* way. At very high energies of the incident particle, classical concepts should hold, and $\sigma^{Ppi}{}_C$ must equal the geometrical cross section, πR^2, multiplied by the "sticking probability" ξ_{Pp} of P to A, i.e.,

$$\sigma^{Ppi}{}_C=\pi R^2\xi_{Pp}\quad\text{for }\lambda\!\!\!^{-}{}_P\ll R. \quad (4)$$

It will be useful to extend the definition of the sticking probability to low energies. That will be done by assuming:

(1) For a given l, the probability of each j and J is given by the statistical weights, i.e.

$$\sigma^{Pplj}{}_J=(2J+1)/(2i+1)(2s+1)(2l+1)\cdot\sigma_{Ppl}. \quad (5)$$

(2) The cross section for each l, σ_{Ppl}, is proportional to the statistical factor, $2l+1$, multiplied with the penetrability P_l of the barrier for particles of that orbital momentum,[3] i.e.,

$$\sigma^{Ppi}{}_C=c\sum_l(2l+1)P_l=cR^2/\lambda\!\!\!^{-}{}_P{}^2=\pi R^2\xi_{Pp} \quad (6)$$

(see (4) and §3, (13c)). Then, comparing (3), (5) and (6) we have

$$\sigma^{Pplj}{}_J=\frac{(2J+1)}{(2i+1)(2s+1)}\pi\lambda\!\!\!^{-}{}^2{}_P\xi_{Pp}P_l=\frac{(2J+1)}{(2i+1)(2s+1)}2\pi^2\lambda\!\!\!^{-}{}^2{}_P\frac{\Gamma^J{}_{Pplj}}{D_J},$$

from which

$$\xi_{Pp}=2\pi\Gamma^J{}_{Pplj}/D_JP_l. \quad (7)$$

This definition of the sticking probability from "correspondence" will be regarded as valid for all energies. It makes ξ nearly independent of energy since the penetrability factor is divided out from Γ and probably the "width without barrier" varies with the energy in about the same way as

[1] Weisskopf, Phys. Rev. **52**, 295 (1937).

[2] Bethe, Rev. Mod. Phys. **9**, 69 (1937). This reference will hereafter be denoted by "I".

[3] This is equivalent to assuming that the formation probability is proportional to the influx of particles, which is appropriate for a sticking probability near unity. If proportionality to the density of the particle wave function near the nucleus had been used instead of just the penetrability, a factor analogous to the inverse of the radial velocity in the classical picture, namely $[1-(l+\frac{1}{2})^2\lambda\!\!\!^{-}{}^2/R^2-zZe^2/ER]^{-\frac{1}{2}}$, would have entered. This would make the probability of sticking proportional to the "time of contact" and would be inconsistent with a unit sticking probability. The presence or absence of such a factor is of little consequence for the excitation functions.

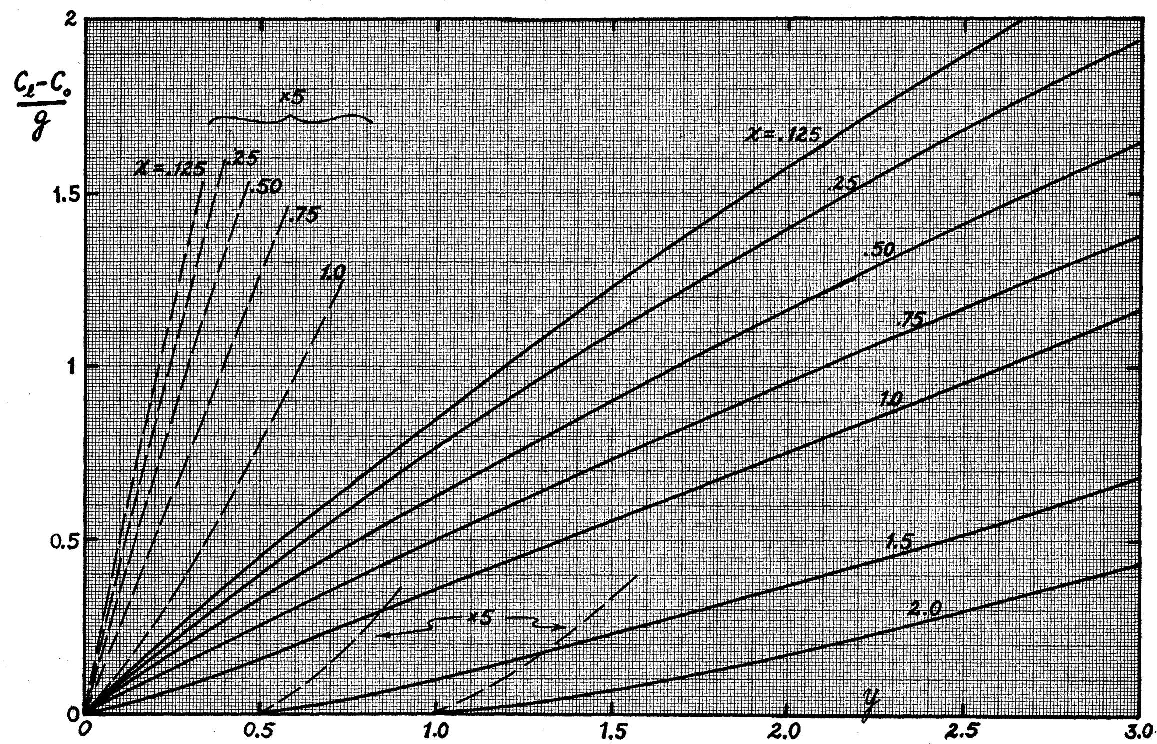

FIG. 1. $(C_l-C_0)/g$ as a function of y for seven values of x. The dashed curves are the same as the solid curves with ordinates multiplied by five.

the spacing D. (I, §54D). It enables the reduction of formula (1) to

$$(\sigma^{Ppi}{}_{Qqi'})_{\text{Av}}=\frac{\pi\lambda\!\!\!^{-2}{}_P\xi_{Pp}\xi_{Qq}}{(2i+1)(2s+1)}\cdot\sum_J\frac{(2J+1)}{\eta_J}\sum_{lj}P_l\sum_{l'j'}P'_{l'},\quad (8)$$

where $$\eta_J=2\pi\Gamma_J/D_J. \quad (8a)$$

The summation over J is to be taken over all values of J available at the given excitation energy. The compound nucleus is usually highly excited (10 MV or more) so that except for very light nuclei the density of levels is great enough for each value of J up to a certain limit to be represented. It does no harm to include values of J up to infinity because, as we shall see, the factor P_l cuts down the contributions of high values of J so much that only values up to $J=3$ or 4 are important even for fairly heavy nuclei.

As it stands, (8) gives the cross section for a single "group" of particles which leave the residual nucleus in a definite excited state q with spin i'. To obtain the total yield of Q, (8) must be summed over all possible residual levels q, each properly weighted. For heavy enough nuclei this is done by multiplying with the number of residual states per energy interval,

$$(2i'+1)dW_B/D_{i'} \quad (9)$$

integrating, and summing over i'. $D_{i'}$ is the average spacing of levels with a given spin i' in the residual nucleus; it is probably almost independent of i'.

§3. THE PENETRABILITY FORMULAE

The penetration probability P_l as calculated by the W-K-B method is well known. It can be written

$$P_l=e^{-2C_l}, \quad (10)$$

in which C_l is a function given in I, Eq. (631). We give $(C_l-C_0)/g$ as a function of x and y in Fig. 1. For C_0, see I, Eq. (600) and Fig. 18. g is the "characteristic orbital momentum:"

$$g=(2MzZe^2R)^{\frac{1}{2}}/\hbar$$
$$=0.262(zZaAr_0/a+A)^{\frac{1}{2}}A^{1/6}, \quad (10a)$$

in which M is the reduced mass of the particle, a and A are the mass numbers of particle and nucleus, respectively, and

$$r_0 = RA^{-\frac{1}{3}} \cdot 10^{+13}, \tag{10b}$$

$$x = E_P/B = E/B', \tag{10c}$$

in which E_P and E are relative and absolute kinetic energies of the particle, while

$$B = zZe^2/R = 1.43_5 zZ/r_0 A^{\frac{1}{3}} \quad \text{MV}, \tag{10d}$$

$$B' = zZe^2(a+A)/RA = 1.43_5 zZ(a+A)/r_0 A^{4/3} \quad \text{MV}, \tag{10e}$$

$$y = l(l+1)/g^2. \tag{10f}$$

With $r_0 = 1.65$, which corresponds to a radius of 10^{-12} cm for the α-radioactive nuclei, the values of g and B' are given in Table I.

Certain approximations for $(C_l - C_0)/g$ will be useful.

$$(C_l - C_0)/g \approx y(1 - \tfrac{1}{2}x) \qquad \text{for } x<1. \tag{11a}$$

$$C_l/g \approx 2(y - x + 1)^{\frac{3}{2}}/3(2x-1) \qquad \text{for } x>1. \tag{11b}$$

For $y < (x-1)$, $C_l = 0$; for $x \geqslant 1$, $C_0/g = 0$, $P_0 = 1$.

For many problems (cf. Eq. (6), (15), (16)) the quantity

$$l_c^2 = \sum_l (2l+1)(P_l/P_0) \tag{12a}$$

is needed. It is shown as a function of x in Fig. 2, for two cases. For sufficiently heavy nuclei a great number of l's will contribute so that the sum may be replaced by the integral

$$l_c^2 = g^2 \int_0^\infty dy P(y)/P(0). \tag{12b}$$

The integral may be extended to infinity except when the available J's are very restricted (§6). With the approximations (11),

$$l_c^2 = g/2 - x \qquad \text{for } x<1, \tag{13a}$$

TABLE I. *Effective heights B' of Coulomb barriers in MV. Characteristic orbital momenta, $g(Z)$.*

	Z / A / NUCLEUS	2 / 4 / He	4 / 9 / Be	10 / 20 / Ne	20 / 40 / Ca	30 / 66 / Zn	50 / 112 / Sn	70 / 174 / Yb	92 / 238 / U
B'	Protons	1.4	1.9	3.4	5.2	6.6	9.1	11.0	12.9
	Deuterons	1.6	2.0	3.5	5.4	6.7	9.2	11.1	13.1
	α-particles	3.6	4.9	7.7	11.2	13.1	18.6	22.4	26.0
g	Protons	0.6	0.9	1.7	2.8	3.7	5.3	6.7	8.2
	Deuterons	0.7	1.3	2.3	3.9	5.2	7.4	9.5	11.5
	α-particles	1.2	2.2	4.5	7.5	10.3	14.7	18.8	22.9

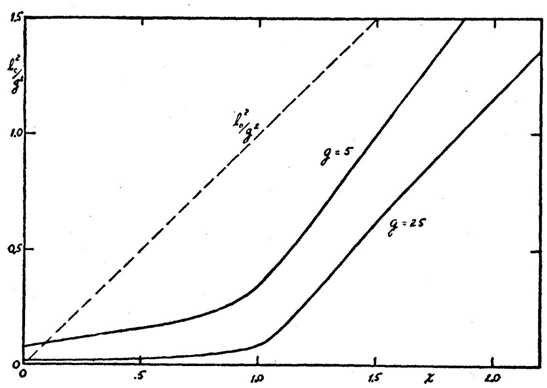

FIG. 2. The orbital momentum factor l_c^2/g^2 as a function of x for two values of g. $g \sim 5$ for deuterons bombarding Ne and $g \sim 25$ for alphas on U. All curves are asymptotic to $l_0^2/g^2 = x$ for very large x.

$$l_c^2 = g^2(x-1) + 0.744 g^{4/3}(2x-1)^{2/3} \qquad \text{for } x>1. \tag{13b}$$

For $x \gg 1$,

$$l_c^2 = (R^2/\lambda\!\!\!^{-2})(1 - B/E) \approx R^2/\lambda\!\!\!^{-2} = l_0^2. \tag{13c}$$

This can also be derived from classical mechanics, $(l+\frac{1}{2})\lambda\!\!\!^{-}$ being identified with the collision parameter. The quantity l_c as given by (13) can easily be shown to represent the "critical orbital momentum" as defined by

$$2(C_l - C_0) = 1. \tag{14}$$

Values of l greater than l_c yield only small contributions, better justifying the extension of the integration in (12) to infinity.

§4. EXCITATION FUNCTIONS FOR MOST PROBABLE REACTIONS

The formula for the yield becomes most simple when the reaction in question is so much the most probable one that

$$\sum_q \Gamma^J{}_{Qq} \approx \Gamma_J.$$

In that case, (8), summed over all residual states q, becomes (cf. (13c))

$$\sigma^{Ppi}{}_Q = \pi R^2 \xi_{Pp} (l_c^2/l_0^2) P_0 \tag{15}$$

after the summations over angular momenta are carried out with proper regard for their conservation. Eq. (15) is essentially the simple G-C-G formula except for the factor l_c^2 which

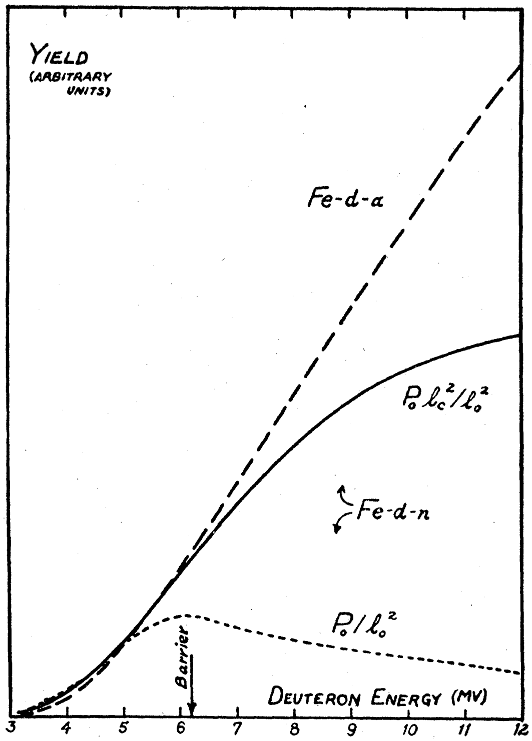

FIG. 3. Comparison of theoretical excitation functions for a most probable process (emission of neutrons) and a less probable one (alphas). Also shown is the excitation function calculated according to the usual G-C-G theory (factor l_c^2 omitted). Both reactions have been observed[9] but measurements of yield have not been yet carried to high enough energies. Ordinates of all curves were made equal at 5 MV.

represents the influence of higher orbital momenta. This factor is practically a constant for $E<B$ (see Fig. 2) and so the simple G-C-G formula gives a correct energy dependence for energies below the barrier. Also because of l_c^2, the yield will continue to rise quite rapidly even after the particles are able to pass over the barrier, contrary to frequent assumptions. An example of this behavior is shown in Fig. 3. When the energy becomes very great ($E \gg B$), (15) goes over into $\pi R^2 \xi_{Pp}$, simply. The usual G-C-G formula has for all energies the factor $1/E$, which (15) shows only for low energies. This point has led in the past to some confusion because of the apparent contradiction to the reasonable expectation that the cross section should go over into the geometrical cross section in the classical limit.

The validity of (15) for all reactions without effective competition accounts for some of the success the G-C-G theory has had in fitting transmutation data. Among the heavy elements ($A>40$), the reactions producing neutrons will usually be the most probable ones (§5). Among the somewhat lighter elements, the most probable reactions may be of almost any variety, depending on which lead to the most stable products. No excitation functions seem to be known at present for any reactions involving this class of elements ($5<Z<20$), which can safely be said to be much more probable than its alternatives. One reason is that, because such reactions usually lead to the most stable products, the observation of radioactivities is not often available as a method of measuring their excitation functions. Among the very light elements, the assumption made in deriving (15), that all J's are available, does not hold and selection rules become important (§6).

At present, there are among the heavy element reactions two sets of data[4] to which (15) is applicable and these seem to contradict each other. Mann[5] finds that a simple G-C-G penetrability function (P_0/E) fits the excitation curve he measured for the Cu^{65}-α-n reaction if he assumes $r_0=1.75$ (see (10b)). On the other hand, Thornton[6] is forced to assume $r_0=1.15$ to obtain approximate agreement of his data on the Ni^{60}-d-n reaction with the simple G-C-G theory. In view of the discrepancy between Mann's and Thornton's data, it may be significant that there seems to be a definite disagreement between Thornton's data and the G-C-G theory at the lowest energies, where the penetrability is almost independent of r_0. The factor l_c^2 plays no great part in the two cases discussed because the measurements were limited to energies below the barriers. Not only Mann's Cu^{65}-α-n curve but also his Cu^{63}-α-n data can be fitted within the theoretical and experimental uncertainties by $r_0=1.65$ (see §3).

Further measurements among the heavy elements would have some importance. In order to

[4] Recently there has been added a third, the measurement of the Se-p-n reaction by the Rochester group (to be published). Calculations for this case have been carried out by Mrs. Weisskopf. She finds that $r_0 \cong 1.50$ fits the data best. We are indebted to Professor Weisskopf for communicating these results to us.

[5] Mann, Phys. Rev. **52**, 405 (1937).

[6] Thornton, Phys. Rev. **51**, 893 (1937).

test the theory, they should be done at the lowest possible energies, first, because there the variation of the penetrability is independent of the choice of r_0, and second, because at low energies the penetrability will be by far the most rapidly varying factor in the cross section and the variation of ξ_{Pp} can more safely be neglected in comparison.

§5. The Influence of Final States

The cross section for a reaction which meets with effective competition must involve the relative probabilities of the various types of emission possible for the same compound nucleus. To simplify the discussion, it will be assumed[7] that there is just one most probable process (e.g. for heavy nuclei, neutron emission). The formula (8) can then be shown to reduce to the cross section (15) for the most probable process, multiplied with

$$(2s'+1)\sum_q \xi_{Qq} l_c'^2 P_0' / (2s''+1) \sum_{q'} \xi_{Q'q'} l_c''^2 \quad (16)$$

$P_0''=1$ because the most probable process (the emission $Q'q'$) is certainly neutron emission;

$$l_c''^2 \approx R^2/\lambda\!\!\!^{-}_N{}^2 \quad (16a)$$

with $\lambda\!\!\!^{-}_N$ the neutron wave-length. (16) is essentially the ratio of the "effective" densities of residual states available for the two processes, Q and Q', assuming that at least the ratio of the sticking probabilities is a constant nearly unity. The word "effective" is used here to denote the reduction of the states available by the penetrability factor, P_0'.

A computation of the density of states in residual nuclei must be based on some model such as the liquid drop model discussed in I, §53C. Although this may have no virtues over other possible models, it still seems to give correctly certain general features, namely the exponential increase of the level density with excitation energy and with mass number, and an approximately correct average spacing of the levels in heavy nuclei. For our purposes, there would be no essential differences arising from the use of other formulae for the density, such as the one involving the exponential of the square root of the excitation and empirical constants.[1]

Computations of the ratio (16), based on the liquid drop model formulae of I, §53C, have been carried out for various excitations of various compound nuclei and the results are shown in Table II. The second column gives the mass numbers A of the compound nuclei, the third their Coulomb barrier heights B in MV. The computations were made for excitations of 10, 15, 20, 25 and 50 MV for the compound nucleus. The first two rows give the deuteron bombardment energy necessary for each of the various excitations for mass numbers 50 and 200. A similar interpretation may be applied to the proton, alpha and neutron energies given in the next six rows, or, instead, each energy value may be regarded as the total energy available for the emission of the corresponding particle. The figures given are based on the semiempirical mass-defect curve which leads to Table XXXVIII of I. The remaining rows of Table II give the ratios of the probabilities of neutron and proton emission and of neutron and alpha-emission, as represented by (16).[8] The ratio of the ξ's was taken as unity.

Table II. *Energies available for various particle emissions starting from given excitations of the compound nucleus. Ratios of neutron emission probabilities to proton and alpha-emission probabilities.*

			Exc. Energy of Compound Nucleus (MV)				
	A	B (MV)	10	15	20	25	50
H^2 energy	50	5.3	—	—	3.5	8.5	33.5
	200	12	—	4	9	14	39
H^1 energy	50	5.3	1	6	11	16	41
	200	12	4	9	14	19	44
He^4 energy	50	11	2.5	7.5	12.5	17.5	42.5
	200	24	14	19	24	29	54
n^1 energy	50	—	0	5	10	15	40
	200	—	3	8	13	18	43
n/p	50		$3\cdot10^4$	12	10	8	7
	100		$5\cdot10^5$	96	70	44	24
	150		$3\cdot10^6$	1400	615	400	110
	200		$1.5\cdot10^7$	16000	6200	2600	410
	240		$2.3\cdot10^8$	32000	15000	12000	520
n/α	50		$2\cdot10^9$	700	84	65	30
	100		$3\cdot10^7$	4800	430	480	100
	150		$3\cdot10^8$	$7\cdot10^7$	$1.3\cdot10^4$	1800	250
	200		$2\cdot10^8$	$7\cdot10^5$	$9\cdot10^5$	18000	1800
	240		$2\cdot10^8$	$5\cdot10^6$	$1.2\cdot10^6$	$1.8\cdot10^5$	23000

[7] Also assumed is that the nuclei involved are heavy enough and the excitations high enough so that all values of the spin of the final nucleus are available both when the process in question takes place and when the most probable emission occurs. This assumption is of little importance for our results.

[8] The summing over states was done wherever possible by the methods suggested by (9) and by I, (346*b*). A few checks by numerical integration showed that these approximations are fair. When the available energies are lower

Table II must not be regarded as giving results with greater accuracy than by a factor of ten or so and then only when applied to elements at the bottom of the "Gamow valley," that is, those having the most stable mass number. In general, less energy is available for radioactive products than indicated in the table and so the probability of forming the radioactive nucleus is reduced accordingly. Contributing to the uncertainties are: the model employed, itself; possible deviations from the smooth mass defect curve which was assumed; the neglect of correlations such as are caused by selection rules.

The first point to be learned from Table II is that the emission of charged particles by nuclei having mass numbers greater than about 100 is practically forbidden. This comes about because neutrons, not having barriers to penetrate, may be emitted with lower kinetic energies than charged particles; then, not only is the region of easily available final states for the residual nucleus greatly widened, but more important, the region of high excitations of the residual nucleus, which is the most densely populated with levels, is easily available solely to the neutron process. On the whole, the experimental data[9] tend to confirm these considerations. Limiting the discussion to nuclei with $A>60$, we find:

(a) Three, not unquestionable, d-α reactions are suggested, with Cu, Zn and Sb, as against a half-dozen d-n reactions (Ni, Se, $Pd^{104,105}$, Sn^{119}, $Sn^{(?)}$). The numerous d-p reactions found must be construed as an argument for the existence of the Oppenheimer-Phillips process of disintegration.[10]

(b) About five n-a (Zn, Ga, Ba, Th, U) and three n-p reactions (Cu^{65}, $Zn^{64,66}$) have been postulated. The Ba-n-α, Th-n-α and U-n-α processes may plausibly be ascribed to short lived alpha-radioactivities following inelastic scattering of the neutron. The others all have $A<70$ so they may still be allowed.

(c) There are no p-α or α-p reactions known for $A>60$, although neutron emission processes have been observed for Ni (2 reactions), Zn(2), As (2), Se(3), Mo(2), Cd(4) and In when bombarded by protons, and for Ni, $Cu^{63,65}$ and As when bombarded by alpha-particles.

than the Coulomb barriers, the most important contributions are made by a few levels for which the penetrability is most favorable. In these cases integration is too crude and so the sums over states were evaluated by adding together the penetrabilities for a few reasonably spaced levels. Often this gave about the same results as the crude integration.

[9] Cf. Livingston and Bethe, Rev. Mod. Phys. **9**, 245 (1937).

[10] Bethe, Phys. Rev. **53**, 39 (1938).

Also shown by the table is the effect of competition on excitation functions. As the energy is increased, the probability of charged particle emission becomes more and more nearly comparable to neutron emission. At extremely high energies, all the processes tend to become equally probable. These effects will be reflected in the excitation functions of the less probable processes by a more rapid rise with energy than for the most probable processes. At the lowest excitations, this added rate of increase will be extremely great. Observations so far have been limited to higher excitations of the compound nucleus (15 MV or more), where the addition to the rate of increase is not quite so pronounced. An example is given in Fig. 3.

The methods of this section are too crude to be applied to elements of mass number smaller than about 50. Tentative calculations showed that the weights (e.g. statistical weights due to spin) given to the individual residual states are quite influential and not easily decided. Only qualitative considerations about the lighter elements seem to be feasible. The most interesting[11] of these deal with the striking decrease which may occur in the probability of a reaction when the bombarding energy is raised beyond a value which makes a competing reaction energetically possible. Haxel[12] reports such an interference of the N-α-p reaction with the yield from N-α-n, and Newson[13] found similar effects in the excitation functions for the reactions C-d-n, N-d-n and O-d-n. Newson points out that the surmounting of the thresholds of the C-d-α and N-d-α reactions are probably responsible for the decrease observed in his first two cases. There is no threshold for a new reaction which could interfere with the O-d-n reaction in the way observed. The low stability of the radioactive F^{17} against emission of protons probably is responsible for the lower probability of detecting it when it is formed with high energies.

§6. The Effect of Selection Rules

Finally, we shall consider the effect of the selection rules on excitation functions. As one might have expected, this effect has its greatest

[11] Bohr and Kalckar, Kgl. Danske Selskab. **14**, 10 (1937).

[12] Haxel, Zeits. f. Physik **93**, 400 (1935).

[13] Newson, Phys. Rev. **51**. 620 (1937).

importance for reactions involving very light nuclei. For these, the formulation of the preceding paragraphs is not strictly valid since it was based on the participation in the process of a great many compound levels at every energy in the relevent range. In its place, the so-called "one-level" formula (I, (646))

$$\sigma=\pi\lambda_P{}^2\frac{2J+1}{(2i+1)(2s+1)}\frac{\Gamma^J{}_{Pp}\Gamma^J{}_{Qq}}{(E-E_r)^2+\frac{1}{4}\Gamma^2{}_J}\qquad(17)$$

is sometimes useful, but it is applicable only in energy ranges for which just one level of the compound nucleus is effective. One may proceed with a detailed analysis of special cases which may decide to which (17) may be applied.

Another difficulty encountered when dealing with light elements is in the calculation of the penetrability itself. The W-K-B method should be expected to fail for the low and thin barriers possessed by the very light nuclei. One might instead make use of the exact wave functions for a Coulomb field calculated by Yost, Wheeler and Breit,[14] but now there arises the question of what boundary conditions to impose (see I, §54A). We have adopted the procedure of Kapur and Peierls,[15] according to which the width of a level depends on the particle energy as

$$\Gamma^J{}_{Ppl_j}\sim E^{\frac{1}{2}}/G_l{}^2,\qquad(18)$$

where G_l is the irregular wave function for a Coulomb field as given in Yost, Wheeler and Breit's paper. For low energies and $l=0$, (18) reduces in its energy dependence to

$$\Gamma^J{}_{Ppl_j}\sim\exp(-2\pi zZe^2/\hbar v),\qquad(18a)$$

with v the velocity of the particle. This is exactly what the W-K-B method gives for low energies (I, (589b)). The procedure we adopt here is quite consistent with our treatment of the penetrability for heavy nuclei (cf. footnote 3).

As a first illustration of how selection rules affect excitation functions, one may take the case of

$$Li^7+H^1\rightarrow 2\ He^4.\qquad(19)$$

The protons responsible for this process must have odd angular momenta since the parity of Li^7 is odd and the alphas obey Bose statistics.

[14] Yost, Wheeler and Breit, Terr. Mag., Dec. (1935). Also Phys. Rev. **49**, 174 (1936).

[15] Kapur and Peierls, Proc. Roy. Soc. **A166**, 277 (1938).

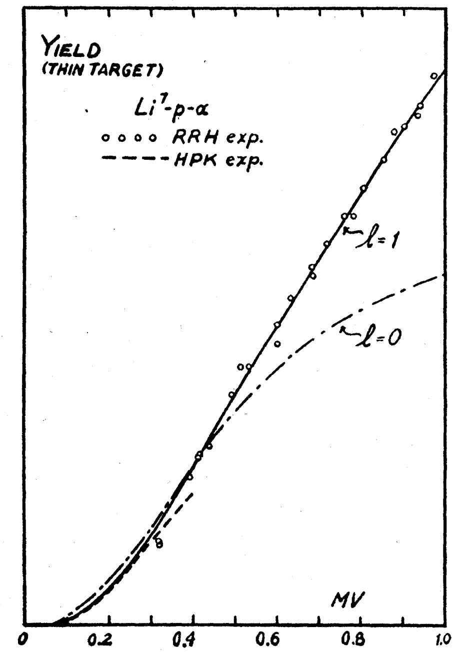

FIG. 4. The circles represent the data of Rumbaugh, Roberts and Hafstad,[16] the short dashed curve that of Herb, Parkinson and Kerst.[16] The solid curve is the theoretical excitation function for $l=1$ and $r_0=1.65$, the dot-dash curve for $l=0$ and $r_0=0$. Curves fitted at 0.4 MV.

The comparison of the theoretical with the experimental excitation functions[16] in Fig. 4 shows that indeed the $l=1$ protons account for the observed variation of the yield much better than $l=0$. A nuclear radius corresponding to $r_0=1.65$ was used for the $l=1$ curve while a zero radius was taken for the $l=0$ case in order to make its comparison as favorable as possible. The form (18) for Γ was employed in computing the $l=1$ function.[17]

Unfortunately, further illustrations of the importance of selection rules are obscured by other considerations. For example, consider the reactions

$$Li^6+H^2\rightarrow 2\ He^4\qquad(20a)$$

and

$$Li^6+H^2\rightarrow Li^7+H^1.\qquad(20b)$$

[16] The experimental data for low energies were taken from Herb, Parkinson and Kerst, Phys. Rev. **48**, 118 (1935). The measurements up to 1 MV are those of Rumbaugh, Roberts, and Hafstad (to be published). We are indebted to the authors and to the Drs. Fleming and Tuve of the Carnegie Institute at Washington for the communication of these results to us.

[17] It was not necessary to assume the existence of a resonance in these calculations in contrast to those of Ostrofsky, Breit and Johnson (Phys. Rev. **49**, 22 (1936)) who employed a one-body model.

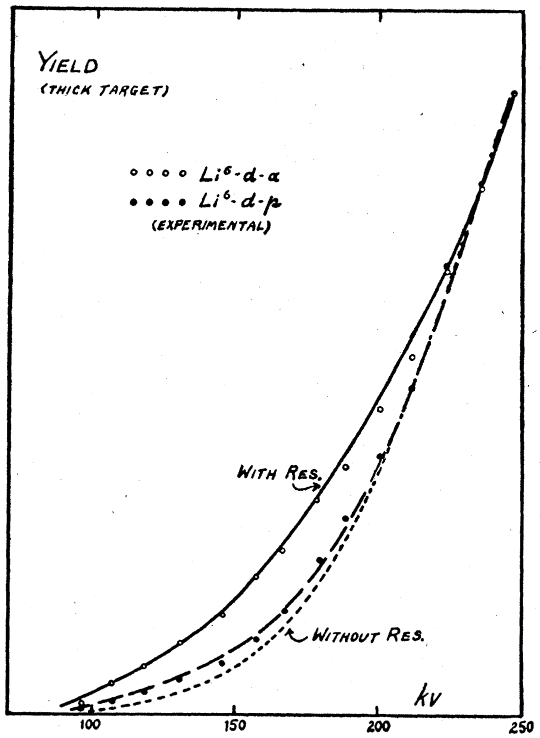

FIG. 5. Circles represent experimental points for the Li^6-d-α reaction, dots for Li^6-d-p, according to Williams, Shepherd and Haxby. The dotted curve is the theoretical excitation function for $l=0$ and no resonance. The solid curve includes a resonance at 40 kv, 50 kv wide. The dashed curve is the sum of the other two made equal at ~160 kv. All curves fitted at 240 kv.

Since the parity of every nucleus involved in (19a) as well as that of the relative motion of the alphas is even, the orbital momentum of the deuteron about Li^6 can only be even. Thus the selection rules require that the yield of protons increases more rapidly with bombarding energy than the alpha-yield, due to the contribution to (20b) of the $l=1$ deuterons which are barred for (20a). Actually, Williams, Shepherd and Haxby[18] do find a steeper curve for the excitation of (20). However, the relative steepness is much too great to be accounted for by the $l=1$ deuterons. With no resonance, the theoretical excitation function (cf. (17)) is given by (18) multiplied with λ^2. For the low energies used, (18a) was found to represent the energy dependence of (18) rather well, i.e., the results depend only slightly on the choice of the radius. Comparison in Fig. 5 shows that the theoretical curves, even with $l=0$, are too steep for both the alpha and proton yields. Even a radius as large as 10^{-12} cm fails to bring about sufficient improvement. A discrepancy in this direction perhaps may indicate that a resonance must be postulated. It was found that a resonance level for the Li^6-d-α reaction at about 40 kv deuteron energy and about 50 kv wide (depending on the position) could remove the discrepancy. It is satisfactory that this resonance is wide compared to the 10 kv or so found for the proton capture resonance widths.[9] Of the three spins, $J=0$, 1 and 2, possible for the compound nucleus with the slow deuterons ($l=0$), the Li^6-d-α resonance certainly has $J=0$ or 2 only. Since the Li^6-d-p reaction may also have $J=1$, only part of it will follow the same excitation function for $l=0$ as Li^6-d-α. We find that by equating the contributions of the cross section without resonance and that with the resonance at 160 kv agreement with the data is obtained. (Fig. 5.)

The excitation function[19] for

$$B^{11}+H^1 \rightarrow 3\,He^4 \tag{20}$$

shows a behavior of the same kind. In this case the postulated resonance level has actually been observed; it lies at 180 kv proton energy and is 11 kv wide. The resonance is most clear for the long range alpha-particles whose emission leads to the ground state of Be^8. From the angular distribution of these alphas it seems plausible[20] that the resonance probably has $J=2$ and is therefore produced by protons of $l=1$. The observed yield curve for (20) is then probably a superposition of an $l=1$ curve showing resonance and a non-resonance function for $l=0$.

The experimental excitation functions for the Be-d-α, Be-d-H^3, Be-d-p and Be-p-α reactions[21] again seem to show too slow a rise compared to the theoretical curve without resonance, but in these cases the differences are too small to be very certain.

[18] Williams, Shepherd and Haxby, Phys. Rev. **52**, 390 (1937).

[19] Williams, Wells, Tate and Hill, Phys. Rev. **51**, 439 (1937).

[20] Oppenheimer and Serber, Phys. Rev. **53**, 636 (1938).

[21] Williams, Haxby and Shepherd, Phys. Rev. **52**, 1031 (1937).