The Nuclear Magnetic Moment of Li^7 by Perturbation Theory

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Under the assumption that the intrinsic magnetic moments of protons and neutrons are not destroyed by the nuclear binding process, and with the adoption of a rather general exchange interaction, the same between all pairs of heavy particles, the magnetic moment of Li^7 is calculated by a perturbation theory which develops from the approximation in which the particles move independently in a central field. The first-order result is that the orbital part of the magnetic moment has a value between $0.26\mu_N$ and $0.30\mu_N$, depending on the details of the interaction, and that the spin part can differ from the proton magnetic moment by not more than $0.03\mu_N$, because of the exchange nature of the like-particle interaction and the small spin dependence of the unlike-particle interaction allowed by the data on scattering. The second-order modification is very small ($0.01\mu_N$ from the doubly excited states) and is due principally to the excitation of the s shell, since states involving only the excitation of the three p particles annul one another in their second-order effect. The modification is positive, corresponding to a limited participation of three protons and four neutrons in the orbital part, rather than one proton and two neutrons as in first order. Cancellation of positive and negative higher order contributions facilitates rapid convergence. If our result is to agree with experiment, the value of the proton magnetic moment must be near the upper limit ($3\mu_N$) of the rather wide range suggested experimentally, which seems unlikely. Such a discrepancy may be associated with the inadequacy of the symmetrical interaction apparent in calculating binding energies.

THE magnetic moment of the Li^7 nucleus is of especial interest because it is complex enough to contain both orbital and spin contributions of the heavy particles in the nucleus, and at the same time is simple enough to be within the reach of calculation. Its magnitude has long been known accurately relative to the magnetic moment of the simpler nucleus Li^6 from hyperfine structure, and the absolute measurement of its value has very recently attained unprecedented accuracy.[1] Calculation of the moment must at present depend on assumptions regarding the nature of heavy-particle interactions. Comparison between calculated and experimental values may serve as a test of the interactions assumed, and of the validity of the concept of intrinsic spin magnetic moments of individual neutrons and protons in nuclei.

First-Order Calculation

The result in the simple first-order, or Hartree, approximation, for the special case of an interaction involving space exchange and space-spin exchange of unlike particles and no exchange of like particles, has been given by Rose and Bethe,[2] following Feenberg and Wigner's treatment of the structure,[3] and was shown to be smaller than the experimental value (relative to Li^6). We shall first carry out the first-order calculation assuming a somewhat more general form of interaction. We assume that the Hamiltonian is symmetrical in all particles,[4] and that the interaction between any two particles may contain space-exchange, space-spin-exchange, nonexchange, and spin-exchange operators as follows

$$U=\sum J(r)\{g_q P^q+gP^{q\sigma}+g_1\mathbf{1}+g_\sigma P^\sigma\}, \qquad g_q+g+g_1+g_\sigma=1. \tag{1}$$

The conditions that there shall exist no exceedingly heavy nuclei with small spin[5] nor with very large total spin[6] demand, respectively,

$$g_q+2g\geqslant 4g_1+2g_\sigma, \tag{2}$$

$$g_q+g\geqslant 2g_1+2g_\sigma. \tag{3}$$

It has been shown in a previous paper[7] that no

[1] Rabi, Millman, Kusch and Zacharias, Phys. Rev. **53**, 495 (1938).
[2] Rose and Bethe, Phys. Rev. **51**, 205 (1937). Although the discrepancy in μ_{Li^7} is small, one calculates essentially $(\mu_{Li^7}-\mu_{s\pi})$, and in it the discrepancy is of the order of 30 percent.
[3] Feenberg and Wigner, Phys. Rev. **51**, 95 (1937).
[4] Breit, Condon and Present, Phys. Rev. **50**, 825 (1936).
[5] Breit and Feenberg, Phys. Rev. **50**, 850 (1936).
[6] N. Kemmer, Nature **140**, 192 (1937).
[7] D. R. Inglis, Phys. Rev. **51**, 531 (1937). Errata: (a) The second term of f_{2020} in (10) should contain τ^{-1}, not τ^{-2}. (b) The small exchange term which arises from the K.E. transformation and is derived in footnote 17, below, was inadvertently neglected in the treatment of Li^6. Eqs. 15 and 17½ of that paper should therefore contain $57\alpha\sigma/12$ instead of $55\alpha\sigma/12$. The change is small enough that it does not affect the minimizing process appreciably,

interaction satisfying these conditions (and the demands of scattering, $g+g_\sigma=g_0\approx 2/9$) seems to give sufficient binding for Li^6, but that the discrepancy is least serious if one selects an interaction satisfying (2) as an equality.[8] The Li^6 binding may also be seen[7] to be greatest by taking (3) approximately as an equality as well, although it is not so sensitive in this respect (compare also reference 6). We shall therefore consider interactions (1) which satisfy (2) and (3) and do not differ very much from that determined by (2) and (3) as equalities, namely, that with

$$g_q,\ g,\ g_1,\ g_\sigma = 8/9,\ -2/9,\ -1/9,\ 4/9, \qquad (4)$$

respectively.[9]

The ground state of Li^7 arises from the configuration $sspp$; ssp (two p neutrons and one p proton outside the closed s shell—symbols after the semicolon (;) refer to protons). With neglect at first of both spin-orbit coupling and the spin dependence of (1), the angular momenta L, S_ν and S_π are separately constants of the motion and the neutrons' ^1S, ^1D and ^3P combine with the proton's ^2p to give [1, 2] ^2PPDF and [3, 2] ^2SPD. Composition of the spin vectors gives three ^2P and two ^2D; the other states occur only singly. The two ^2P arising from the neutrons' ^1S and ^1D are mixed by a nondiagonal matrix element, even when we neglect the spin dependence of (1), and give rise to the ground state.[3] The third ^2P arising from the neutrons' ^3P is admixed somewhat by the spin dependence of (1). Since spin-orbit coupling[10] is actually very much smaller than the interactions (1) we may neglect its nondiagonal elements. L and S and their projections therefore remain good quantum numbers and we need consider from this configuration only the three substates of the three ^2P with $M_L=1$, $M_S=\frac{1}{2}$. We take $M_S=\frac{1}{2}$ rather than $-\frac{1}{2}$ because the spin-orbit coupling is of such a sign as to make these three substates give rise to the ground state.[10]

The space factors of these wave functions are[2, 11]

$$\begin{aligned} P^1(D_\nu p_\pi) &= \{(ac+ca-2bb)a \\ &\qquad +3(ab+ba)b+6aac\}/(60)^{\frac{1}{2}}, \\ P^1(S_\nu p_\pi) &= (ac+ca+bb)a/3^{\frac{1}{2}}, \\ P^1(P_\nu p_\pi) &= \{(ac-ca)a+(ab-ba)b\}/2, \end{aligned} \qquad (5)$$

where we use large letters S, P, D to denote a composite orbital angular momentum and small letters for the orbital angular momentum of one particle, followed by superscripts which refer to the corresponding M_L or m_l. We have further abbreviated the different projections of the single-particle p function thus: $p^1=a$, $p^0=b$, and $p^{-1}=c$. We preserve the order: first the two p neutrons, then the p proton, and omit here the s particles.

In calculating the matrix of the interaction for these states we may avoid until later specializing the shape of the potentials by using the

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so the consequent change in the energy is 4.4 mc^2 (increasing the discrepancy slightly). (c) In the last line of page 543, $\Sigma\Delta n_{\xi}$ should read $\Delta n_{\xi\pi}$ for φ_0 less symmetrical than that of Li^6, so the conclusion about higher order orbital contributions applies to Li^6 (compare reference 2) but not to Li^7 in which we are here interested. (d) Professor Margenau, whose comment on the present manuscript has also been appreciated, has kindly pointed out that the number of states of the type 01, 0, 1; 01, 0, 1 in Table IV should not be 12. It should, in fact, be only 3, so $|E_D^{(2)}|$ there deduced is about 4 mc^2 too great. Compare the conclusion, that the symmetrical Hamiltonian seems inadequate, with that of Rarita and Present, Phys. Rev. **51**, 788 (1937).

[8] A similar conclusion anent (2) was also reached from rougher treatment of heavier nuclei by Volz, Zeits. f. Physik **105**, 537 (1937).

[9] The similar values (10, −2, −1, 5)/12, from $g_0=1/4$, have recently been used by Heisenberg in another connection, Naturwiss. **25**, 749 (1937).

[10] D. R. Inglis, Phys. Rev. **50**, 783 (1936); W. H. Furry, Phys. Rev. **50**, 785 (1936); G. Breit, Phys. Rev. **51**, 248 (1936).

[11] These are easily derived by the rather well-known method employing the operator which we shall call n, representing the sum of the combinations p_x-ip_y of the projections of orbital angular momentum of the several particles. Since this obeys the selection rules $\Delta L=0$ and $\Delta M_L=-1$, it enables one to get the wave functions of known L and M_L from those of the same L and higher M_L. Writing out the operator and applying it to the p^{ml}, one finds directly $na=-b$, $nb=c$. For the two neutrons one then has, normalizing,

$$\begin{aligned} &D^2=aa, \\ &D^1=-(ab+ba)/2^{\frac{1}{2}}, \\ &D^0=(2bb-ac-ca)/6^{\frac{1}{2}}; \\ \text{then by orthogonality}\quad &P^1=(ab-ba)/2^{\frac{1}{2}}, \\ \text{whence, applying } n,\quad &P^0=(ac-ca)/2^{\frac{1}{2}}, \\ \text{and by orthogonality}\quad &S^0=(ac+ca+bb)/3^{\frac{1}{2}}. \end{aligned}$$

Similarly for the three particles composing $D_\nu p_\pi$, we have (within normalizing factors):

$$\begin{aligned} &F^3=aaa, \\ &F^2=(ab+ba)a+aab, \\ &F^1=(ac+ca-2bb)a-2(ab+ba)b+aac, \\ &D^2=(ab+ba)a-2aab, \\ &D^1=(ac+ca-2bb)a+(ab+ba)b-2aac, \\ &P^1=(ac+ca-2bb)a+3(ab+ba)b+6aac \\ &\quad=6^{\frac{1}{2}}D^0a+2^{\frac{1}{2}}3D^1b-6D^2c, \end{aligned}$$

and for $P_\nu p_\pi$:

$$\begin{aligned} &D^2=(ab-ba)a, \\ &D^1=(ac-ca)a-(ab-ba)b, \\ &P^1=(ac-ca)a+(ab-ba)b. \end{aligned}$$

relations (9) and (10) of reference 3; namely[12] $L=(xx|J|xx)$, $K=(xx|J|yy)$, and $L-2K=(xy|J|xy)$, where x has the angle dependence of the normalized wave function which it represents. These lead to the convenient relations[12a]

$$\begin{aligned}&(aa|J|aa)=(ac|J|ac)=L-K,\\&(ab|J|ab)=L-2K,\\&(bb|J|bb)=L,\\&(ac|J|bb)=(ab|J|ba)=K,\\&(ac|J|ca)=2K,\end{aligned}\tag{6}$$

which are used to reduce the matrix elements in considering successively the interactions of the various pairs of particles, with the possible orthogonality of the functions of the other particle. One must of course also consider the spin in calculating the matrix elements of the terms in $P^{q\sigma}$ and P^{σ}. The spin factor of each of the first two lines of (5) is ${}^{12}S=(\alpha\beta\alpha-\beta\alpha\alpha)/\sqrt{2}$, antisymmetric in the spin coordinates of the two neutrons (here α and β are the usual Pauli spin functions for $m_s=\pm\frac{1}{2}$, respectively). The last line of (5) has the spin factor ${}^{32}S=(\alpha\beta\alpha+\beta\alpha\alpha-2\alpha\alpha\beta)/\sqrt{6}$ (as is easily seen by the method of footnote 11, for example). The spin-exchange operator between one of the neutrons and the proton then has the matrix elements

$$({}^{12}S|P_{\nu\pi}{}^{\sigma}|{}^{12}S)=\tfrac{1}{2},\quad ({}^{32}S|P_{\nu\pi}{}^{\sigma}|{}^{32}S)=-\tfrac{1}{2},\quad ({}^{12}S|P_{\nu\pi}{}^{\sigma}|{}^{32}S)=\pm\tfrac{3}{4}.\tag{7}$$

The $\pm$ sign is $+$ for one neutron and $-$ for the other. A similar $\pm$ sign occurs in the space factor $(P(Sp)|J|P(Pp))$ with or without space exchange, because of the antisymmetry of $P(Pp)$ in the neutrons. The contributions of the two neutrons are thus additive in the "singlet-triplet" matrix elements of $JP^{q\sigma}$ and JP^{σ}, although the corresponding elements of JP^{q} and J are zero, as required to leave S_{ν} a constant of the motion with spin-independent forces. The diagonal elements of $P_{\nu\nu}{}^{\sigma}$ are ±1 according to the symmetry. Because of the possibility that one might later desire to abandon the symmetrical Hamiltonian, we list separately the matrix elements of the like-particle and unlike-particle interactions, thus calculated. For the unlike-particle terms of (1) in the representation (5) we have the symmetric sub-matrix:

${}^2P({}^1D^2p)$	$g_q(L+14K)/3$ $+g(L+14K)/6$ $+g_1(2L-K/3)$ $+g_\sigma(L-K/6)$		
${}^2P({}^1S^2p)$	$\{g_q(2L-2K)$ $+g(L-K)$ $+g_14K$ $+g_\sigma2K\}5^{\frac{1}{2}}/3$	$g_q(2L+4K)/3$ $+g(L+2K)/3$ $+g_1(2L-8K/3)$ $+g_\sigma(L-4K/3)$	
${}^2P({}^3P^2p)$	$-\{gL$ $+g_\sigma3K\}5^{\frac{1}{2}}/2$	$g(L-3K)$	$g_q(L+2K)$ $+g(-L/2-K)$ $g_1(2L-K)$ $+g_\sigma(-L+K/2)$

(8)

We first write a matrix of six rows and six columns, labeled *aca*, *caa*, etc., for the terms in $P_{37}{}^{q}$ and $\mathbf{1}_{37}$ between the first p neutron and the p proton (most of the elements are zero from orthogonality). We then combine them using the coefficients listed in (5), then multiply by two to take account of the second p neutron in all cases except where the elements are zero as noted just under (7), above, and finally obtain the elements of $P^{q\sigma}=P^{q}P^{\sigma}$ and of P^{σ} by multiplying the elements of P^{q} and $\mathbf{1}$ by the factors given in (7). The like-particle interaction is of course diagonal in this representation, having the diagonal elements [as most easily calculated from (6) and the simple sum rule:

$D=aa$; $D+P=ab+ba$; $D+P+S=ac+ca+bb$

—all representing diagonal elements of JP^{q} or J (compare reference 3)]:

${}^2P({}^1D^2p)$	${}^2P({}^1S^2p)$	${}^2P({}^3P^2p)$
$g_q(L-K)$ $+g(-L+K)$ $+g_1(L-K)$ $+g_\sigma(-L+K)$	$g_q(L+2K)$ $+g(-L-2K)$ $+g_1(L+2K)$ $+g_\sigma(-L-2K)$	$g_q(-L+3K)$ $+g(-L+3K)$ $+g_1(L-3K)$ $+g_\sigma(L-3K)$

(9)

Adding the diagonal matrix (9) to (8) one has a secular problem which may be specialized by introducing relations between the various coefficients from other sources. By specializing to the Hartree model with oscillator potential hole, one may estimate[3] $K=L/7$ (see below), but it might be well to consider the possibility of considerable deviation from that estimate. We should also allow considerable latitude in the selection of the g's. Since one choice of interactions[2] gives too small a magnetic moment, and since the small admixture of ${}^2P({}^3P^2p)$ can only decrease the calculated moment (the proton spin and the neutron magnetic moments "point backwards"); we may first ascertain whether there is a possibility of finding an interaction which gives a large enough moment in first order by neglecting the

[12] Derivation: subject $(x'x'|J|x'x')$ to the rotation $x'=x\cos+y\sin$ and equate it to $(xx|J|xx)(\cos+\sin)^2$.

[12a] F. Hund, Zeits. f. Physik **105**, 220 (1937).

admixture of $^2P(^3P^2p)$. The preliminary secular problem is then merely quadratic, involving the matrix elements:

$$\begin{array}{c|c|c} & & \\ {}^2P(^1D^2p) & \begin{array}{l} g_q(4L+11K)/3 \\ +g(-5L+20K)/6 \\ +g_1(3L-4K/3) \\ +g_\sigma(-5K/6) \end{array} & \\ \hline {}^2P(^1S^2p) & \begin{array}{l} \{g_q(2L-2K) \\ +g(L-K) \\ +g_1 4K \\ +g_\sigma 2K\}5^{\frac{1}{2}}/3 \end{array} & \begin{array}{l} g_q(5L+10K)/3 \\ +g(-2L-4K)/3 \\ +g_1(3L-2K/3) \\ +g_\sigma(-10K/3) \end{array} \end{array} \qquad (10)$$

We wish to calculate the orbital contribution μ_L to the magnetic moment, the spin contribution being the same ($\mu_{s\pi}$) from each of the two states involved in the approximate wave function

$$C_1{}^0\,{}^2P(^1D^2p)+C_2{}^0\,{}^2P(^1S^2p). \qquad (11)$$

From (5), or from the very simple use of the sum rule and vector model (which thus serves to verify (5)), it is apparent that the orbital[13] contribution of the state $P(Sp)$ is $+1$ (the unit of magnetic moment here and henceforth being the heavy magneton $\mu_N=\mu_B/1840$) and that of the state $P(Dp)$ is $-(\frac{1}{2})$ so the first-order orbital contribution is, with $(C_1{}^0)^2+(C_2{}^0)^2=1$, approximately

$$\mu_L{}^0=1-(3/2)(C_1{}^0)^2. \qquad (12)$$

With the space-exchange term alone ($g=g_1=g_\sigma=0$) and with $K=L/7$, to illustrate by the simplest example, one has from (10) the secular equations

$$C_1{}^0(13-\epsilon/K)+C_2{}^0 4\sqrt{5}=0,$$
$$C_1{}^0 4\sqrt{5}+C_2{}^0(15-\epsilon/K)=0,$$

whence $\epsilon/K=14\pm 9=23$ for the ground state and $C_2{}^0/C_1{}^0=5^{\frac{1}{2}}/2$ so $(C_1{}^0)^2=4/9$ and $\mu_L{}^0=\frac{1}{3}$. The results of the same calculation with different selections of the coefficients are given in Table I.

[13] We neglect the "wobble of the rest of the nucleus" in calculating in the usual simple manner the orbital magnetic moment of a single proton. A factor of about 6/7, which one might insert in the final calculated μ_L on classical grounds to account for this effect, would not significantly alter the conclusions reached below. This approximation amounts to taking $H'=-U-\frac{1}{2}(\sigma\alpha)^2\Sigma r^2$ instead of the more complicated expression (3), reference 7, and we do this also in calculating the second-order effects, of which only the order of magnitude will be important. The terms in ∇, here neglected, reduce the effect of the doubly excited states in reference 7, so that the convergence there appeared to set in only beyond the quadruply excited states. Here we may expect the quadruply excited states to contribute considerably less than the upper limit which we obtain for the doubly excited states by neglecting those terms.

In the last line the coefficients are different for like-particle pairs ($g_1=1$) and unlike-particle pairs ($g_q=0.78$, $g=0.22$), corresponding to the unsymmetrical interaction used by Rose and Bethe,[2] for comparison. For these the correction arising from the participation of neutron spin due to the admixture of the $^2P(^3P^2p)$ is[2] $\delta\mu_s=-0.07$. This correction is much smaller for the symmetrical interaction (1), because the leading term of the neutron-neutron interaction contains the space-exchange operator which, together with the Pauli antisymmetry, strongly separates the state involving the $^3P_\nu$ from the others (compare reference 3, Table II).

The calculation of the admixture of the state $^2P(^3P^2p)$ involves all of the matrix (8)+(9):

$$\left|\begin{array}{ccc|ccc} a & b & d & & & \\ b & c & e=\dfrac{K}{27} & 278 & 88\sqrt{5} & 3\sqrt{5} \\ d & e & f & 88\sqrt{5} & 295 & -24 \\ & & & 3\sqrt{5} & -24 & 90 \end{array}\right|$$

for the coefficients of Eq. (4) (the sixth line of Table I), for example. One sees here that the nondiagonal elements d and e, responsible for the admixture, are indeed small, and they alter ϵ and C_2/C_1 very little.[14] The same is true of the other cases considered. Treating the admixture as a perturbation,[14] we have

$$\epsilon=(a+c)/2+b+(a-c)^2/8b$$

and[15]

TABLE I. *Results of the calculations of the orbital contribution to the magnetic moment for various selections of coefficients.*

	L/K	g_q	g	g_1	g	ϵ/K	$C_2{}^0/C_1{}^0$	$\mu_L{}^0$	$\{\mu_{\mathrm{Li}}{}^{7(1)}-\mu_{s\pi}\}$
1	7	1	0	0	0	23	1.12	1/3	1/3
2	7	0.8	0.2	0	0	18.4	1.06	0.29	0.28
3	7	0.8	0	0	0.2	18.4	1.05	0.29	0.28
4	7	0.53	0.22	0.25	0	18	1.04	0.28	0.27
5	7	0.8	−0.1	0	0.3	18.4	1.05	0.29	0.26
6	7	8/9	−2/9	−1/9	4/9	17.9	1.05	0.28	0.28
7	5	8/9	−2/9	−1/9	4/9	13.2	1.01	0.26	0.25
8	10	8/9	−2/9	−1/9	4/9	24.9	1.07	0.30	0.29
9	7	(reference 2)				21.8	1.07	0.30	0.23

[14] In order to justify treating the admixture as a perturbation, one may solve the cubic secular problem using the coefficients of equation (4). One finds $27\epsilon/K=483.8$, $C_1{}^2=0.476$, $C_2{}^2=0.523$, $C_3{}^2=0.00104$, $\delta\mu_s=-0.0068$ (the same by the approximation in the text), $\mu_L=0.286$ (compare with $\mu_L{}^0=0.284$).

[15] The coupling of $S_\nu=1$ to $S_\pi=\frac{1}{2}$ gives $^4S^{\frac{3}{2}}$ with $\mu_s=2\mu_{s\nu}+\mu_{s\pi}$. Then $^4S^{\frac{3}{2}}$, with M_S one-third as great, has $\mu_{sz}=(2\mu_{s\nu}+\mu_{s\pi})/3$. But $M_S=\frac{1}{2}$ can be attained by putting $M_{S\nu}=1$, $m_{s\pi}=-\frac{1}{2}$ or $M_{S\nu}=0$, $m_s=\frac{1}{2}$, so the sum of the μ_{sz}'s is $2\mu_{s\nu}$. Subtracting we have for $^2S^{\frac{1}{2}}$, $\mu_{sz}=(4\mu_{s\nu}-\mu_{s\pi})/3$. This substituted for $\mu_{s\pi}$ to an extent $C_3{}^2$ gives

$$\delta\mu_s=(4/3)(\mu_{s\nu}-\mu_{s\pi})C_3{}^2.$$

$$\delta\mu_S=(4/3)(\mu_{sv}-\mu_{s\pi})\times\{bd+(\epsilon-a)e\}^2/\{[b^2+(\epsilon-a)^2](\epsilon-f)^2\}.$$

The values of $\delta\mu_S$ are so small that the exact values of μ_{sv} and $\mu_{s\pi}$ are unimportant: the excess of the Li^7 moment over the proton moment is calculated as $\mu_L{}^0+\delta\mu_S$, with $\mu_{sv}-\mu_{s\pi}=-4.85$, and is evaluated in the last column of Table I. The very small[14] effect of the admixture on μ_L is there neglected. The Li^7 moment being[1] $\mu_{Li^7}=3.265\pm0.016$, the proton moment[16] would have to be at least 2.96 if our assumptions and first-order calculation are to agree with experiment.

As yet we have not specialized the Hartree model but have only shown that alteration of the form of the fictitious potential, which alters L/K, would not affect the first-order result appreciably (Table I). As a starting point for a second order calculation we shall need a definite set of wave functions, which may be determined by choosing a quadratic zero-order potential $V^0=\frac{1}{2}(\sigma\alpha)^2\Sigma r^2$ and varying σ to make the first-order energy a minimum.[7] Assuming henceforth that $J(r)=-Be^{-\alpha r^2}$, we see that the matrix (8)+(9) accounts for the interactions between the p-particles, with $L=-Buf_{1111}=-Bu\{(1-1/\tau)^2+2/\tau^2\}$ and $K=-Buf^2{}_{1100}=-Bu/\tau^2$, in the notation of reference 7 ($\tau=\sigma+2$, $u=(\sigma/\tau)^{\frac{3}{2}}$). The rest of the interaction is independent of the orientation of the p orbits, so it is simply the s–s and s–p part of $(ssbb;\ ssb|U|ssbb;\ ssb)$ for each of the diagonal elements, and this part is, by the methods of reference 7, equal to

$$3Bu\{(2-2g_0)f_{0000}+(-1-g+5g_1+3g_\sigma)f_{1010}+(4-2g-5g_1-6g_\sigma)f_{1001}\}=3Bu\{1-3g+5g_1+g_\sigma+(5-g-10g_1-9g_\sigma)/\tau\}.$$

This together with the lowest solution ϵ of the determinant arising by specializing (10) to the case (4):

$$\begin{vmatrix}(28L+82K-27\epsilon) & (14L-10K)\sqrt{5}\\ (14L-10K)\sqrt{5} & (35L+50K-27\epsilon)\end{vmatrix}=0$$

gives the first-order energy for that case (neglecting the small effect of $^2P(^3P^2p)$), when added to the kinetic[17] and Coulomb terms $6\alpha\sigma+0.7(\alpha\sigma)^{\frac{1}{2}}$, and this may be minimized by trial of several values of σ. Taking $B=92\ mc^2$, $\alpha=22$, one finds the minimum of the first-order energy at $\sigma=1.2$ or $\tau=3.2$, $E^{(0)}+E^{(1)}=-30.4\ mc^2$. The minimizing value of τ is very insensitive to the selection of the coefficients in (1), as is shown by the fact that it retains the same value (within 0.03) when we consider instead the quite different case of the fourth row of Table I, for which $E^{(0)}+E^{(1)}=-31.5\ mc^2$. If we judge by the Li^6 calculations, the minimizing value of τ is also sufficiently insensitive to the choice of B and $\alpha(\partial\tau_{\min}/\partial\alpha\sim0.03)$ that trial of other reasonable values would not be expected to alter the following result significantly. This insensitivity arises essentially from the fact that $\sigma=\tau-2$ is a factor in the minimizing process, but only τ enters the ratio of coefficients in the secular problem.

Second-Order Calculation

The higher order contributions to the binding energy are arithmetically additive because the energy differences between excited states and the ground state all have the same sign, and the increasing number of smaller contributions makes it necessary to compute a great number of contributions to obtain a fair approximation to the energy.[7] The higher order contributions to the magnetic moment, on the contrary, occur with both signs with approximately equal abundance, so that the large number of small contributions from highly excited states very nearly cancel out, and the convergence may be expected to be much more gratifying in calculating magnetic moments than energies. The first-order magnetic moment is not a difference of two large terms, as is the first-order energy, so it is expected to be better from this point of view also.

The first-order wave function of the ground state is, from the above calculation,

[16] The upper limit of the value 2.85 ± 0.15 (Kellogg, Rabi and Zacharias, Phys. Rev. **50**, 472 (1936)) is compatible with our assumptions but is far above the value 2.46 ± 0.1 (Estermann, Simpson and Stern, Phys. Rev. **52**, 535 (1937)). The latter would be more appropriate here, μ_{Li^7} being measured by its coupling to an external field, if h.f.s. were due in part to some nonmagnetic cosine coupling (L. A. Young, Phys. Rev. **52**, 138 (1937)).

[17] The kinetic energy term $-(3/7)(ssbb;\ ssb|\Sigma\nabla^2|ssbb;\ ssb)$ is equal to $81\alpha\sigma/14$, since $(n|\partial^2/\partial x^2|n)=-(n+\frac{1}{2})\alpha\sigma$. There is also a small exchange term in the kinetic energy arising from the cross terms $\nabla_a\cdot\nabla_b/N$ and the antisymmetry. It amounts to $-(1|\partial/\partial x|0)(0|\partial/\partial x|1)/N=\alpha\sigma/2N$ for each like-particle, like-spin $s-p$ pair, or $3\alpha\sigma/14$ altogether, for Li^7.

$$0.67^2P(^1D^2p)+0.74^2P(^1S^2p)+0.069^2P(^3P^2p)$$
$$=0.67(60)^{-\frac{1}{2}}\{(5.94ac+5.94ca+2.94bb)a$$
$$+3(ab+ba)b+6aac\}(\alpha\beta-\beta\alpha)\alpha/2 \quad (13)$$
$$+0.034\{(ac-ca)a+(ab-ba)b\}$$
$$\times\{(\alpha\beta+\beta\alpha)-2\alpha\alpha\beta\}/6^{\frac{1}{2}}$$

for the case (4). The coefficients 5.94 and 2.94 are so nearly equal to 6 and 3, respectively, that we may safely simplify the wave function by taking them exactly equal. (This approximation is good for various selections of the g's and L/K, C_2^0/C_1^0 being almost constant in Table I.) If, further, we neglect the small triplet term we have the approximate wave function[18]

$$\{2(aac+aca+caa)+bba+bab+abb\}/\sqrt{15} \quad (14)$$

with the singlet spin factor $(\alpha\beta-\beta\alpha)\alpha/2$. For it $\mu_{L0}=\frac{1}{3}$, as is apparent from the symmetry in the three particles, one of which is a proton.

Since the second-order contributions need not be computed with as great relative accuracy as the first-order, we may use this approximate first-order wave function in reckoning the second-order contributions (and later estimate the errors so committed). We shall also begin by considering only the leading term in P^q in (1) and discuss the possibility of spin effects later. We thus are concerned with the contributions of the higher states to the projected orbital magnetic moment μ_L. Although it is feasible to calculate in terms of excited states described by the quantum numbers L and M_L, it is simpler to carry out the equivalent expansion in terms of the excited states having the individual particle moments l_i and m_{l_i} as quantum numbers.[19] The excited states, to contribute, must have the same parity (doubly excited, quadruply excited states, etc.) and the same M_L as has the ground state. The symmetry of the space wave function (14) simplifies the problem tremendously, because the possibility of exciting the three p particles is also symmetric in their coordinates, so the two neutrons and one proton are admixed equally from such excitations[20] by a symmetrical interaction (1), and μ_L remains $\frac{1}{3}$.

The second-order contribution $\delta\mu_L$ therefore arises from the possibility of exciting the s shell. Excitations involving the s shell may be grouped in three classes, arising from terms of the ground state[21] by the excitations $ss\rightarrow pp$, $s\rightarrow d$, or $sp\rightarrow pd$. Let us consider first the excitations $ss\rightarrow pp$, of which only $ss\rightarrow ac$ contribute to $\delta\mu_L$. Here the number of contributing states is further limited by the fact that some of the p states to which an s particle might be excited are already "occupied." Arising from the terms [21] in aac we have the products

$+-$	$+-$	$+-$	$+$
ss	aa;	ac	c
cs	aa;	as	c
cs	aa;	sa	c
sc	aa;	as	c
sc	aa;	sa	c
ss	ac;	ca	a
ca	ac;	ss	a
cs	ac;	sa	a
sa	ac;	cs	a
sa	ac;	sc	a
ss	ca;	ca	a
ac	ca;	ss	a
sc	ca;	sa	a
as	ca;	sc	a
as	ca;	cs	a

[18] Exact for $g=g_\sigma=0$, as follows from considerations of Feenberg and Phillips, Phys. Rev. **51**, 597 (1937), and kindly communicated by Dr. Phillips. It is remarkable that the spin-dependence leaves it as good an approximation as it does.

[19] The Cartesian oscillator wave functions used for calculating the binding energy of Li^6 are not so convenient here where the angular momenta are important.

[20] For example, consider the second-order contributions of states arising from an excitation $p\rightarrow f$. If the f particle have $m_l=3$, we have the three states f^3cc and its permutations, with the same spin factor as has (14), each having $M_L=1$ and the same matrix element $H_{0a}'=(f^3c|J|aa)\,4/\sqrt{15}$ with (14). Since the operator for the proton orbital moment is diagonal in these functions, the effect of H_{0a}' is to add $\mu_{La}H_{0a}'^2/(E_a-E_0)^2$ to the first-order value μ_{L0}, but also to subtract $\mu_{L0}H_{0a}'^2/(E_a-E_0)^2$ by altering the normalization. (That is, the property of the excited state is to a small extent substituted for that of the ground state.) Since f^3cc and cf^3c each have $\mu_L=-1$ and ccf^3 has $\mu_L=3$, their average is $\frac{1}{3}$, which is μ_{L0} so the net effect of the admixture is zero.

[21] When considering the s shell explicitly, the term aac in (14) is to be written $ssaa$; ssc. The corresponding antisymmetric function with spin contains many permutations, P, of $s^+s^-a^+a^-$; $s^+s^-c^+$, and corresponding to each of them there is an excited product $P(d^{2+}s^-a^+c^-$; $s^+s^-c^+)$ which bears the same relation to it as does $d^{2+}s^-a^+c^-$; $s^+s^-c^+$, for example, to the original term. It is therefore quite arbitrary whether we expand in antisymmetric combinations of these products, or in the products singly, interacting with (14) antisymmetrized and suitably normalized, or whether we consider only the excited products arising directly from

(fifteen of them—for each permutation of the original *aac*, five: there are three places for the excited *c* if the excited *a* forms a like-particle, like-spin pair with the original *c*, otherwise two places). Of these the second and last are duplicates, differing only by like-particle exchanges, and likewise the fourth and ninth, and the seventh and twelfth. Taking one of each pair, we find that these three states have their average μ_L equal to μ_{L0} so contribute nothing to $\delta\mu_L$. The other nine states all have the same matrix element with the ground state, $H_{0a}'=2(ac|J|ss)/\sqrt{15}=2Bu/(\tau\sqrt{15})$; the evaluation is easily carried out[22] by use of (10), reference 7. Their average μ_L is proportional to $\Sigma m_{l\pi}/9=5/9$, so their contribution to the magnetic moment is

$$\delta\mu_L=9(5/9-\mu_{L0})(H_{0a}'/(E_a-E_0))^2=2(Bu/\alpha\sigma\tau)^2/15=0.0083$$

TABLE II. *Influence of the excitations s→d.*

TYPE	NR.	$\{(\mu_L)_{Av}-1/3\}$	$15^{\frac{1}{2}}H_{0a}'$	$\frac{\delta\mu_L\cdot 60\alpha^2\sigma^2}{\{(1-1/\tau)Bu/\tau\}^2}$
$d^2s\,ac;\ ss\ \ c$	6	2/3	$4(d^2c\|J\|as)$	32
$ss\ \ aa;\ sd^{-2}\,a$	1	−4/3	$6(d^{-2}a\|J\|cs)$	−24
$d^1s\,ab;\ ss\ \ c$	12	1/3	$2(d^1b\|J\|as)$ $+(d^1c\|J\|bs)$	36
$ss\ \ aa;\ sd^{-1}\,b$	3	−2/3	$2(d^{-1}a\|J\|bs)$ $+2(d^{-1}b\|J\|cs)$	−32
$d^2s\,bb;\ ss\ \ c$	9	4/9	$(d^2c\|J\|as)$	1
				13

one of the six terms of (14) by excitation (or alteration) of not more than two of the single-particle functions (and having the same spin order, for $g=g_\sigma=0$). With the latter, somewhat simpler, convention we keep the normalization (14) but remember antisymmetry: if such a state as d^2sca; *ssc* differ from a term of (14), such as *ssaa*; *ssc*, only by two like-particle, like-spin functions, then in addition to the direct term, $(d^2c|JP^q|sa)$, in the matrix element H_{0a}' there is the exchange term, $-(d^2c|JP^q|as)$, arising from the fact that the complete approximate wave function replacing (14) is antisymmetric in those two functions. (The direct term is contributed again by the term *ssca*; *ssa* of (14) in the case mentioned.)

[22] Using

$$d^{\pm 2}=(H_{200}-H_{020}\pm 2iH_{110})e^{-\rho^2/2}/\sqrt{6},$$
$$d^{\pm 1}=(H_{101}\pm iH_{011})e^{-\rho^2/2}/\sqrt{2},$$
$$d^0=(H_{200}+H_{020}-2H_{002})e^{-\rho^2/2}/\sqrt{6},$$
$$a,\ c=(H_{100}\pm iH_{010})e^{-\rho^2/2}/\sqrt{2},$$
$$b=H_{001}e^{-\rho^2/2},\quad s=H_{000}e^{-\rho^2/2},$$

where $H_{n_\xi n_\eta n_\zeta}$ is a product of three Hermite polynomials appearing in (5), reference 7, and $\rho^2=\xi^2+\eta^2+\zeta^2$, one finds $(ac|J|ss)=(bb|J|ss)=Bu/\tau$ and $2^{\frac{1}{2}}(d^2c|J|as)=(d^1b|J|as)=(d^1c|J|bs)=3^{\frac{1}{2}}(d^0a|J|as)=-(3^{\frac{1}{2}}/2)\times(d^0b|J|bs)=Bu(1-1/\tau)/\tau$ and $2^{\frac{1}{2}}(d^2c|J|sa)=(d^1b|J|sa)=(d^1c|J|sb)=(d^0a|J|sa)=-(3^{\frac{1}{2}}/2)(d^0b|J|sb)=Bu/\tau^2$. The further relations $(d^2c|J|as)=(d^{-2}a|J|cs)$, etc., the two differing only in the sign of the ζ axis, are apparent.

with $\tau=3.2$. Next, we have twenty-seven states arising from the terms in *bba* by the excitation $ss\to ac$; nine of the thirty-six permutations (three of *bba* times twelve of *ssac*) are "excluded" by having two like-particle, like-spin *a*'s. If all thirty-six were allowed, the excitation $ss\to ac$, being permuted, would make no net contribution to $\Sigma m_{l\pi}$ so μ_L would be due to the original *p* particles, and average μ_{L0}. The total excess over μ_{L0} is thus proportional to the negative of $\Sigma m_{l\pi}$ for the excited particles for the nine excluded states, which is 2. Since for these 27 states, $H_{0a}'=(ac|J|ss)/\sqrt{15}$, we have for them

$$\delta\mu_L=2(Bu/2\alpha\sigma\tau)^2/15=0.0021.$$

Next we consider the excitations $s\to d$, which may be accompanied by a change of m_l of one of the original *p* particles, to keep Σm_l unaltered. Arising from the three terms[21] in *aac* of (14), there are twelve states which involve the excitation $sa\to d^2c$, for example. They are of two types, six of the type d^2sac; *ssc* and six of the type d^2sca; *ssc*, since the matrix element H_{0a}' depends upon whether or not the excited function d^2 and the altered *p* function *c* (either of the *c*'s) form a like-particle, like-spin pair.[21] The latter type, having d^2 and *c* paired, contribute nothing to $\delta\mu_L$, because the excitation $sa\to d^2c$ of two like-particles does not change $\Sigma m_{l\pi}$ and the pair d^2c may be permuted with the other *a* and *c* just as freely as could the *a* which it replaces, leaving the average μ_L equal to μ_{L0} for this type. The same is true for all types with the excited and altered functions as a like-particle, like-spin pair, so we need not consider them further. The states of the former type, d^2sac; *ssc*, have an average $m_{l\pi}$ of $-\frac{1}{3}$ from the *p* functions and $2(\frac{1}{2}+\frac{1}{6})=4/3$ from the d^2 (which may either be in the sixth place or paired with *a*), so their average μ_L is 1. Similarly for the other [23] types of the class $s\to d$ we have the results[22] listed in Table II. We have yet to consider the excitations $sp\to pd$. They differ from those we have just considered, $s\to d$ or what we might call $sp\to dp'$, by interchange of the excited and the altered functions. This does not alter the situation which makes a type with these functions as a like-particle, like-spin pair con-

[23] Only those excitations which change the individual-particle projections m_l alter the projected magnetic moment μ_L.

tribute nothing. The interchange alters the types of Table II in such a way as to give the results listed in Table III. The contribution of the states listed in Tables II and III is

$$\delta\mu_L=\{13(1-1/\tau)^2-5/\tau^2\}(Bu/\alpha\sigma\tau)^2/60 \\ =(6.4-0.5)/10^3=0.0059.$$

The entire second-order contribution of the states arising by double excitation from the terms of (14) (with the s shell), when only the leading (space-exchange) term in (1) is considered, is thus $\delta\mu_L=0.015$ nuclear magnetons, for $\tau=3.2$.

Second-Order Spin Effects

The second-order effect of the main term of the interaction is thus so small that an estimate of order of magnitude should suffice for the rest of the terms. We must give attention to two possibilities that might make the second-order spin effects larger than the effects of the spin-independent terms; we must see that the spin dependence does not leave large groups of contributing terms, but destroy their tendency, noted above, to oppose one another in $\delta\mu$, and we must see that the large difference between the proton and neutron moment does not make the spin contributions too large. Although we do not want to consider that g and g_σ are individually not larger than their sum $g_0\approx 0.2$, we do tentatively consider them as individually smaller than the coefficient of the main term, g_q. Even so, they are together effectively only about as large as their sum (since the space-exchange does not change the sign nor order of magnitude of the matrix elements).[22] The second-order contributions to $\delta\mu_S$ are proportional to the squares of the matrix elements, so they contain a factor of about g_0^2 to make them much smaller than the $\delta\mu_L$ calculated above, and this should be more than sufficient to compensate the excess of $(\mu_{s\pi}-\mu_{s\nu})\approx 5$ over $\mu_{l\pi}\approx 1$, if the cancellation is approximately as general for the spins as for the orbital momenta. In a preliminary way one may thus expect $\delta\mu_S$ to be smaller than $\delta\mu_L$ in magnitude. The sign is expected to be negative, since $\delta\mu_S$ is due to a participation in the spin by the neutrons.

We proceed to investigate briefly the cancellation in $\delta\mu_S$. The spin-dependent terms of (1) have matrix elements with states having two of the opposite m_s, in the order $+-+-$; $+-+$, exchanged. If the two both refer to protons, or to neutrons, the state contributes nothing to $\delta\mu_S$, but if one refers to a proton and the other to a neutron, the state contributes $\pm(\mu_{s\pi}-\mu_{s\nu})\times(H_{0a}'/(E_a-E_0))^2$. Among the excitations $ss\rightarrow pp$, for example, one finds the state $c^-s^-a^+a^-$; $s^+a^+c^+$ to cancel the effect of the state $s^+c^+a^+a^-$; $a^-s^-c^+$, and similarly for all of these states except $s^+a^+c^+a^-$; $c^-s^-a^+$ of which $\delta\mu_S$ is very small, containing g_0^2, and negative. Among the excitations $s\rightarrow d$, those states in which the altered m_s are both from the s shell of course cancel. Of those having an altered m_s from the p shell, we have, arising from the term aac of (14), besides two canceling pairs, the states

$$\begin{array}{llll} sd^{0+} & aa; & ss & c^- \\ ss & ab^+; & d^{1-}s & c \\ ss & bc^+; & d^{2-}s & c \end{array}$$

(where only the altered m_s are indicated) and from the term aca, besides three canceling pairs, the states

$$\begin{array}{llll} ss & ac^+; & d^{0-}s & a \\ ss & ab^+; & d^{-1-}s & c \\ sd^{2+} & ac; & ss & c^- \end{array}$$

and similarly three from the term caa. The matrix element H_{0a}' for the first of these nine states, for example, is $2\{g(d^0c|J|cs)+g_\sigma(d^0c|J|sc)\}/\sqrt{15}$ and their total contribution is $\delta\mu_S=(11/30)\times\{g(1-1/\tau)+g_\sigma/\tau\}^2(Bu/\alpha\sigma\tau)^2(\mu_{s\nu}-\mu_{s\pi})$ which is about -0.0026 for $g=g_0$ and about -3×10^{-5} for (4). For the states arising from the terms in bba, the situation is similar, but the lack of the factor 2 in these terms of (14) makes their contribution about one-fourth as large. Thus the $\delta\mu_S$ from these states is in magnitude not more than about half as large as the $\delta\mu_L$ from the states of Table II, which also arise by the $s\rightarrow d$

Table III. *Influence of the excitations $sp\rightarrow pd$.*

Type	Nr.	$\{(\mu_L)_{Av}-1/3\}$	$15^{\frac{1}{2}}H_{0a}'$	$\delta\mu_L\cdot 60(\alpha\sigma\tau^2/Bu)^2$
$cs\,ad^2$; $ss\,c$	12	$-1/3$	$2(d^2c\|J\|sa)$	-8
$ss\,d^{-2}a$; $sa\,a$	3	$2/3$	$2(d^{-2}a\|J\|sc)$	4
$bs\,ad^1$; $ss\,c$	12	0		
$cs\,ad^1$; $ss\,b$	12	$-1/3$	$(d^1c\|J\|sb)$	-4
$ss\,aa$; $sb\,d^{-1}$	3	0		
$ss\,ad^{-1}$; $sa\,b$	6	$2/3$	$(d^{-1}a\|J\|sb)$	4
$cs\,bb$; $ss\,d^2$	9	$-2/9$	$(d^2c\|J\|sa)$	-1
				-5

excitation, in spite of the fact that the individual contributions of the states listed to $\delta\mu_S$ are all of the same sign, in contrast to Table II.

The states arising from other types of excitation are also not numerous, and their contribution is, similarly, small and negative. The excitation $s^{\pm}p^{\mp}\rightarrow p^{\mp}d^{\pm}$ differs from $s^{\pm}p^{\mp}\rightarrow d^{\mp}p^{\pm}$, considered above, principally by an interchange which exchanges the roles of g and g_σ. The type $s^{\pm}p^{\mp}\rightarrow s^{\mp}f^{\pm}$ contributes nothing, the excited states being symmetric in two like s particles. The excitation of the p shell alone does give a small contribution here: there are five contributing states of the type $p^{\pm}p^{\mp}\rightarrow f^{\mp}p^{\pm}$ and five of the type $p^{\pm}p^{\mp}\rightarrow d^{\mp}d^{\pm}$, the matrix elements of which are of a higher order in $1/\tau$ than those considered above, and therefore smaller. Without more detailed calculation one may safely conclude that the second-order modification of (14) by the doubly excited states due to any reasonable spin dependence of the interaction (1) leads to a $-\delta\mu_S$ which is considerably smaller than the $\delta\mu_L$ due to the spin-independent terms. The more exact first-order wave function (13) differs from (14) principally by the inclusion of a term arising from ${}^3P_\nu$ with a coefficient about one-tenth as large as that of either of the other two terms. The effect of this term and the leading term in P^q of (1) is to make (13) have matrix elements with excited states arising from triplets. These states thus contribute to $\delta\mu$ independently, and their contributions contain a factor of the order $(C_3/C_1)^2\approx 10^{-2}$ compared to the contributions calculated above. These are therefore unimportant, in spite of the fact that spin magnetic moments are somewhat larger than orbital magnetic moments. The effect of the term arising from a triplet in (13) with the spin-dependent terms of (1) is to modify some of the matrix elements calculated above, but the modification contains a factor of the order $g_0C_3/C_1\approx 1/50$ so the influence of these terms is not great enough to change significantly the very small value of $\delta\mu$ already calculated.

The more exact wave function (13) differs slightly from the approximation we have used (14) also in the coefficients of the terms arising from the singlets. Although the exact symmetry of (14) greatly reduces the labor involved in computing $\delta\mu_L$, it could not be expected to make any essential difference in the magnitude of the result. (The fact that the terms with $m_{l\pi}=1$ are relatively smaller in (13) has two opposing effects: it reduces μ_{L0} and thereby increases $\delta\mu_L$ from all the excited states, and it decreases the matrix elements preferentially for excited states having $m_{l\pi}=1$, and thus tends to decrease $\delta\mu_L$). For (14), $\delta\mu_L=0.015$, as we have seen, and the spin effects give rise to a smaller, and negative, $\delta\mu_S$, which makes $\delta\mu=0.01$. The slightly altered (probably greater) $\delta\mu_L$ from (13) would probably leave $\delta\mu=0.01$, but one may more safely take 0.02 as an upper limit for $\delta\mu$ due to doubly excited states in second order.

In determining the sum of the large number of smaller contributions due to more highly excited states (of the nth order in $1/n\tau$ for n-tuple excitation) and higher orders, the cancelation of positive and negative contributions, so important in our considerations above, would be expected to be even more complete, making the convergence very rapid (in contrast to the computation of binding energies,[7] where the contributions all have the same sign). In the absence of a complete investigation of the convergence, it seems very unlikely that the final result would differ from the first-order result by more than 0.04. This, with the results of Table I, places a lower limit of the values of the proton magnetic moment with which our assumptions are compatible at 2.92 nuclear magnetons,[16] 0.33 below the lower limit[1] of the observed μ_{Li^7}.

In perturbation theory, the magnitude of the second-order contribution may be considered as a fairly reliable criterion of the accuracy of a first-order result. The result of our second-order calculation is, broadly speaking, that the first-order (Hartree) calculation seems to give with good accuracy the value of the magnetic moment of Li^7 *which follows from our assumed interaction.* In the first-order calculation, the orbital moment is due almost equally to the three p particles, making $\mu_L\approx\frac{1}{3}$. If μ_L were due equally to all seven particles, one would expect it to be 3/7, and the small positive value of the second-order $\delta\mu_L$ (arising from excitation of the s shell) may be interpreted as indicating a slight tendency toward that situation.