An Experimental Investigation of de Broglie's Equation

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The validity of de Broglie's equation is checked experimentally for electrons of 24 to 64 electron-kilovolts energy. The experiment consists essentially of deflecting a beam of cathode rays by an electrostatic field and, keeping the deflection constant, determining the relation between the intensity of the deflecting field, V, and the de Broglie wave-length, λ, of the deflected electrons. On the assumption that the motion of the electrons is in accordance with the laws of relativistic mechanics, and that the de Broglie wave-length is inversely proportional to the momentum, it is shown theoretically that the expression $\lambda^2 V(1+h^2/m_0^2c^2\lambda^2)^{\frac{1}{2}}$ should be independent of λ. Within the limits of experimental error this expression is found to be independent of λ, thereby demonstrating that the de Broglie equation, $\lambda = h/mv$, combined with the relativity expression for m, is adequate to describe the facts.

I

SINCE the discovery in 1927 of the phenomenon of electron diffraction, many experimenters have been attracted to this field. The great usefulness of electron diffraction as a *method* of research soon became apparent and, as a consequence, electron diffraction, itself, as a *subject* of research was somewhat neglected. In particular, few experiments[1–9] have been performed to check the fundamental relation given by de Broglie's equation,

$$\lambda = (h/m_0v)(1-v^2/c^2)^{\frac{1}{2}}.$$

The present research was undertaken in order to obtain an increase of precision in checking the theory by experiment. The method used avoids the necessity of measuring high voltages or determining the geometry and absolute intensity of deflecting fields of force, and thus eliminates the principal sources of error encountered by earlier experimenters.

II

The diffraction apparatus used to investigate the validity of the de Broglie relation is shown schematically in Fig. 1.

[1] G. P. Thomson, Proc. Roy. Soc. **117**, 600 (1928); **119**, 651 (1928); **125**, 352 (1929).
[2] A. Reid, Proc. Roy. Soc. **119**, 663 (1928).
[3] R. Ironside, Proc. Roy. Soc. **119**, 668 (1928).
[4] M. Ponte, Ann. de physique **13**, 395 (1930).
[5] E. Rupp, Ann. d. Physik **9**, 458 (1931); **10**, 927 (1931).
[6] F. Kirchner, Ann. d. Physik **11**, 741 (1931).
[7] R. v. Meibom and E. Rupp, Ann. d. Physik **13**, 725 (1932); **17**, 221 (1933).
[8] J. Gnan, Ann. d. Physik **20**, 361 (1934).
[9] J. V. Hughes, Phil. Mag. **19**, 129 (1935).

The beam of electrons produced in the cold cathode discharge tube is collimated by two diaphragms placed 8.9 cm apart. The circular openings in these diaphragms are 0.01 cm in diameter. The collimated beam passes between two mutually insulated plates which constitute a velocity selector. These plates are about 11.4 cm long, 1.9 cm wide and separated from each other by 0.475 cm. A final diaphragm, located 2.5 cm beyond the deflecting plates, is provided with a hole 0.01 cm in diameter. The specimen holder is securely fastened to this diaphragm. Another diaphragm, provided with a 0.041-cm slit, is located 1.9 cm in front of the final diaphragm. This diaphragm serves to prevent secondary electrons, produced in the velocity selector, from passing through the final hole.

A sheet of lead foil separates the diffraction and camera chamber from the velocity selector chamber, thus assuring a field-free space in the former. The total distance from the specimen holder to the photographic plate is 29.46 cm. A baffle, located approximately midway between the specimen holder and the photographic plate prevents reflections from the walls of the tube.

A pair of Helmholtz coils, of 12-inch radius, is mounted around the diffraction apparatus in such a position as to neutralize effectively the earth's magnetic field in the region between the anticathode and the specimen holder.

Two stages of mercury diffusion pumps, backed by a Cenco vacuum pump, serve to evacuate the velocity selector and camera

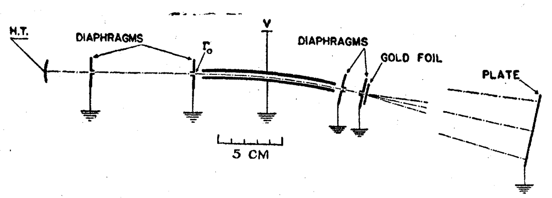

FIG. 1. Schematic diagram of electron diffraction apparatus.

chambers. The pressure in these parts is kept below 10^{-6} mm of Hg, during operation. A separate diffusion pump, operating into the same fore-vacuum, is used for the discharge tube. The pressure in the discharge tube is regulated to give the correct current flow by means of an adjustable leak from the fore-vacuum.

The high tension for the discharge tube is obtained from a 70,000-volt transformer. A commutator type rectifier provides full-wave rectification. An autotransformer in the primary circuit permits the desired control of the voltage. Two large Leyden jars, across the high tension lines, serve to by-pass the high frequency components introduced by the commutator.

The deflecting voltage, used by the velocity selector, is obtained from a power pack of conventional design. The filter circuit of this power supply includes six microfarads capacity. Since the maximum current drawn from this power pack is only 300 microamperes, the 60-cycle ripple is negligible. Line fluctuations in the primary power lines are reduced to a minimum by means of a saturated iron type voltage regulator. A high resistance potentiometer permits the necessary voltage adjustment. The maximum voltage available is about 850 volts.

A 300-volt, megohm Weston voltmeter, provided with a suitable precision wire-wound multiplier, serves for the determination of the deflecting voltage. The voltmeter was carefully calibrated against a Weston laboratory standard voltmeter (0.1 percent accuracy).

A comparator was used to measure the diameters of the diffraction rings registered on the photographic plates.

III

In this section there will be developed certain theoretical relationships which serve as the basis of the experimental work.

Consider an electron traveling through a potential field, $V\phi(\mathbf{r})$, where V is a scalar parameter independent of $\mathbf{r}$. If the accelerating field $\nabla(V\phi)$ is not great, the motion of the electron is "quasi-stationary," and in accordance with the equation

$$e\nabla(V\phi)=m_0 d/dt\{(1-v^2/c^2)^{-\frac{1}{2}}d\mathbf{r}/dt\}. \tag{1}$$

This may be transformed into

$$Ve\nabla\phi=m_0\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}}v^2\frac{d^2\mathbf{r}}{ds^2}+m_0\frac{dv}{ds}\left\{\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}}v+\left(1-\frac{v^2}{c^2}\right)^{-\frac{3}{2}}\frac{v^3}{c^2}\right\}\frac{d\mathbf{r}}{ds}. \tag{2}$$

Recalling that the vectors $d\mathbf{r}/ds$ and $d^2\mathbf{r}/ds^2$ are at any point mutually perpendicular, and that $d\mathbf{r}/ds$ is a unit vector, we get by scalar multiplication,

$$Ve\nabla\phi\cdot\frac{d^2\mathbf{r}}{ds^2}=m_0\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}}v^2\left(\frac{d^2\mathbf{r}}{ds^2}\right)^2, \tag{3}$$

and

$$Ve\nabla\phi\cdot\frac{d\mathbf{r}}{ds}=m_0\frac{dv}{ds}\left\{\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}}v+\left(1-\frac{v^2}{c^2}\right)^{-\frac{3}{2}}\frac{v^3}{c^2}\right\}. \tag{4}$$

At points where the curvature of the trajectory does not vanish, Eq. (3) gives

$$\frac{e}{m_0c^2}\frac{\nabla\phi\cdot d^2\mathbf{r}/ds^2}{(d^2\mathbf{r}/ds^2)^2}=R=V^{-1}\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}}\frac{v^2}{c^2}. \tag{5}$$

By integration, Eq. (4) gives

$$\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}}=\left(1-\frac{v_0^2}{c^2}\right)^{-\frac{1}{2}}+\frac{Ve(\phi-\phi_0)}{m_0c^2}, \tag{6}$$

in which v_0 is the velocity of the electron at a point where the potential is $V\phi_0$.

Equation (5) may now be written,

$$R=\frac{\left\{\left(1-\frac{v_0^2}{c^2}\right)^{-\frac{1}{2}}+\frac{Ve(\phi-\phi_0)}{m_0c^2}\right\}^2-1}{V\left\{\left(1-\frac{v_0^2}{c^2}\right)^{-\frac{1}{2}}+\frac{Ve(\phi-\phi_0)}{m_0c^2}\right\}}. \tag{7}$$

If we now impose the condition that $d^2\mathbf{r}/ds^2$ (at a point where $\phi=\phi_0$) be independent of the energy of the electron, we get

$$V^{-1}(1-v_0^2/c^2)^{-\frac{1}{2}}(v_0^2/c^2)=R_0\neq f(v_0). \quad (8)$$

Substitution of Eq. (8) in Eq. (7) results in

$$R=R_0+2\frac{e(\phi-\phi_0)}{m_0c^2}-\frac{e(\phi-\phi_0)}{m_0c^2}\cdot\frac{v_0^2}{c^2}$$
$$+\text{higher powers of } e(\phi-\phi_0)/m_0c^2. \quad (9)$$

The first two terms on the right of this expression are independent of the energy of the electron. Under the conditions of the experiment to be described the remaining terms are less than 0.07 percent of R_0.[10] Thus, if Eq. (8) is satisfied, $d^2\mathbf{r}/ds^2$ is, within the limits of experimental error,[11] independent of the energy of the electron. That is, the differential equation of the trajectory,

$$d^2\mathbf{r}/ds^2=F(d\mathbf{r}/ds, \mathbf{r}), \quad (10)$$

is independent of the electron's energy. If the point of entry of the electron into the field, $V\phi$, and the direction of the electron's velocity[12] at that point are also independent of the energy, the whole trajectory will be independent of the electron's energy. The construction of the apparatus insures that these boundary conditions are satisfied. From the de Broglie relation

$$\lambda=(h/m_0v)(1-v^2/c^2)^{\frac{1}{2}}, \quad (11)$$

the third condition, Eq. (8), may be written

$$\lambda^2V(1+h^2/m_0^2c^2\lambda^2)^{\frac{1}{2}}=h^2/m_0^2c^2R_0=\text{const.} \quad (12)$$

The fact that this equation represents the condition that the path of the electron is independent of λ (or the energy) constitutes a verification of de Broglie's relation, (11).

It will be noted that the development given above makes no assumptions concerning the form of the electric field other than that $e(\phi-\phi_0)/m_0c^2$ is small compared to R (Eq. (9)). This greatly simplifies the experimental application of the developed relations.

IV

From Fig. 1 it is clear that at the point, $\mathbf{r}_0$, the cathode rays travel in a direction fixed by the two collimating diaphragms, and that in the space between the two plates the electric field at any point is directly proportional to the voltage applied between the plates. It is also evident from the unidirectional character of the electric field that a single point, beyond the deflecting field, will serve to determine uniquely the whole trajectory of the beam passing through that point. Thus, the beam that passes through the final diaphragm hole satisfies the conditions which were shown, in Part III, to lead to the expression $\lambda^2V(1+h^2/m_0^2c^2\lambda^2)^{\frac{1}{2}}=\text{constant}$.

Eighteen photographs were taken to check the validity of this equation. The energies of the cathode rays were between 24 and 64 electron-kilovolts. Gold was used as the diffracting material.

In the analysis of these plates the following sources of error must be considered:

(1) *The measurement of the deflecting voltage.*—Because of the calibration of the voltmeter against the Weston laboratory standard voltmeter, the voltage readings are reliable to about 0.1 percent of their full scale reading. (Full scale reading with multiplier—850 volts.) An error in the value of the multiplier resistance would have no significant effect on the results, since it would

TABLE I. *Experimental results.*

PLATE	V (e.m.u.)	λ (cm)	λ^2V (e.m.u.)	$\lambda^2V(1+h^2/m_0^2c^2\lambda^2)^{\frac{1}{2}}$ (e.m.u.)
11–2	8.080×10^{10}	4.713×10^{-10}	1.795×10^{-8}	2.019×10^{-8}
18–5	8.000	4.730	1.790	2.012
18–2	7.789	4.812	1.804	2.020
8–2	7.469	4.907	1.799	2.007
18–4	7.200	5.013	1.810	2.011
16–5	6.920	5.127	1.819	2.013
16–2	6.754	5.188	1.818	2.007
8–4	6.451	5.321	1.827	2.008
11–3	6.051	5.510	1.837	2.008
18–3	5.703	5.694	1.849	2.009
8–1	5.509	5.808	1.858	2.014
16–3	5.440	5.841	1.856	2.010
16–4	4.926	6.183	1.883	2.023
11–5	4.469	6.490	1.882	2.009
11–1	3.989	6.919	1.909	2.024
16–1	3.520	7.366	1.910	2.011
18–1	3.217	7.711	1.913	2.006
11–4	3.046	7.881	1.892	1.979

[10] The approximate values of the quantities involved are: $R_0=3\times10^{-12}$ e.m.u., $e(\phi-\phi_0)/m_0c^2(\text{max.})=1\times10^{-14}$ e.m.u., $v_0^2/c^2(\text{max.})=0.2$.

[11] An error of 0.07 percent in the experimental determination of V would result in an error of 0.07 percent in the curvature along the *whole* electron path. Since the precision of the measurement of V is not sufficient to detect such an error, the uncertainty due to variation of curvature with electron energy is, *a fortiori*, less than the accidental errors of measurement.

[12] These are the boundary conditions for the differential equation (10).

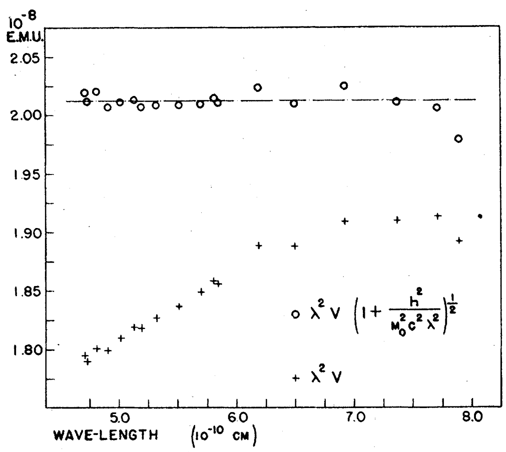

FIG. 2. Results of the experimental test of de Broglie's equation.

merely change the value of the constant determined.

(2) *Determination of the wave-length.*—The only appreciable errors in the determination of the wave-length are due to the measurement of the diffraction ring diameters. Small errors in the plate distance or in the assumed lattice constant of gold, since they have nearly equal effects for all plates, merely change the value of the experimentally determined constant. The probable error in the determination of λ, calculated from the statistical deviations of various diameter measurements, is considerably less than 0.1 percent for most of the plates.

The experimental results are tabulated in Table I and shown graphically in Fig. 2. It is evident that the function

$$\lambda^2 V(1+h^2/m_0{}^2c^2\lambda^2)^{\frac{1}{2}}$$

is actually found to be a constant for electrons in the range of energies from 24 to 64 electron-kilovolts.[13]

The average value of the quantity

$$\lambda^2 V(1+h^2/m_0{}^2c^2\lambda^2)^{\frac{1}{2}},$$

as measured on seventeen plates, is 2.0124×10^{-8} e.m.u. with a mean deviation of 0.0045×10^{-8}. The average value of the first eight plates, taken in order of increasing λ, is 2.0121×10^{-8} with a mean deviation of 0.0039×10^{-8}. The average of the last eight plates is 2.0132×10^{-8} with a mean deviation of 0.0052×10^{-8}.

We note that the precision of the experiment, statistically determined, is in fair agreement with the anticipated errors of measurement—the constant is reliably determined to within about 0.2 percent. Since the constant is approximately proportional to λ^2, we may conclude from this experiment that the de Broglie wave-length of an electron, in the energy range of 24 to 64 electron-kilovolts, is inversely proportional to the momentum to within about 0.1 percent. This is, then, a close confirmation of the de Broglie relation.

For the sake of comparison, Table I and Fig. 2 also include the values of the product $\lambda^2 V$. *Without* relativity this quantity would be constant.

V

The constant of proportionality in de Broglie's equation was not determined in the present experiment. The method used, however, might be extended in the following manner to obtain the value of this important constant.

If electrons of moderate energy were employed, it would be possible to measure with precision both the discharge tube voltage and the voltage impressed on the deflecting plates. A Faraday collector would serve to indicate when the electron beam actually passed through all the diaphragms. (No diffracting material would be used.) The constant, R_0, of the apparatus could then be calculated from Eq. (8) since the energy of the electrons as well as the necessary deflecting voltage would be known. The constant of Eq. (12), as determined by the present experiment, would then be a definite function of universal constants. The determination of this combination of universal constants would depend, then, only upon the measurement of moderate voltages and the measurement of wave-lengths by a diffraction experiment.

The writer wishes to acknowledge his indebtedness to Professor C. B. Bazzoni, under whose direction this investigation was undertaken, for kindly encouragement and advice.

[13] Plate 11–4 is obviously discordant and is not included in the summary. The deviation of this plate from the average is more than seven times the average deviation of the remaining plates. This plate was considerably fainter than the others.