Blackett and J. G. Wilson, and Ruhlig and Crane, but is in disagreement with the results of Crussard and Leprince-Ringuet for cosmic-ray electrons.

The author wishes to express his sincere gratitude to Professor J. M. Cork and Dr. R. L. Thornton for the privilege of working under their guidance with the University of Michigan cyclotron, and for many helpful suggestions which have facilitated the progress of this work. He is deeply indebted to Dr. H. R. Crane for the original discussion which led to the undertaking of this particular problem and for his constant encouragement. The financial support which made possible the carrying out of this investigation came through the courtesy of the Horace H. Rackham Endowment Fund, which the author gratefully acknowledges.

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# Calculations on a New Neutron-Proton Interaction Potential

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(Received April 16, 1938)

On the basis of a neutron-proton interaction potential of the form $Ce^{-r/\alpha}/r$ numerical calculations for the deuteron and for the scattering of neutrons by protons have been made.

## INTRODUCTION

There has recently been suggested,[1] on theoretical grounds, a new neutron-proton interaction potential of the form $J(r) = -(C/r)\exp(-r/\alpha)$, where $\alpha = \hbar/M_0c$, $M_0$ being the mass of the heavy electron. Of the two unknown constants, $\alpha$ determines the range, $A = C\cdot\alpha$ the strength of the interaction. If this potential is to give the correct energy[2] for the ground state of the deuteron ($E_0 = 2.17$ Mev) a relation between $A$ and $\alpha$ must be fulfilled. For other two-constant potentials, namely the spherical well, inverse fifth power, exponential, and Gaussian type interaction, the dependence of the magnitude of the forces on the range of the forces and the neutron-proton scattering cross sections have previously been calculated.[3] In this paper the same calculations are made for the new potential.

[1] H. Yukawa, Proc. Phys-Math Soc. Japan **17**, 48 (1935); **19**, 1084 (1937). N. Kemmer, Nature **141**, 116 (1938). H. J. Bhabha, Nature **141**, 117 (1938).

[2] H. A. Bethe, Phys. Rev. **53**, 313 (1938).

[3] H. S. W. Massey and R. A. Buckingham, Proc. Roy. Soc. **163**, 281 (1937). Morse, Fisk and Schiff, Phys. Rev. **50**, 748 (1936); **51**, 706 (1937).

## 1. THE GROUND STATE OF THE DEUTERON

The differential equation for the eigenfunction, $\psi_0$, of the ground state of the deuteron is[4]

$$\frac{\hbar^2}{M}\frac{d^2u_0}{dr^2}+(-E_0)u_0=-A\frac{e^{-r/\alpha}}{r/\alpha}u_0;\quad u_0=r\psi_0,$$

where $E_0$ is taken positive.

The transformation $r=\alpha x$ yields

$$u''+a(e^{-x}/x-\epsilon_0)u=0, \tag{1}$$

where $$a=A\alpha^2M/\hbar^2,\quad \epsilon_0=E_0/A; \tag{2}$$

$u(x)=u_0(r)$, and primes indicate differentiation with respect to $x$. The problem is now to determine, for given values of $a$, the constant $\epsilon_0$ which yields a wave function $u$ satisfying the boundary condition.

In the range between $x=0$ and $x=\frac{1}{2}$ (the value $\frac{1}{2}$ is chosen for convenience) the integration of (1) is made by using the first six terms of a power series development of $u(x)$. The boundary condition that $u$ be zero ($\psi_0$ be finite) at $x=0$ is

[4] H. A. Bethe and R. F. Bacher, Rev. Mod. Phys. **8**, 82 (1936).

TABLE I. *Values of $\epsilon_0$, $\alpha$ and $A$ for various values of $a$. Values in parentheses are extrapolated.*

| | | | | | | |
|---|---|---|---|---|---|---|
| $a$ | 1.683 | 2.00 | 2.30 | 2.50 | 2.70 | (2.78) |
| $\epsilon_0$ | 0 | 0.010 | 0.032 | 0.050 | 0.073 | (0.108) |
| $\alpha$ ($\times 10^{13}$) | | 0.62 | 1.18 | 1.54 | 1.93 | (2.39) |
| $A$ (Mev) | | 217 | 67.8 | 43.4 | 29.7 | (20) |

then automatically satisfied. For the range beyond $x=\frac{1}{2}$ Eq. (1) may be put in the form

$$p'+p^2+a(e^{-x}/x-\epsilon_0)=0, \quad \text{where} \quad p=u'/u. \quad (3)$$

This equation was integrated numerically from large distances in to $x=\frac{1}{2}$ by the method of Runge-Kutta.[5] The value of $p$ for large $x$ must approach $-(a\epsilon_0)^{\frac{1}{2}}$ in order that the wave function decrease exponentially at large distances. This was used as the boundary condition at $x=7.5$, where $e^{-x}/x$ is $10^{-4}$ and can be neglected. Values of $\epsilon_0$ were tried for a given value of $a$, until a value was found that would make the numerical solution join smoothly with the power series solution at $x=\frac{1}{2}$. The values of $\epsilon_0$ for given values of $a$ are listed in Table I. By using the known energy[2] of the ground state of the deuteron, $E_0=2.17$ Mev, the corresponding values of $A$ and $\alpha$ have been calculated and are also listed in Table I. The values in parentheses are interpolated or extrapolated.

The equation for the singlet state is: $u''+a(e^{-x}/x)u=0$. This is obtained from (1) by setting $\epsilon_0=0$ since the binding energy for the singlet state is approximately zero. Eq. (3) is replaced by:

$$p'+p^2+a(e^{-x}/x)=0.$$

Different values of $a$ were tried until one was found that made the two solutions join smoothly at $x=\frac{1}{2}$. This value of $a$ gives a fixed relation between $\alpha$ and $A$ for the singlet state. Its value is $a=1.683$.

## 2. SCATTERING

In order to calculate the scattering for neutrons on protons, the wave equation is written for positive energy:

$$u''+a(e^{-x}/x)+\epsilon)u=0; \quad a=A\alpha^2M/\hbar^2, \quad \epsilon=E/A, \quad (4)$$

$E$ being the energy of the neutrons in a coordinate system fixed with respect to the center of gravity. (If $E_n$ be the energy of the neutrons in a system fixed relative to the protons,[4] $E_n=2E$.) The solution[6, 4] of Eq. (4) for large values of $x$ is asymptotically $u=c\sin[(a\epsilon)^{\frac{1}{2}}x+\delta]$; and the total scattering cross section[4] is $\sigma=4\pi(\hbar^2/ME)\sin^2\delta$.

The phases, $\delta$, were calculated for different values of $E_n$, by using the same type of series solution as before, out to $x=\frac{1}{2}$, and integrating numerically from there out to $x=6.5$ by the Runge-Kutta[7] method for second-order differential equations. The values of $\alpha$ and $A$ were obtained from Table I for the triplet state; and, for the singlet state, $A$ was calculated for those same values of $\alpha$ from $a=1.683$. Table II gives the results. The subscripts "$t$" and "$s$" refer to triplet and singlet scattering, respectively.

TABLE II. *Values of $\delta_0$ calculated for the triplet and singlet states.*

| $E_n$ (Mev) | $\alpha\times 10^{13}$ (cm) | $\delta_{0t}$ | $\delta_{0s}$ |
|---|---|---|---|
| 2.2 | 0.62 | 2.382 | 1.418 |
| | 1.93 | 2.304 | 1.243 |
| | (2.39) | (2.277) | — |
| | (2.24) | (2.286) | — |
| 5.4 | 2.39 | 1.877 | — |
| | 2.24 | 1.849 | — |
| 13.7 | 2.39 | 1.520 | — |
| | 2.24 | 1.525 | — |
| 21.5 | 0.62 | 1.688 | 1.248 |
| | 1.93 | 1.487 | 0.862 |
| | (2.39) | (1.417) | — |
| | (2.24) | (1.439) | — |

Table III gives a comparison of the results with the results of Massey and Buckingham[3] for the exponential and spherical well potentials. By

TABLE III. *Comparison of the present results for the phases $\delta$ with the results of Massey and Buckingham for the exponential and spherical well potentials.*

| $J(r)$ | $\alpha\times 10^{13}$ cm | $E_n=2.2$ | 5.4 | 13.7 | 21.5 Mev |
|---|---|---|---|---|---|
| $J=C\ \ r<\alpha$ <br> $J=0\ \ r<\alpha$ | (3.107) | (2.277) | (1.914) | (1.461) | (1.226) |
| $(C/r)e^{-r/a}$ | 2.39 | (2.277) | 1.877 | 1.520 | (1.417) |
| $Ce^{-2r/a}$ | 2.4 | 2.277 | 1.930 | 1.532 | 1.330 |

[5] C. Runge and H. König, *Numerisches Rechnen* (J. Springer, 1924) p. 286.

[6] Mott and Massey, *The Theory of Atomic Collisions* (Oxford, 1933) Ch. II.

[7] See reference 5, p. 311.

interpolation, the values of $\alpha$ for the $e^{-x}/x$ and spherical well potentials were adjusted to give the same phase and therefore scattering cross section as the exponential interaction for 2.2 Mev neutrons. The phases for higher energies were then calculated for these values of $\alpha$. Only the triplet phases were considered since the effect of the singlet state can be neglected[4] above $E_n=2$ Mev. According to the table the values of the phases for the spherical well and exponential potentials diverge slightly but run almost parallel for different values of $E_n$. However, the phases for the $e^{-x}/x$ interaction first go below and then cross over to go above the phases of the other two potentials.

The observed[8] cross sections for the scattering of neutrons by protons are considerably smaller than those calculated with the usually accepted range of about $2\times10^{-13}$ cm. Ranges as small as $0.5\times10^{-13}$ cm would give agreement, but the experiments are at present too uncertain to draw conclusions.

[8] J. Chadwick, Proc. Roy. Soc. **A142**, 1 (1933). W. H. Zinn, S. Seely and V. W. Cohen, Bull. Am. Phys. Soc. **13**, 14 (1938). R. Ladenburg and M. H. Kanner, Phys. Rev. **51**, 1022 (1937). E. T. Booth and C. Hurst, Proc. Roy. Soc. **A161**, 248 (1937).

## Conclusion

It appears that there is little possibility of distinguishing between this and other potentials even by means of scattering experiments over a large range of neutron energies. With the cross sections for the exponential and $e^{-x}/x$ potentials adjusted to be approximately equal for 21.5 Mev neutrons, the total cross sections differ by only 6 percent for 2.2 Mev neutrons.

The ranges of force considered correspond to heavy electron masses ranging from about $700m_e(\alpha=0.5\times10^{-13}$ cm) to $150m_e(\alpha=2.4\times10^{-13}$ cm). Since it is generally accepted[4] that the range of the interaction in any case cannot be much larger than $2\times10^{-13}$ cm, it appears that a lower limit to the mass of the heavy electron would be about 100 electron masses, on the assumption that this potential is the correct one.

The authors wish to thank Professor E. Teller for the suggestion of the problem and the interest taken in it.

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# The Scattering of Fast Electrons in Gases *

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(Received January 27, 1938)

The angular distribution of elastically scattered electrons of energies between 49.5 and 87.7 Kev has been measured in argon, neon, and helium over the range 0.3° to 6°. Comparison of the results with the theoretical curves calculated from the Hartree functions with the "Born approximation" shows good agreement in the case of argon, and in neon except at the smallest angles, but there is no agreement at all in the case of helium. The discrepancies are very similar to those found at the smallest angles in studies of scattering of electrons of a few hundred volts energy, and it is felt that the explanation in terms of polarization of the atomic field by the passing electron may be accepted here also.

SINCE the pioneer work of Lenard[1] carried out more than forty years ago, various problems in connection with the passage of electrons through matter have attracted the attention of physicists down to the present time. The scattering of electrons (elastic and inelastic)

* The work here reported forms part of a dissertation presented to the faculty of Princeton University in candidacy for the degree of Doctor of Philosophy. The complete dissertation is on file in the Princeton University library. A preliminary report was given at the Washington meeting of the American Physical Society, 1937; Kuper, Phys. Rev. **51**, 1024A (1937).

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[1] Lenard, Wied. Ann. **51**, 225 (1894); **56**, 255 (1895).