

Then we obtain for the magnetic field in this region

$$\begin{aligned} H_z &= 1 - \frac{1}{2}t_1^2 e^{-2\pi(x+U_0)} \cos 2\pi z, \\ H_x &= \frac{1}{2}t_1^2 e^{-2\pi(x+U_0)} \sin 2\pi z. \end{aligned} \quad (27)$$

For the uncorrected field of plane parallel pole faces the field inhomogeneity is one-tenth percent at $x=1.8$. For the field with ring shims in the case $b=0.125$, $a=0.095$ we find from Fig. 4 that this same inhomogeneity occurs at $x=0.80$. Thus if the exit slit is placed at this distance (0.4 the magnetic gap) from the edge, the magnetic field over the entire region of motion of the ions will be homogeneous within the required degree of accuracy.

Finally, we may return to a consideration of the assumptions made at the beginning of this section. First of all the assumption of low reluctance of the iron will in general be fulfilled rather well. Of course, it is sufficient if only the cyclotron lids and not the large magnets themselves be of low reluctance, high permeability iron. Secondly, the assumption that the shims be placed inside the chamber against the lids rather than in the air gap need not impose any restriction on the applicability of the results obtained here. Since the shims used to make the field homogeneous may be inserted at the time of construction it should perhaps be not inconvenient actually to place them inside the chamber.

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Some Experiments on the Magnetic Properties of Free Neutrons

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The behavior of partly polarized beams of slow neutrons as regards their precession on passing through homogeneous magnetic fields has been investigated. From the experiments it is concluded that the neutron has a magnetic moment not far from $2 \times 1/1840$ Bohr magneton and that the sign is negative. Further, the precession of neutrons inside magnetized iron was investigated; it was found that the field accounting for the observed rate of precession is more than 100 times the actual field strength H and actually of the order of magnitude of the magnetic induction B .

1. INTRODUCTION

THAT a neutron should have a magnetic moment at all, seems somewhat surprising, on account of its being electrically neutral. On the other hand, from the magnetic moments of the proton^{1, 2} (2.5 to 2.8 n.m., 1 n.m. = 1 nuclear magneton = $1/1840$ Bohr magneton); and the deuteron^{2, 3} (0.85 n.m.), a magnetic moment μ_n of the neutron, of about 2 n.m., can be deduced;⁴

the sign of μ_n should be negative, that is, the relative position of spin and magnetic moment should be the same as in the (negative) electron. A tentative explanation of this moment, based on the Fermi theory of beta-decay, has been offered by Wick.⁵

A way of measuring, at least roughly, the magnetic moment of free neutrons has been pointed out by Bloch.⁶ He showed that the magnetic interaction between neutrons and electrons must have a measurable influence on the scattering of slow neutrons by magnetic atoms or ions (provided the neutron has a magnetic moment of the order of 2 n.m.). Of special interest is the scattering of neutrons from a ferromagnetic substance in

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¹O. R. Frisch, O. Stern, *Zeits. f. Physik* **85**, 4 (1933); I. Estermann, O. Stern, *Zeits. f. Physik* **85**, 17 (1933); I. Estermann, O. C. Simpson, O. Stern, *Phys. Rev.* **52**, 535 (1937).

²I. I. Rabi, J. M. B. Kellogg, J. R. Zacharias, *Phys. Rev.* **46**, 157, 163 (1934); **50**, 472 (1936).

³I. Estermann, O. Stern, *Phys. Rev.* **45**, 761 (1934).

⁴H. A. Bethe, R. F. Bacher, *Rev. Mod. Phys.* **8**, 91, 205 (1936).

⁵G. C. Wick, *Att. Acad. Lincei* **21**, 170 (1935); see also reference 4.

⁶F. Bloch, *Phys. Rev.* **50**, 259 (1936).

which, by means of a magnetic field, the magnetic momenta of a large number of electrons are adjusted parallel among themselves; then the scattering should depend on the spin orientation of the neutrons relative to that of the electrons. One should therefore be able to produce partly "polarized" beams of neutrons by sending them through a piece of magnetized iron; a second, similar piece might be used as an "analyzer."

The existence of such an effect was demonstrated by Hoffman, Livingston, and Bethe;⁷ they found that the total transmission, for slow neutrons, of a system of two iron bars (of 1 cm thickness each) was 2.3 percent smaller when the bars were magnetized antiparallel instead of parallel. The amount of the difference was in fair agreement with a calculation⁷ based on the assumption $\mu_n = 2$ n.m.; the agreement may be regarded as an argument for the correctness of this assumption. Similar results have been reported by Beyer, Carroll, Dunning and Powers.⁸⁻¹⁰

It was soon realized that the more or less irregular magnetic stray fields prevailing along the path of the neutrons would complicate the course of the phenomenon by causing frequent reorientation of the polarized neutrons. On the other hand, the systematic study of this reorientation might permit one to determine the magnitude and also the sign of the magnetic moment μ_n of the neutron. This was pointed out by Rabi,¹¹ who calculated the reorientation caused by a special type of an inhomogeneous magnetic field. Independently the author¹² also demonstrated the existence of this reorientation effect by using a homogeneous field perpendicular to the polarizing and analyzing fields, and showed that it behaves roughly as one would expect from the assumption $\mu_n = 2$ n.m.

Using a similar arrangement, the authors also proved¹³ that the sign of μ_n is actually negative,

as expected from the moments of proton and deuteron. The same result was found by Powers, Carroll, Beyer and Dunning¹⁴ who used an arrangement similar to the one suggested by Rabi.¹¹

A third paper of the authors¹⁵ was devoted to the study of the precession of neutrons inside magnetized iron, where the question of some theoretical interest is, whether the magnetic field strength H or the magnetic induction B (or perhaps something in between) is responsible for the rate of precession.

In the following we shall give a more detailed account of the experiments the results of which have been published in the three preliminary notes,^{12, 13, 15} and some considerations and calculations which have to do with the problem.

2. GENERAL REMARKS

In the classical theory a particle possessing a magnetic moment μ and an angular momentum J precesses in a magnetic field H like a gyroscope, with an angular velocity $\omega = H\mu/J$. According to quantum mechanics, however, this picture cannot in general be used, and in particular fails entirely in cases like the Stern-Gerlach effect

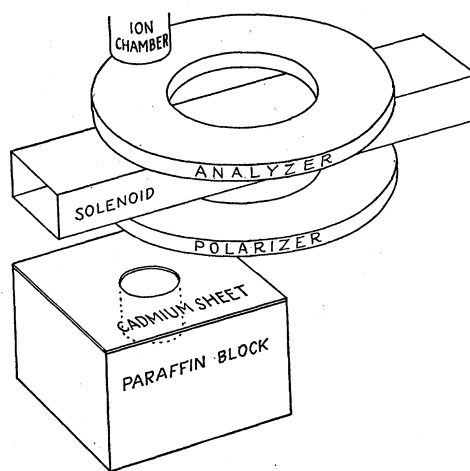


FIG. 1. Experimental arrangement for the demonstration of the precession of neutrons. The path of the neutrons from the hole in the paraffin block to the ionization chamber was almost entirely surrounded by boron and cadmium sheets (not shown in the figure) to suppress scattered neutrons.

⁷ J. G. Hoffman, M. Stanley Livingston, H. A. Bethe, *Phys. Rev.* **51**, 214 (1937).

⁸ J. R. Dunning, P. N. Powers, H. G. Beyer, *Phys. Rev.* **51**, 51 (1937).

⁹ P. N. Powers, H. G. Beyer, J. R. Dunning, *Phys. Rev.* **51**, 371 (1937).

¹⁰ P. N. Powers, H. Carroll, J. R. Dunning, *Phys. Rev.* **51**, 1112 (1937).

¹¹ I. I. Rabi, *Phys. Rev.* **51**, 652 (1937).

¹² O. R. Frisch, H. von Halban, J. Koch, *Nature* **139**, 756 (1937).

¹³ O. R. Frisch, H. von Halban, J. Koch, *Nature* **139**, 1021 (1937).

¹⁴ P. N. Powers, H. Carroll, H. Beyer, J. R. Dunning, *Phys. Rev.* **52**, 38 (1937).

¹⁵ O. R. Frisch, H. von Halban, J. Koch, *Nature* **140**, 360 (1937).

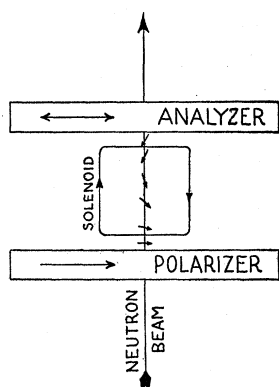


FIG. 2. Schematic diagram, illustrating principle of precession experiment.

where we have to do with a different motion of the particles in the separate stationary states corresponding to different orientations of the spin axes relative to the magnetic force. Still, as can be simply verified by a direct quantum-mechanical analysis,¹⁶ the precession picture retains, in accordance with the correspondence argument, its validity in all such cases where, as in the phenomena considered below, the translative motion of the particles is not essentially influenced by their angular orientation in the field.

A particle with angular momentum $\frac{1}{2}\hbar$ and magnetic moment μ in a magnetic field H precesses with an angular velocity $\omega = 2\mu H/\hbar$. If the velocity of the particle is v , then it moves a distance $l_R = v/\omega = v\hbar/2\mu H$, while rotating through one radian. Assuming $\mu = 2$ n.m. $= 1.10^{-23}$ c.g.s. units, we find $\omega = 2 \cdot 10^4 H$; for an average "thermal" neutron with $v = 2 \cdot 10^5$ cm/sec. we get $l_R \cdot H = 10$, where l_R is measured in cm and H in gauss.

This value of $l_R \cdot H$ is very much larger (about 10^4 times) than the values encountered in experiments on the reorientation of atoms.¹⁷ In molecular beam experiments, unless special arrangements are made, the direction of the magnetic field changes only very slightly over a distance l_R along the path of the particles, and the spins therefore follow the magnetic field lines adiabatically, precessing within a very narrow cone around them. Only if the field is made very weak

and at the same time strongly inhomogeneous at some point of the beam, are the spins "shaken off the field lines" and actual precession occurs. Of course, all this may be expressed, as is usually done, in terms of space quantization and non-adiabatic transitions.

In analogous experiments with neutrons the field need be neither so weak nor so strongly inhomogeneous, because $l_R \cdot H$ is so much larger. Furthermore, it is possible, in the case of neutrons, to study the especially simple case of precession in a sharply limited homogeneous "precession field" which is e.g. perpendicular to the polarizing and analyzing fields; in order to separate the different fields, a layer of copper winding carrying a suitable current may be applied, since a thin layer of copper is almost transparent for neutrons.

3. DEMONSTRATION OF THE PRECESSION OF NEUTRONS

A. Experimental arrangement

Our first experiment¹² was designed to demonstrate the existence of the precession and to measure its rate in a rough way. The arrangement (Fig. 1) consisted of a paraffin block containing a neutron source (400 mg Ra+Be), the polarizer, the precession field solenoid, the analyzer, and the detecting boron-lined ionization chamber connected to an amplifier and mechanical counter. The principle of the method is shown in the schematic Fig. 2.

The polarizer was a flat iron ring (20 cm outer, 11 cm inner diameter, 0.8 cm thickness) which was wound all over with one layer of 1.5 mm copper wire (enameled). In such a symmetrical arrangement the magnetizing current should produce practically no field outside the iron (the fields in the neighborhood of the single wires can be neglected). Actually, when a current of 10 amp. was passed through the windings, irregular fields of several gauss were observed near the surface of the iron rings, presumably on account of slight local variations of the magnetic properties. We decided therefore to switch on the current of 10 amp. only for a few seconds every time the magnetization was reversed, and then to switch it off again, and use the remanent field only. In this case the stray fields were consider-

¹⁶ Compare., e.g., C. G. Darwin, Proc. Roy. Soc. **117**, 258 (1927).

¹⁷ O. R. Frisch, T. E. Phipps, E. Segrè, O. Stern, Nature **130**, 892 (1932); O. R. Frisch, E. Segrè, Zeits. f. Physik **80**, 610 (1933); J. M. B. Kellogg, I. I. Rabi, J. R. Zacharias, Phys. Rev. **50**, 472 (1936).

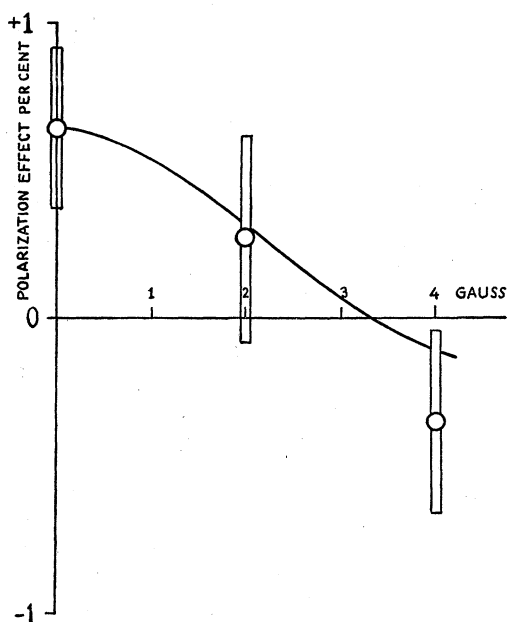


FIG. 3. Result of precession experiment. Polarization effect plotted as a function of precession field.

ably smaller and could not have caused appreciable precession.

The precession field coil was made with rectangular cross section, 3.5×5 cm, and 35 cm long, and consisted of a single layer of 0.65 mm copper wire, wound on a brass frame. The field was calculated from the formula for an infinitely long solenoid.

The analyzer was made identical with the polarizer.

The boron-lined ionization chamber was of the "double-decker" design, in order to get a large useful boron surface. It was connected to a transformer-coupled high gain amplifier acting on a mechanical counter; the resolving power of this arrangement was sufficient since only about 100 neutrons per minute were counted.

The experiment consisted in taking alternative counts, of 7 minutes each, with the magnetization of the polarizer reversed after each count; the analyzer was kept with the same magnetization all the time. Runs were made with three different currents through the precession field solenoid. Each run had to last about 50 hours in order to have a small statistical error. The whole experiment was carried through entirely automatically by means of an arrangement of relays and rotat-

ing switches, controlled by a clock. Every 7.5 minutes the counting was stopped during half a minute; within this time, a picture of the mechanical counter (and some control instruments) was taken on a motion-picture film, and the polarizer current was reversed and switched off again. Because of the short cycle of counting, slow variations of the sensitivity of the amplifier (which actually occurred) were practically eliminated.

B. Results and discussion

In the first run, with no current in the precession field solenoid, the number of neutrons recorded was 0.65 ± 0.28 percent larger with the fields in the polarizer and analyzer parallel than with antiparallel fields. In the second run, with a precession field of 2 gauss, a difference of 0.29 ± 0.36 percent was recorded, while in the third run, with 4 gauss precession field, the difference was -0.36 ± 0.31 percent; the negative sign indicates that the recorded number of neutrons was smaller with parallel than with antiparallel fields, in the third run. The errors have been calculated in the standard way from the square roots of the numbers of neutrons counted. It was found repeatedly that the fluctuations were not larger than those to be expected as purely statistical fluctuations. In this experiment, $1.2 \cdot 10^6$ neutrons were counted, on the whole.

Supposing, for the moment, that all the neutrons in the beam had the same velocity v_0 , the polarization effect (difference in intensity, with parallel and antiparallel fields) would be proportional to the cosine of the angle $\varphi = 2\mu Hl/hv_0$ through which the neutrons precess on their way (of length l) through the field H . On account of the velocity distribution of the neutrons, the polarization effect as a function of H will be not a cosine but a superposition of cosines with different periods, which is roughly equivalent to a damping of the cosine function which corresponds to the mean velocity. We have calculated this function (see curve C, Fig. 5) making a number of assumptions which are discussed in §7.

In Fig. 3 the observed differences are plotted together with the theoretical curve, and it is seen that they agree within the very large experimental errors. Obviously it would be hopeless to

discuss these results any further and to try and enclose the magnetic moment of the neutron between definite limits. We started a new experiment with an improved arrangement, taking counts with five different values of the precession field, but from a fortnight's continuous counting we found differences showing the same general trend but still slightly smaller than those reported here, perhaps merely because of an adverse statistical fluctuation. Substantial improvement in the accuracy would have required such unreasonably long periods of counting that we decided to postpone a repetition of the experiment until a time when stronger sources should be available.

The maximum polarization effect found by us (0.65 percent) must be multiplied by 2 in order to take account of the fact that about half of the recorded neutrons were faster than one volt (went through 0.5 mm of cadmium) and, therefore, should not contribute to the polarization effect. The resulting figure, 1.3 percent, is smaller than the one found by Hoffman, Livingston, and Bethe,⁷ (2.3 percent). This difference is understandable since we used a field of 10,000 gauss in polarizer and analyzer (the remanent field) while Hoffman, Livingston and Bethe used a field of 15,000 gauss.

4. SIGN OF THE MAGNETIC MOMENT OF THE NEUTRON

A. Experimental arrangement

In order to determine the sign of the magnetic moment of the neutron it is necessary to find the direction of precession. This is obviously not possible with the symmetrical arrangement of our first experiment, and the arrangement was therefore changed in the following way (see Fig. 4): (1) the analyzing iron ring was rotated through an angle of 90° , with the beam as an axis, so that the polarizing and analyzing fields were perpendicular to each other (and to the neutron beam); (2) the precession field solenoid was removed and a short (2.2 cm long, 5 cm diameter) coil was inserted producing a field parallel to the beam; this field was found (by means of a small flip coil) to be fairly homogeneous, presumably on account of the iron below and above.

The experimental procedure this time was to reverse the precession field alternately while the rings were kept magnetized in the same direction all the time. The direction of precession changes with the precession field, and the neutron intensity recorded must be larger when the neutrons precess in the same direction as one would have to rotate the polarizer in order to make the polarizing and the analyzing fields parallel (see Fig. 4). From the direction of the precession field for which the transmitted neutron intensity is larger than for the opposite direction, one can deduce the sign of the magnetic moment of the neutron.

In order to get a large effect, the magnetizing current on the rings was left on (2 amp.) all the time. Consequently the stray fields were quite strong. By rotating the rings independently in their own plane, however, a position was found where the stray field was nearly homogeneous and parallel to the beam, and of the right magnitude (3 gauss) to turn the neutrons by 90° , on the average. This field could be reversed by passing a suitable current through the solenoid. A large number of counts (of 7 minutes each) were then taken with the solenoid current alternately on and off.

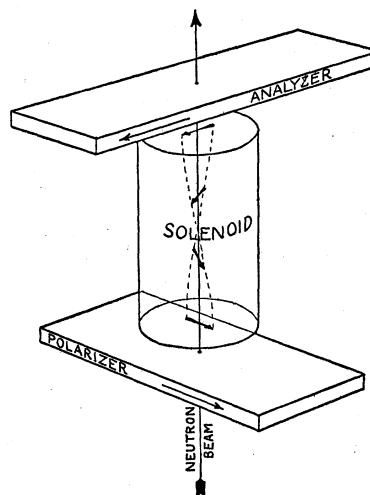


FIG. 4. Determination of the sign of the magnetic moment of the neutron, schematic diagram of the arrangement. If the neutrons precess in the way indicated, a larger fraction of them goes through the analyzer than when the current in the solenoid is reversed and the neutrons correspondingly precess in the opposite way. For the sake of clearness, the solenoid is made much longer in Fig. 4 than it was in the actual experiment.

B. Results

A difference of 1.05 ± 0.24 percent was recorded between the intensities observed with the two directions of the precession field. The amount of the difference is in accordance with the assumption $\mu_n = 2$ n.m., although the arrangement is not well suited for measuring the magnitude of μ_n . From the sign of the difference it followed that the neutrons precess in the direction of the (positive) current in the solenoid; from this again it follows that the sign of the magnetic moment of the neutron is negative, that is, the relative direction of spin and magnetic moment is the same as in the (negative) electron.

Shortly after our first publication, the same result was published by Powers, Carroll, Beyer and Dunning,¹⁴ who had used a type of an unsymmetrical inhomogeneous field suggested by Rabi.¹¹ The negative sign had been generally expected on account of the moments of the proton and the deuteron (see §1).

5. PRECESSION OF NEUTRONS INSIDE MAGNETIZED IRON

A. The problem

Rossi¹⁸ and Mott-Smith¹⁹ have tried to detect the deflection which cosmic-ray electrons undergo when passing through magnetized iron. For the interpretation of such experiments it is essential to know, whether the amount of deflection corresponds to the magnetic induction B (about 20,000 gauss at saturation) or the magnetic field strength H (in general only a few gauss). Weizsäcker²⁰ has examined the question with the help of Dirac's theory and found that the induction B is effective.

We have put to ourselves the analogous question: Is the rate of precession of neutrons inside magnetized iron determined by H or B ? So far, no theoretical investigation of the problem as such has been published; but Bloch²¹ made some interesting remarks from which it appears that the theoretical treatment of this and some

similar problems would require a much more intimate knowledge of the interaction between electrons and neutrons than is available at present. In this state of affairs we thought that even a crude experiment might be of interest.

B. Experimental arrangement

The arrangement was in principle similar to our first one, but yet different in several respects. As polarizer and analyzer we used straight iron bars of cross section 1×5 cm, and 80 cm length, carrying a single layer of 0.65 mm copper wire; by means of iron pieces at the ends they were connected to form a single magnetic circuit. The magnetic induction in the bars was about 14,000 gauss. The space between them was investigated with a small flip coil, and the neutron beam was sent through at a point where the stray field was weak enough to cause no disturbance.

A long, flat solenoid (7×0.7 cm cross section, 15 cm long) was placed across the beam, similar to the first arrangement, and a strip of thin (0.15 mm) iron sheet inside the coil; the ends of the strip were connected by a strong iron yoke (made of round iron of 1 cm diameter) to close the magnetic circuit. The current through the solenoid was alternately set at two different values corresponding to magnetic fields of 2.8 gauss and 35 gauss, respectively; the polarizing and analyzing fields remained unchanged (they were bound to be antiparallel since they formed a magnetic circuit).

In order to enhance the polarization effect we used a low temperature source of neutrons,¹⁰ consisting of an ice block at liquid-air temperature, of similar shape as the paraffin block used before.

C. Results

To check the whole arrangement, a run was made with the iron strip removed. With a precession field of 2.8 gauss, along a path of only 7 mm, the neutrons should hardly precess at all and the transmission of the system should have the low value corresponding to the polarizing and analyzing fields being antiparallel. With the precession field at 35 gauss, however, most of the neutrons precess by more than 360° and the beam should be practically depolarized, and consequently the

¹⁸ B. Rossi, *Rend. Lincei* **11**, 478 (1930); *Nature* **128**, 300 (1931).

¹⁹ L. M. Mott-Smith, *Phys. Rev.* **37**, 1001 (1931); **39**, 403 (1932).

²⁰ C. F. v. Weizsäcker, *Ann. d. Physik* **17**, 869 (1933).

²¹ F. Bloch, *Phys. Rev.* **51**, 994 (1937).

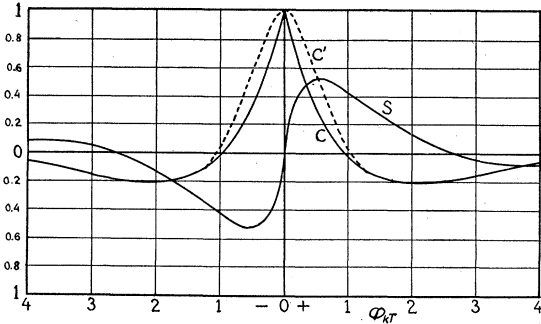


FIG. 5. Curves representing the precession of thermal neutrons.

transmission of the system should lie halfway between the values for parallel and antiparallel fields. Such a difference was actually observed; the intensity was 0.83 ± 0.26 percent larger with 35 gauss than with 2.8 gauss. This figure of 0.83 is to be compared with one-half of 0.65, the result in our first experiment (see §3B). It should be remembered that in this first experiment the polarizing field was alternately reversed. The improvement by a factor of 2 or 3 is to be attributed partly to the cooling, and partly to the larger magnetic induction in polarizer and analyzer (14,000 instead of 10,000 gauss).

The iron strip was now inserted (and closed by the yoke) and a second run, quite similar to the first one, was carried out. If the rate of precession in the iron was determined by the field strength H , then the iron strip should make no difference at all, because the field strength is the same within the iron as below and above it. If it is B that matters, however, then the neutrons should precess many times on their way through the iron sheet and be completely depolarized; since the iron is practically saturated even with the weak field of 2.8 gauss, the neutrons reaching the analyzer should be depolarized and the intensities transmitted the same both for 2.8 and 35 gauss. This is actually what we found: the difference was -0.05 ± 0.24 percent, that is, zero within the limits of error.

It is concluded that the field accounting for the rate of precession of neutrons inside magnetized iron is certainly much larger than H ; a numerical discussion shows that it is probably larger than 500 gauss, or, in other words, larger than $0.03 \cdot B$.

6. CALCULATION OF THE PRECESSION OF NEUTRONS WITH MAXWELLIAN VELOCITY DISTRIBUTION

We assume the velocity distribution by the neutron beam and the velocity dependence of the indicating system to be such that $f(v)dv$ is the number of neutrons with velocities between v and $v+dv$ counted per unit time, with polarizer and analyzer removed. With the polarizer (of thickness x) in place (and magnetized) the number of neutrons is reduced to

$$J_0(v)dv = \frac{1}{2}f(v)dv(e^{-x\mu_1(v)} + e^{-x\mu_2(v)}),$$

where μ_1 and μ_2 are the attenuation coefficients for parallel and antiparallel neutrons, respectively.

If now the analyzer (also of thickness x) is brought in place, the transmitted intensity depends on its position:

$$\text{parallel } J_1(v)dv = \frac{1}{2}f(v)dv(e^{-2x\mu_1(v)} + e^{-2x\mu_2(v)}), \quad (1)$$

$$\text{antiparallel } J_2(v)dv = f(v)dv \cdot e^{-x[\mu_1(v)+\mu_2(v)]}. \quad (2)$$

Introducing $\mu_1(v) = \mu + \beta(v)$ and $\mu_2(v) = \mu - \beta(v)$ (we may assume that μ does not depend on v) we find

$$\begin{aligned} [J_1(v) - J_2(v)]dv \\ = 2f(v)dv \cdot e^{-2\mu x} \cdot \sinh^2[x \cdot \beta(v)]. \end{aligned} \quad (3)$$

If the analyzer forms an angle φ with the direction of polarization, (3) must be multiplied with $\cos \varphi$. We assume that the analyzer forms an angle α with the polarizer, and that the neutrons pass through a magnetic field H (perpendicular to both polarizer and analyzer) and precess through an angle $\varphi_p = 2\mu_n l H / \hbar v$. Then we have $\varphi = \varphi_p - \alpha$, and hence, if we integrate over v

$$\begin{aligned} J_1 - J_2 &= \text{const.} \int f(v)dv \cdot \sinh^2[x \cdot \beta(v)] \\ &\quad \cdot [\cos \alpha \cos \varphi_p(v) + \sin \alpha \sin \varphi_p(v)] \\ &= \text{const.} \left[\cos \alpha \int f(v)dv \cdot \sinh^2[x \cdot \beta(v)] \cdot \cos \varphi_p(v) \right. \\ &\quad \left. + \sin \alpha \int f(v)dv \cdot \sinh^2[x \cdot \beta(v)] \cdot \sin \varphi_p(v) \right] \\ &= \text{const.} [\cos \alpha \cdot C + \sin \alpha \cdot S]. \end{aligned}$$

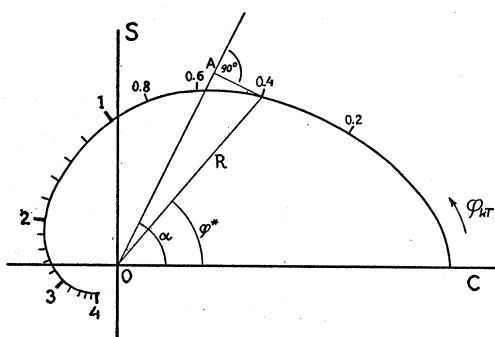


FIG. 6. Curve representing the precession of thermal neutrons. The distance OA indicates the polarization effect obtained for a given value (e.g., 0.4) of φ_{kT} and an angle α between polarizer and analyzer.

The functions C and S have been calculated (by numerical integration) and are plotted in Fig. 5. The abscissa is $\varphi_{kT} = H \cdot l \cdot 2\mu_n / \hbar(2kT/m)^{1/2}$, the angle through which neutrons with an energy of kT precess on their way l through the field H . For neutrons with a magnetic moment of 2 n.m. and thermal energy distribution at room temperature we have $\varphi_{kT} = 0.088 lH$. The scale of ordinates is chosen so that $C(0) = 1$.

In calculating the functions C and S , we have made the following assumptions:

(1) $f(v)dv = \text{const. } v^2 e^{-mv^2/2kT} dv$, which means Maxwellian distribution of the neutrons and $1/v$ sensitivity of the boron chamber. The integrations were carried from zero to infinity.

(2) $\sinh [x\beta(v)] \sim x\beta(v)$, which is correct for the larger part of the velocity spectrum where $x\beta(v) \ll 1$ (with $x \sim 1$ cm).

(3) $\beta(v) = \text{const.}/v$. This assumption⁷ should be correct for the mean part of the spectrum, but tends to overemphasize the importance of the slowest neutrons. We have therefore recalculated a part of the function C for neutrons of room temperature, using for $\beta(v)$ the more accurate expression⁷

$$\text{const. } [K + 1 - (2K + 1)^{1/2}] / K^2 (K = 0.43 \cdot 10^{-10} \cdot v^2).$$

It is seen (Fig. 5, curve C') that the initial slope is less steep, but the intersection with the axis

of abscissae is not considerably shifted. It should be remembered, however, that the expression for $\beta(v)$ depends on the density distribution of those electrons which are responsible for the ferromagnetism of the polarizer material. Since this density distribution is not too well known, any determination of μ_n from the precession of neutrons with a wide velocity band would be somewhat uncertain. For high accuracy, experiments with neutrons of homogeneous velocity will be required.

In Fig. 6 a somewhat different representation of the functions C and S is given. Corresponding values of C and S are plotted as abscissae and ordinates, respectively, and the parameter φ_{kT} is noted along the curve. The polarization effect for any angle α between polarizer and analyzer and for any value of φ_{kT} may then be obtained in the way indicated in the figure. For a given value of φ_{kT} the largest polarization effect is obtained when the analyzer is placed in the direction of the vector R ($\alpha = \varphi^*$). The vector R represents therefore the "mean spin direction" in the beam, and its length is proportional to the polarization effect. If all the neutrons in the beam had the same velocity $v = v_{kT} = (2kT/m)^{1/2}$, then φ^* would be equal to φ_{kT} and the corresponding vector R would rotate at a uniform rate without changing in length, and would represent directly the precession of the neutrons. The velocity distribution of the neutrons has the effect that, firstly, the amount of R decreases with increasing φ_{kT} ("depolarization") and, secondly, φ^* is not proportional to φ_{kT} , but increases more slowly at large values of φ_{kT} ; this is because the slowest neutrons are the first to be depolarized, and the remaining faster ones require stronger fields for precessing through the same angle.

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