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The Saturation Requirements for Nuclear Forces

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The inequalities between the coefficients M, H, B, V, representing the proportions of Majorana, Heisenberg, Bartlett and ordinary interactions in the symmetric Hamiltonian, arising through considerations of saturation and of the instability of odd-odd nuclei heavier than N^{14} are represented graphically (Fig. 1). The allowed values of M, H, B, V correspond to a part of a plane bounded by three straight lines. These inequalities are sufficient for saturation at infinitely high density of nuclear particles and all conditions derivable from the necessity of saturation for the potential energy in the high density condition are derivable from those listed. The limits set on the interactions in the ^3P and ^1P states by the inequalities are discussed. The results are summarized in Table I, Eqs. (7) and (8) and in the adjoining section. The sufficiency of the conditions is discussed in Section 2. The physical limitations are gone into in Section 3.

IT is customary at present to use in nuclear theory a symmetric Hamiltonian consisting of interactions between pairs of nuclear particles represented by potentials of the following types: Majorana exchange (exchange of space coordinates), Heisenberg exchange (exchange of space and spin coordinates), Bartlett exchange (exchange of spin coordinates), ordinary non-exchange potential. It is, of course, not certain that the actual forces are of this type and that the saturation of the nuclear forces owes its origin to a preponderance of exchange forces rather than to saturation phenomena of other types.[1] It appears to be, nevertheless, of interest to systematize the available information[2] regarding the possible proportions of the different types of exchange forces as obtainable from considerations of stability of heavy nuclei (saturation) and the instability of nuclei heavier than N^{14} with an odd number of neutrons N and an odd number of protons Z. A brief elementary discussion is given below of the inequalities that must exist between the four kinds of potentials together with a diagrammatic representation of the values that are possible according to present evidence. A proof is then given by means of group character formulas that the elementary method contains all of the information derivable from requirements of saturation.

[1] G. Gamow and E. Teller, Phys. Rev. **51**, 289 (1937).
[2] G. Breit and E. Feenberg, Phys. Rev. **50**, 850 (1936); D. Inglis and L. A. Young, Phys. Rev. **51**, 525 (1937); E. Wigner, Phys. Rev. **51**, 947 (1937); N. Kemmer, Nature **140**, 192 (1937); E. Feenberg, Phys. Rev. **52**, 667 (1937); cf. also related discussion by H. Volz, Zeits. f. Physik **105**, 537 (1937).

1. NECESSARY CONDITIONS

The potential energy between a pair of nuclear particles will be represented by

$$[MP^M+HP^H+BP^B+V\cdot 1]J(r);\quad J(0)<0, \quad (1)$$

where M, H, B, V are ordinary dimensionless numbers and P^M, P^H, P^B are, respectively, the Majorana, Heisenberg and Bartlett exchange operators. The function $J(r)$ is taken to be negative in the vicinity of $r=0$. The coefficients M, H, B, V will be supposed to be connected by

$$M+H+B+V=1. \quad (2)$$

The potential energy in the stable ^3S state of the deuteron is $J(r)$. In the ^1S state scattering experiments indicate a smaller degree of attraction represented by the potential $qJ(r)$ with $q\sim 1/2$. Thus

$$M-H-B+V=q<1. \quad (3)$$

If a definite value of q, say $q=3/5$ is assumed, the above relations restrict the four quantities M, H, B, V so that only two of them are independent. The quantities M and H will be used as independent variables and a diagram using M and H as coordinates will be employed. The interactions in the ^3P and ^1P states will be written as $(^3P)J(r)$, $(^1P)J(r)$. Positive values of (^3P) and (^1P) correspond to attractions in these states. These quantities are given by

$$(^3P)=-M-H+B+V=-2M-2H+1, \quad (3.1)$$

$$(^1P)=-M+H-B+V=-2M+2H+q. \quad (3.2)$$

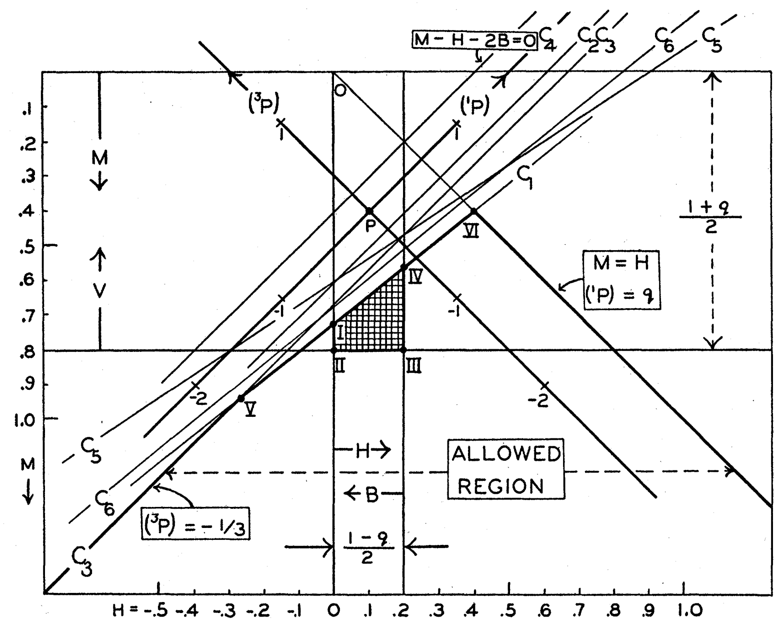

FIG. 1. Graphical representation of inequalities. Origin of M, H coordinate system is at 0. Origin of B, V coordinate system is at III. Origin of (^3P), (^1P) coordinate system is at P. To any point in the plane there correspond the values of M, H, B, V, in these coordinate systems. The inequalities $c_n \leqq 0$, $H+2B-M\leqq 0$, $H-M\leqq 0$ exclude all points above the lines $c_n=0$, $H+2B-M=0$, $H-M=0$. The lines $c_n=0$ are marked by c_n in the figure. The region allowed by all the inequalities considered here lies below the lines c_3, c_1 and $M=H$. The shaded area I, II, III, IV is that part of the allowed region for which M, H, B, V are positive. The scale of (^3P), (^1P) differs from that of M, H, B, V and is marked in the coordinate system (^3P), (^1P). Diagram is drawn for $q=3/5$.

One can express also M, H, B, V in terms of (^3P) and (^1P) as independent variables:

$$M=\tfrac{1}{4}[1+q-(^3P)-(^1P)], \tag{3.3}$$

$$H=\tfrac{1}{4}[1-q-(^3P)+(^1P)], \tag{3.4}$$

$$B=\tfrac{1}{4}[1-q+(^3P)-(^1P)], \tag{3.5}$$

$$V=\tfrac{1}{4}[1+q+(^3P)+(^1P)]. \tag{3.6}$$

Denoting the projection of total spin along an arbitrary axis by $\sigma\hbar/2$ and distinguishing between protons and neutrons by suffixes π and ν one has the following inequalities

$$c_1=4V-M-2H+2B=-5M-4H+3+q\leqq 0;\quad \sigma_\nu=\sigma_\pi=0;\quad Z=N \tag{4.1}$$

$$c_2=2V-M-2H+B=-3M-3H+\tfrac{1}{2}(3+q)\leqq 0;\quad \sigma_\nu=\sigma_\pi=0;\quad Z=0 \tag{4.2}$$

$$c_3=2V-M-H+2B=-3M-3H+2=c_2+\tfrac{1}{2}(1-q)\leqq 0;\quad \sigma_\nu=N;\quad \sigma_\pi=Z;\quad Z=N \tag{4.3}$$

$$c_4=V-M-H+B=-2M-2H+1=\tfrac{1}{3}(-1+2c_3)\leqq 0;\quad \sigma_\nu=N;\quad Z=0 \tag{4.4}$$

$$c_5=2V-M-H+B=-3M-2H+\tfrac{1}{2}(3+q)\leqq 0;\quad \sigma_\nu=N;\quad \sigma_\pi=-Z;\quad Z=N \tag{4.5}$$

$$c_6=8V-3M-4H+5B=-11M-9H+\tfrac{1}{2}(13+3q)\leqq 0;\quad \sigma_\nu=N;\quad \sigma_\pi=0;\quad Z=N. \tag{4.6}$$

The first four inequalities are not new[2] and some additional conditions are known as well.[2] It will be shown below that with the value of q indicated by present experimental data all the inequalities are consequences of (4.1), (4.3) and (5.2). If one assumes the inequalities (5), conditions (4.2), (4.4), (4.5) and (4.6) do not give any additional information. On the other hand, if the inequalities (5) are not used then (4.5) gives new information inasmuch as it is sufficient for the validity of (4.1) and (4.3) in the region of small M and large H.

The above six inequalities may be obtained by using trial wave functions contained in a volume that is sufficiently small to make $J(r)$ equal to $J(0)$. The trial wave functions give an expectation value for the energy which is higher than that corresponding to an accurate solution of the wave equation. The numbers of neutrons and protons, N and Z, are contained in this expression as N^2 and Z^2 as well as in lower powers of N and Z. The sum of all terms in N^2 and Z^2 must, therefore, be positive or zero. Otherwise the energy in the most stable condition will go to $-\infty$ as $-N^2$ or $-Z^2$. The conditions (4.1) to (4.6) are obtained in this manner and by neglecting the Coulomb energy which is justifiable[2] in view of its relatively small value. In addition the nonexistence of stable odd-odd nuclei heavier than N^{14} gives[2]

$$H-M+2B=-M-H+1-q\leqq 0 \quad \text{(parallel spin of } \nu \text{ and } \pi \text{ in unstable nucleus)} \qquad (5.1)$$

$$H-M\leqq 0 \quad \text{(antiparallel spin of } \nu \text{ and } \pi \text{ in unstable nucleus)} \qquad (5.2)$$

The variables H, M are used in Fig. 1 as Cartesian coordinates of a point in a plane. Any one of the inequalities $c\leqq 0$ has the geometrical significance of dividing the H, M plane into an allowed part below the line $c=0$ and a forbidden part above that line. Only the part of the plane below the six lines given by the inequalities (4.1)$\cdots$(4.6) and the two lines corresponding to (5.1) and (5.2) can represent a satisfactory set of M, H, B, V. The six lines corresponding to $c_n=0$ are marked as c_n in the figure. The two lines corresponding to Eqs. (5.1) and (5.2) are labeled by these equations. It is seen that for $q=3/5$ the permissible region of the plane is bounded by the three lines

$$c_3=0, \qquad c_1=0, \qquad H=M. \qquad (6)$$

The axis of H is also the negative of the axis of B with $B=0$ which comes at $H=\frac{1}{2}(1-q)$. Similarly the values of V increase upwards along the M axis with $V=0$ coming at $M=\frac{1}{2}(1+q)$. In the same figure a rotation through $\pi/4$ gives lines of constant (^3P) and (^1P). The values of (^3P) increase as one moves up and to the left, those of (^1P) as one moves up and to the right. The scales for (^3P) and (^1P) are shown on these axes. Unity is seen to be represented by a smaller length in these coordinates than in the $M-H$ system. As has been noticed by Feenberg,[2] the inequality $c_3\leqq 0$ determines an upper limit of $-1/3$ for (^3P) which is of interest for the scattering of protons by protons. According to this the repulsion between protons in the ^3P state should be greater than that corresponding to a potential $-J(r)/3$. The condition is apparent in the diagram. The line $c_3=0$ is one of the bounding lines of the allowed region and for it $(^3P)=-1/3$.

An upper limit is set for (^1P) by the condition $H=M$. For this line $(^1P)=q$ and only smaller values of (^1P) are allowed. If it is supposed that, in addition, the interaction is such[3] that $c_1=0$ then (^1P) must be greater than that corresponding to the intersection of $c_1=0$ with $c_3=0$, which is $-3q$. The latter condition $[(^1P)\geqq -3q]$ was first given by Feenberg. If one similarly restricts the allowed region for (^3P) by the requirement $c_1=0$, then $H<M$ imposes the additional condition $(^3P)\geqq -\frac{1}{9}(4q+3)$. Thus for $c_1=0$

$$-1/3\geqq (^3P)\geqq -(4q+3)/9, \qquad (7)$$

$$q\geqq (^1P)\geqq -3q. \qquad (8)$$

Still closer limits on (^3P) are set by requiring that V, M, H, B should all be positive. The corresponding region is shaded in Fig. 1 and is seen to be a trapezoid. Its vertices are the points I, II, III, IV and the points at the intersection of $c_1=0$ with $c_3=0$ and $H=M$ are

TABLE I. *Values of (^3P), (^1P), V, M, H, B for points (I), (II), (III), (IV), (V), (VI).*

Point	Intersecting Lines	(^3P)	(^1P)	V	M	H	B
(I)	$c_1=0$, $H=0$	$\frac{-2q-1}{5}$	$\frac{3q-6}{5}$	$\frac{3q-1}{10}$	$\frac{q+3}{5}$	0	$\frac{1-q}{2}$
(II)	$H=0$, $V=0$	$-q$	-1	0	$\frac{1+q}{2}$	0	$\frac{1-q}{2}$
(III)	$V=0$, $B=0$	-1	$-q$	0	$\frac{1+q}{2}$	$\frac{1-q}{2}$	0
(IV)	$B=0$, $c_1=0$	$\frac{-2-q}{5}$	$\frac{3-6q}{5}$	$\frac{3-q}{10}$	$\frac{1+3q}{5}$	$\frac{1-q}{2}$	0
(V)	$c_1=0$, $c_3=0$	$-\frac{1}{3}$	$-3q$	$\frac{1-3q}{6}$	$\frac{1+3q}{3}$	$\frac{1-3q}{3}$	$\frac{1+3q}{6}$
(VI)	$c_1=0$, $H=M$	$\frac{-4q-3}{9}$	q	$\frac{7q+3}{18}$	$\frac{q+3}{9}$	$\frac{q+3}{9}$	$\frac{3-11q}{18}$

[3] D. Inglis, Phys. Rev. **51**, 531 (1937); E. Feenberg, Phys. Rev. **51**, 777 (1937).

designated by V and VI respectively. In Table I are given the values of (^3P), (^1P), V, M, H, B for these six points. The values of V, M, H, B are very much restricted if one excludes negative values of these quantities. The simultaneous requirements of V, M, H, $B \geqq 0$ and $c_1=0$ determine (^3P) particularly well and place it between the values for the points I and IV. For $q=3/5$ these limits give $-11/25>(^3P)>-13/25$ and for $q=1/2$ they are $-2/5>(^3P)>-1/2$. If one removes the requirement $c_1=0$ but keeps $V, \cdots, B\geqq 0$, then the points I and III set the limits on (^3P) which give for $q=3/5$, $-11/25>(^3P)>-1$. Similarly removing the requirement $c_1=0$ but keeping $V, \cdots, B\geqq 0$ the value of (^1P) is seen to lie between the limits set by points II and IV. Thus $-1<(^1P)<\frac{1}{5}(3-6q)$ which for $q=3/5$ becomes $-1<(^1P)<-3/25$. The requirement of positiveness of M, H, B, V is seen to restrict (^3P) within a narrower region than (^1P).

2. Sufficient Conditions

The inequalities (4.1)$\cdots$(4.6) have been obtained by considering states with special spin orientations that have been listed. It is not obvious that they correspond to the most stringent conditions on V, M, H, B because by a suitable change of the wave function it may be possible to lower the expectation value of the energy. It will be shown in this section that the requirement of the absence of terms in the expectation value of the potential energy, that go to $-\infty$ as $-N^2$ and $-Z^2$, in the approximation of long range forces, gives conditions on V, M, H, B that follow from the inequalities (4.1), (4.3), (5.2). This means that these inequalities are sufficient for saturation of the potential energy at high particle densities where by saturation is meant the impossibility of finding a trial wave function which gives, at high particle densities, a negative expectation value for the potential energy varying quadratically with the number of particles. For certain functions $J(r)$ it can be shown then by an argument analogous to one previously used[4] that the same holds for every wave function independently of its extension in space. Since the expectation value of the kinetic energy is positive these conditions are also sufficient for the saturation of the total energy.

For wave functions with a small spatial extension the $J(r)$ in (1) may be replaced by $J(0)$. The problem of finding a trial wave function with the lowest expectation value for (1) is then reduced to that of finding the eigenfunction of

$$-MP^M-HP^H-BP^B-V\cdot 1$$

which corresponds to the lowest eigenvalue. Since the four operators in this sum commute, there is a complete set of eigenfunctions common to all four of them. The lowest eigenvalue is obtained for that eigenfunction for which the sum of the four eigenvalues is lowest.

It has been shown by Dirac[5] that the eigenfunctions of P^B correspond to definite values of the total spin S. Similarly, according to Feenberg and Phillips,[6] the eigenfunctions for P^H have definite isotopic spin[7] T and those for P^M are known[7] to have definite "multiplicity." The part of the potential energy which is proportional to the square of the total number of particles becomes

$$E=-A^2J(0)\{-\tfrac{1}{2}V+[\tfrac{1}{8}+\tfrac{1}{2}(P^2+P'^2+P''^2)]M+(\tfrac{1}{4}+\bar{T}^2)H-(\tfrac{1}{4}+\bar{S}^2)B\}. \quad (9)$$

Here[8] $A=N+Z=$atomic number, $\bar{S}=S/A$, $\bar{T}=T/A$, $S=$particle spin in units $\hbar$, $T=$isotopic spin, $P=$greatest $\bar{S}$ possible for the multiplet, $P'=$greatest $\bar{T}$ possible for this $\bar{S}$, $P''=$greatest Y/A possible for these $\bar{S}$, $\bar{T}$.

Since $M>0$ (see diagram), it will suffice to consider the following cases:

(1) $M>0, H>0, B<0$;
(2) $M>0, H>0, B>0$;
(3) $M>0, H<0, B>0$.

The case $M>0$, $H<0$, $B<0$ need not be considered because $B+H=\frac{1}{2}(1-q)>0$, which is impossible for $H<0$, $B<0$. For each of the

[4] E. Wigner, Proc. Nat. Acad. **22**, 662 (1936).

[5] Cf. P. A. M. Dirac, *The Principles of Quantum Mechanics*, second edition (Oxford, 1935) §61.

[6] E. Feenberg and M. Phillips, Phys. Rev. **51**, 597 (1937).

[7] E. Wigner, Phys. Rev. **51**, 947 (1937).

[8] E. Wigner, Phys. Rev. **51**, 106 (1937). The notation used in the present paper differs from that of this reference in referring to S, T, Y of Eqs. (11) of the latter as PA, $P'A$, $P''A$.

above cases the expression for the energy will be minimized by choosing suitably $\bar{S}$, $\bar{T}$, P, P', P''. It will then be required that E, as given by Eq. (9), be >0. For any other choice of $\bar{S}$, $\bar{T}$, P, P', P'' it follows that $E>0$. In this way one obtains, therefore, sufficient conditions for $E>0$ and it is necessary for nuclear stability that all such conditions be fulfilled. Since the actual energy may be lower than (9) these conditions may not be sufficient for stability but it appears likely that they are,[4] since this has been shown to be the case for a special class of functions $J(r)$.

For case (1) the minimum E is obtained for $P=P'=P''=\bar{T}=\bar{S}=0$ which leads on substitution into Eq. (9) to $c_1<0$ and is therefore contained in the preceding inequalities. For case (2) one may consider: (a) $M-2B>0$ and (b) $M-2B<0$. For (a) the combination

$$\tfrac{1}{2}P^2M-\bar{S}^2B=P^2(\tfrac{1}{2}M-B)+(P^2-\bar{S}^2)B$$

is a minimum when $P=\bar{S}=0$. The minimum of E corresponds, therefore, to $P=\bar{S}=P'=P''=\bar{T}=0$ which gives $c_1<0$ as in case (1). For (b) the combination $\frac{1}{2}P^2M-\bar{S}^2B$ is a minimum when P has its maximum and $P^2-\bar{S}^2$ has its minimum value. Since P is the largest possible $\bar{S}$, the minimum $P^2-\bar{S}^2=0$ and the maximum $P=1/2$ which corresponds to a parallel orientation of all the particle spins. The best values are thus $P=\bar{S}=1/2$. For these values the minimum E corresponds to $P'=P''=\bar{T}=0$. Substituting into Eq. (9) one finds then $c_3<0$ in agreement with the fact that in the inequality (4.3) all particle spins were taken to be parallel.

For case (3) it is convenient to rearrange the expression for E:

$$E=-A^2J(0)[\tfrac{5}{8}M+\tfrac{1}{2}H-\tfrac{1}{8}(q+3)]+E_v, \qquad (9.1)$$

$$E_v=-A^2J(0)[\tfrac{1}{2}M(P^2+P'^2+P''^2)+H(\bar{T}^2+\bar{S}^2)-\tfrac{1}{2}(1-q)\bar{S}^2]. \qquad (9.2)$$

The variable terms are contained in E_v. Here $H<0$, $-\frac{1}{2}(1-q)<0$. Hence, for fixed P, P', P'', a minimum is obtained for $\bar{T}=P'$, $\bar{S}=P''$. For these values Eq. (9.2) becomes

$$E_v=-A^2J(0)\{[H+\tfrac{1}{2}M-\tfrac{1}{2}(1-q)]P^2+(H+\tfrac{1}{2}M)P'^2+\tfrac{1}{2}MP''^2\}. \qquad (9.3)$$

In this expression the coefficient of P'' is positive since $M>0$. The coefficient of P^2 is necessarily less than that of P'^2. Therefore, there are three possibilities: (a) $H+\frac{1}{2}M-\frac{1}{2}(1-q)>0$, $H+\frac{1}{2}M>0$; (b) $H+\frac{1}{2}M-\frac{1}{2}(1-q)<0$, $H+\frac{1}{2}M<0$; (c) $H+\frac{1}{2}M-\frac{1}{2}(1-q)<0$, $H+\frac{1}{2}M<0$. For (a) the minimum E is obtained if $P=P'=P''=0$. In this case, therefore, $\bar{S}=\bar{T}=0$ as well. The minimum E is the same as in case (1) and one obtains $c_1<0$. For (b) the minimum E is obtained by using maximum $P=\frac{1}{2}$ and minimum $P'=P''=0$ as in (2b). The condition $c_3<0$ is, accordingly, obtained again.

For (3b) the coefficients of P^2 and P'^2 are both negative. In order to obtain a low E it is now advantageous to use large values of P and P' and at the same time P'' must be made as small as possible. It is not possible, however, to vary P, P', P'' independently of each other as the minimum of E is approached. In fact the minimum for Eq. (9.3) with independent variations of P, P', P'' corresponds to $P=0$, $P'=1/2$, $P''=0$ which is an impossible set of these quantities since $P\geqq P'\geqq P''$. The way in which P, P', P'' may vary can be inferred[8] from their expression in terms of $\Lambda_4\geqq\Lambda_3\geqq\Lambda_2\geqq\Lambda_1\geqq 0$.

$$P=\frac{1}{2A}(\Lambda_4-\Lambda_3-\Lambda_2-\Lambda_1);$$

$$P'=\frac{1}{2A}(\Lambda_4-\Lambda_3+\Lambda_2-\Lambda_1)$$

$$P''=\frac{1}{2A}(\Lambda_4-\Lambda_3-\Lambda_2+\Lambda_1);$$

$$A=\Lambda_4+\Lambda_3+\Lambda_2+\Lambda_1.$$

If here $\Lambda_1>0$ one can decrease Λ_1 and increase Λ_4 by an equal amount. This leaves A, P'' unchanged and increases both P and P' which gives a lower energy. Thus $\Lambda_1=0$ will include the most favorable cases. With this restriction $P''=P+P'-\frac{1}{2}$ and

$$E_v=-A^2J(0)\left\{[H+M-\tfrac{1}{2}(1-q)]P^2+(M+H)P'^2+M(PP'+\tfrac{1}{8})-\frac{M}{2}(P+P')\right\}. \qquad (9.4)$$

Further

$$\Lambda_4/A=P+P'>\Lambda_3/A=\tfrac{1}{2}-P'>\Lambda_2/A=\tfrac{1}{2}-P>0. \qquad (9.5)$$

The above inequalities (9.5) confine the possible P, P' to a triangular region in the P, P' plane bounded by the lines $P=\frac{1}{2}$, $P=P'$, $2P'+P=\frac{1}{2}$. In this triangle the values of P, P' can be varied arbitrarily. If one uses the facts that in the present case $H<0$, $M+2H<0$ while according to conditions already derived $M+H>0$ one finds by means of Eq. (9.4), after a somewhat lengthy but quite elementary discussion, that the minimum E corresponds to $P=1/2$, $P'=0$, $P''=0$ which gives $c_3<0$ as in (3b). This exhausts the possibilities.

3. Final Remarks

It should be observed that the conditions (4) have been derived on the assumption of the absence of other nuclear forces than those given in Eq. (1). In particular forces of the "many body" type could bring about saturation in spite of the inequalities (4) not being fulfilled. Even if under ordinary circumstances these forces should be negligible they may possibly be of special importance in the state of high density considered for saturation.[9] A similar remark applies[2] to the influence of a velocity dependence of the potential.

It should also be pointed out that the inequalities (4) are obtained by a consideration of an idealized state of high density which would be the actual state, if the inequalities (4) were not satisfied. On the other hand, inequalities (5) follow from the consideration of actual nuclei and involve several approximations. One can derive them on the basis of the high density model also but this certainly is not the condition in actual nuclei. All other considerations on this question are based, apart from some, rather rough approximations, on the assumption that H and B are small compared[10] with M and should not be used, therefore, to establish the inequality $H<M$. It is rather the apparent agreement of these considerations with experiment that gives one confidence in the inequality $H<M$. The underlying experimental rule[11] holds without exception for[12] $A>14$. In fact, this seems to indicate that H is materially smaller than M, as otherwise accidental factors would be likely to make at least some of the heavier odd-odd nuclei stable.

On the other hand, information regarding the approximate symmetry of the nuclear Hamiltonian arises out of experimental material with ordinary rather than high nuclear densities. It is more closely related to the inequalities (5) than to (4) and one may be skeptical about the latter from this point of view.

Very little is known about the space dependence of the function $J(r)$ and still less about $J(0)$. It is possible that the coefficients M, H, B, V should be multiplied by different functions $J(r)$ and there is a chance that the four $J(r)$ are roughly the same for distances of the order 2×10^{-13} cm but that the $J(0)$ are widely different. If such should be the case the saturation inequalities (4) will have very little meaning.

One of us (E.W.), would like to express his indebtedness to the Wisconsin Alumni Research Foundation for its support of this work.

[9] J. A. Wheeler [Phys. Rev. **52**, 1083 (1937), especially p. 1106], has investigated the saturation properties of many particle forces.

[10] The following qualitative arguments suggest that H and B separately are small compared with M and hence neither of them has a large negative value. (1) The empirical mass differences $N^{13}-C^{13}+C^{14}-N^{14}=0.0026$, $C^{11}-B^{11}+Be^{10}-B^{10}=0.0027$, $He^{3}-H^{3}+He^{6}-Li^{6}=0.0039$ give approximate values of $-2(H+B)\bar{J}(r)$ in N^{14}, B^{10}, Li^{6}. The last value is not very reliable experimentally and can be expected to be great on account of the loose packing of He^{6}. The first two are seen to be practically the energy difference ${}^1S-{}^3S$ in H^2. If H and B were separately large such close agreement would not be expected for then the wave functions would be changed appreciably from the symmetry required by the preponderance of M. (2) Considerations of the kind as the one leading to Table II in footnote 7 and the apparent existence of S^{36} and Ca^{46} [A. O. Nier, Phys. Rev. **53**, 282 (1938)] may be considered as qualitative arguments supporting the above view. From this point of view (5.1) is a more restrictive assumption than (5.2) inasmuch as in the region of positive H and B a greater part of the plane is eliminated by (5.1). Small negative values of H or B cannot be excluded by the above arguments. Nevertheless, the set of values $M=14/12$, $H=-7/12$, $B=10/12$, $V=-5/12$ proposed by Volz, reference 2, contradicts the inequality $c_3<0$.

[11] W. D. Harkins, Phys. Rev. **19**, 135 (1922). J. Am. Chem. Soc. **39**, 856 (1917); **45**, 1426 (1923); J. Frank. Inst. **194**, 165, 329, 521, 645, 783 (1922); **195**, 67, 553 (1923); Phil. Mag. **42**, 305 (1921).

[12] For $A\leqq 14$, the conditions are different and odd-odd nuclei can be stable even if H is quite small.