Magnetic Field Corrections in the Cyclotron

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(Received March 15, 1938)

It is pointed out that in order to obtain magnetic fields which will allow the ions in the cyclotron to attain high energies it is first necessary to render the field homogeneous within an accuracy of about one part in a thousand. The correction (insertion of "shims") for the most important source of inhomogeneity, namely the decrease in the field near the edge of the pole pieces, is considered. A general method is given for determining the dimensions for any shape of these shims. Results are given for the dimensions of shims in the form of a ring of rectangular cross section placed at the edge of the cyclotron chamber. The magnetic field resulting from the use of such shims is determined and compared with the field of plane parallel pole faces. For the special case of a ring whose thickness is 1/16 the magnetic gap, the field is homogeneous within the required accuracy up to a distance from the edge of the poles of 0.4 of the magnetic gap. The uncorrected field is homogeneous only up to a distance from the edge equal to 0.9 of the magnetic gap.

I. Introduction

It has been realized from the beginning of cyclotron technique that great homogeneity of the magnetic field is absolutely essential for the maintenance of resonance between the Larmor rotation of the ions and the oscillation of the accelerating electric field.[1] A deviation of one percent, e.g., will throw the ions completely out of resonance after 100 accelerations so that they will actually be decelerated instead of accelerated. Now the field produced by plane parallel magnetic poles is always inhomogeneous near the pole edges (cf. Fig. 4 below). Unless a great part of the area of the pole faces is to be sacrificed it is necessary to introduce "shims" near the edge which will have the effect of increasing the field in their neighborhood and thus render the field more homogeneous.

This shimming must be done rather accurately because over-correction must definitely be avoided. An over-correction, i.e., a magnetic field increasing with the distance r from the center of the cyclotron chamber, leads to strong magnetic defocusing of the ions, as was shown in a previous paper,[2] and therefore to enormous losses of intensity. The magnetic field must not increase radially at any point except possibly near the center where the electric field provides some focusing.

Even more exact design of the field is necessary if very high energy ions are to be obtained from the cyclotron. It was shown in I that the impossibility of simultaneously maintaining *exact* resonance and focusing of the ions imposes a limit on the energy obtainable. At the same time it was shown that magnetic fields can be devised which make this energy limit rather large.[3] In order to attain the highest energies, a field such as is given in Fig. 5 of I should be used. However, all such fields are somewhat critical in the sense that they necessarily do not differ by very much from a defocusing field. Hence the deviations from the prescribed field of the field finally obtained must be kept very small, of the order or less than one part in a thousand.

The success in attaining high energies will depend very much on the elimination of undesirable inhomogeneities in the field. The most important source of such inhomogeneity is the decreasing field due to the finite size of the pole pieces. In fact, since the desired field has been defined only inside the cyclotron chamber, the shims required to produce this field can be determined only if the effect of the pole edges is made negligible in the region where the ions move. The following section contains the calculation of the size of shims required to compensate for this edge effect as far out from the center of the chamber as is possible.[4]

[1] Of course, this is apart from the considerations of obtaining very high energies.

[2] M. E. Rose, Phys. Rev. **53**, 392 (1938); hereafter referred to as I.

[3] In the best case, field given in Fig. 5 of I, the energy is $E=2.12\ (V_0AZ)^{\frac{1}{2}}$ Mev where V_0 is the accelerating dee voltage in kilovolts, A and Z the atomic weight and atomic number of the ion.

[4] Of course, there will in general be some inhomogeneity due to the nonuniformity usually present in the large iron

II. Shims for Homogeneous Field

In the following we shall make two assumptions: The first is that the reluctance of the iron can be neglected so that the faces of the magnets (or lids of the cyclotron chamber) form equipotential surfaces.[5] The second assumption is that the shims may be placed on the inner surfaces of the lids of the cyclotron chamber rather than in the air-gap below and above the lids.

Since the magnets are supposed to be axially symmetrical we should introduce cylindrical coordinates with the origin at the center of the cyclotron chamber. Then the magnetic potential V is a solution of the Laplace equation written thus:

$$\partial^2 V/\partial r^2+(1/r)\partial V/\partial r+\partial^2 V/\partial z^2=0, \qquad (1)$$

where z is the vertical coordinate. However for our present purpose, only the field near the edge where r is large is of importance. In this case the second term in (1) will be smaller than the other terms by a factor of order magnetic gap divided by diameter of magnetic poles[6] and hence may be neglected.

In this case we have a two-dimensional Cartesian problem to which we may apply the Schwarz transformation method.[7] We introduce a Cartesian coordinate system in a plane through the z axis, with origin at the edge and in the median plane. Denoting (radial) distances measured inward by x we have

$$\partial^2 V/\partial x^2+\partial^2 V/\partial z^2=0. \qquad (2)$$

We introduce the complex variable

$$\zeta=x+iz \qquad (3)$$

and the potential U conjugate to V (U=constant giving the magnetic lines of force). For our Cartesian problem we have the Cauchy-Riemann relations between the potentials:

$$\partial U/\partial x=-\partial V/\partial z, \quad \partial U/\partial z=\partial V/\partial x. \qquad (4)$$

magnets, and possibly to irregularities arising from machining or assembling the magnets, etc. Obviously corrections for these inhomogeneities must be carried out empirically and it will be assumed that this can be done to the required degree of accuracy.

[5] No currents in the cyclotron chamber.

[6] Cf. the result (27) for the field obtained below and the remark preceding (15). The ratio of the second and first term of (1) is $1/2\pi$ times gap/diameter which for the usual dimensions is of the order of one percent.

[7] Cf. e.g., F. Kottler, *Handbuch der Physik*, Vol. 12, p. 480.

Hence the general solution of (2) is the imaginary part of the complex function W defined by

$$W(\zeta)=U(x, z)+iV(x, z). \qquad (5)$$

The procedure now consists in the establishment of a connection by conformal mapping between an auxiliary variable t and W (potentials) on the one hand and between t and ζ (coordinates) on the other. By elimination of t the potentials or magnetic field can be obtained in terms of the coordinates. The variable t is to assume real values on the equipotential surfaces and particular values are to be assigned at all places where the slope of these surfaces changes i.e., at all the corners of the pole faces. Only three of these particular values (say 0, 1 and ∞) are independent, the remaining values being completely determined by the special geometry. If we regard the pole faces as forming the boundary of a polygon and if the particular value t_n is assigned to the nth corner where the internal angle of the polygon is α_n, then the connection between ζ and t is[8]

$$d\zeta/dt=C_1\prod_n(t-t_n)^{\alpha_n/\pi-1}, \qquad (6)$$

where C_1 is a constant to be determined from the boundary conditions.

As an example we may consider the case of plane parallel poles. In the finite part of the ζ plane there is, of course, one corner on each pole to which the values $t_n=\pm 1$ may be assigned. At this corner the internal angle $\alpha_n=3\pi/2$. At $x=0$, $z=\pm\infty$ we can set $t_n=\pm\infty$ and at $x=\infty$, $t_n=0$, and $\alpha_n=0$. Then we have

$$d\zeta/dt=C_1(t^2-1)^{\frac{1}{2}}/t. \qquad (6a)$$

Since in all cases there are only two equipotential surfaces we have, regardless of geometry, for the connection between W and t (cf. Fig. 1, also Fig. 73 reference 7).

$$dW/dt=C_2/t, \qquad (7)$$

where again C_2 is a constant to be determined from the boundary conditions.

It is convenient to introduce as unit of potential the magnetic potential on the (lower) pole face so that we have as boundary conditions

$$W(t=1)=-i, \quad W(t=-1)=+i. \qquad (8)$$

[8] The index n can assume a series of discrete or a continuous range of values. The latter case is pertinent when the slope of the shims varies continuously.

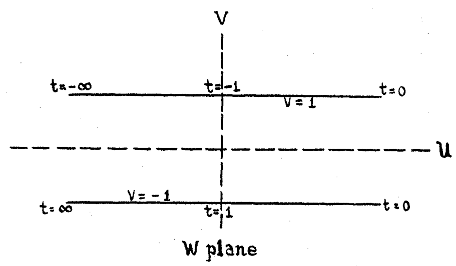

FIG. 1. Conformal mapping of the t plane on the W plane. The real t axis coincides with the equipotentials $V=\pm 1$.

Then we find from (7) and (8)

$$W=-(2/\pi)\log te^{\pi i/2}, \qquad (9)$$

or by the use of (5)

$$t=\exp -\pi/2[U+i(1+V)]. \qquad (10)$$

From the symmetry with respect to the median plane, the values t_n at corresponding corners of upper and lower pole faces are opposite in sign. At these corresponding corners the angles α_n will be the same. In addition, taking $t_n=0$ between the poles at $x=\infty$ at which point $\alpha_n=0$ we may write (6) in the form

$$d\zeta/dt=(C_1/t)\prod_n(t^2-t_n^2)^{\alpha_n/\pi-1}, \qquad (11)$$

where the index n runs over the values on one pole face only.

Introducing the components of the magnetic field

$$H_x=-\partial V/\partial x, \quad H_z=-\partial V/\partial z \qquad (12)$$

we find from (4) and (10)

$$H_z-iH_x=-(2/\pi)(td\zeta/dt)^{-1}, \qquad (13)$$

or from (11)

$$H_z-iH_x=-(2/\pi C_1)\prod_n(t^2-t_n^2)^{1-\alpha_n/\pi}. \qquad (14)$$

We use as unit of length half the magnetic gap so that for $x=\infty$, $t=0$ the magnetic field becomes

$$H_z=1, \quad H_x=0. \qquad (15)$$

We may then determine the constant C_1 and obtain

$$H_z-iH_x=\prod_n(1-t^2/t_n^2)^{1-\alpha_n/\pi}. \qquad (16)$$

Thus it is seen that the magnetic field may be obtained as a function of t or from (10) as a function of the potentials without integration.

From this result we see that the inhomogeneity of the field can be expressed as a power series in t^2. Further, since the field is in any case almost homogeneous inside the cyclotron chamber, U will be proportional to x within very small correction terms (cf. 24, 25 below). Hence from (10) the inhomogeneity is

$$1-H_z=k_1e^{-\pi x}\cos\pi z+k_2e^{-2\pi x}\cos 2\pi z+\cdots, \qquad (17)$$

where $k_1, k_2\cdots$ are coefficients depending on the parameters t_n and hence on the geometry. In general the coefficient k_1 will not vanish and for plane parallel pole face $k_1\sim\frac{1}{4}$ so that the inhomogeneity at $x=1$ in this case is about one percent. To make the field more homogeneous the dimensions of the shims may be made such that k_1 vanishes. In this case the inhomogeneity $k_2e^{-2\pi x}\cos 2\pi z+\cdots$ decreases very rapidly and will be negligible for x larger than unity.

Application to ring shims

A great variety of shapes of shims would be equally suitable for accomplishing our purpose. In many cases it would be possible to make the main term in the inhomogeneity ($\sim e^{-\pi x}$) vanish. Of course, in the various cases, the size of the remaining inhomogeneity, i.e., the coefficient of $e^{-2\pi x}$ would depend on the geometry in different ways. However, since nothing is gained by complication, we consider here only the simplest case of a single ring shim of rectangular cross section on each pole face. The outer diameter of the ring is to be equal to the pole radius, the width of the ring a and its thickness b, cf. Fig. 2. The parameters t_n and internal angles α_n are:

ζ	t_n	α_n	
$\pm i(1-b)$	± 1	$3\pi/2$	
$a\pm i(1-b)$	$\pm t_1$	$3\pi/2$	(18)
$a\pm i$	$\pm t_2$	$\pi/2.$	

In addition at $\zeta=\pm i\infty$, $t=\pm\infty$ and at $\zeta=\infty+iz$, $t_n=0$. From (16) we find

$$H_z-iH_x=(1-t^2/t_2^2)^{\frac{1}{2}}(1-t^2)^{-\frac{1}{2}}(1-t^2/t_1^2)^{-\frac{1}{2}}. \qquad (19)$$

The coefficient of t^2 in the expansion of this expression vanishes if

$$t_2^2=t_1^2/(1+t_1^2) \qquad (20)$$

and with this result we have

$$H_z-iH_x=1-t^4/2t_1^2+\cdots. \qquad (21)$$

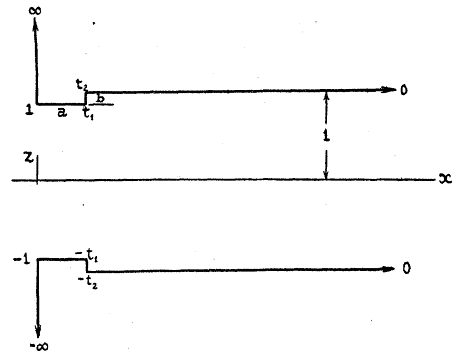

FIG. 2. Conformal mapping of the t plane on the ζ plane. The real t axis coincides with the surfaces of the magnets.

To make the inhomogeneity small, it is desirable that t_1 be not too small. This will mean as may be seen below that a should be small; e.g. $a \approx 0.1$ would be quite satisfactory.

For $d\zeta/dt$ we have from (13)

$$d\zeta/dt = -(2/\pi t)(1-t^2)^{\frac{1}{2}} \times (1-t^2/t_1^2)^{\frac{1}{2}}(1-t^2/t_2^2)^{-\frac{1}{2}}. \quad (22)$$

Upon integrating (22) from $t=1$ to $t=t_1$ we obtain a as a function of t_1. Again integrating (22) from $t=t_1$ to $t=t_2$ we obtain b as a function of t_1, using (20). Eliminating t_1 we have the thickness b in terms of the width a. This final result is given in Fig. 3.

The magnetic field on the median plane is obtained from (22), (13) and (10) with $V=0$.

$$H_z = (1+e^{-\pi U}/t_2^2)^{\frac{1}{2}}(1+e^{-\pi U})^{-\frac{1}{2}} \times (1+e^{-\pi U}/t_1^2)^{-\frac{1}{2}}. \quad (23)$$

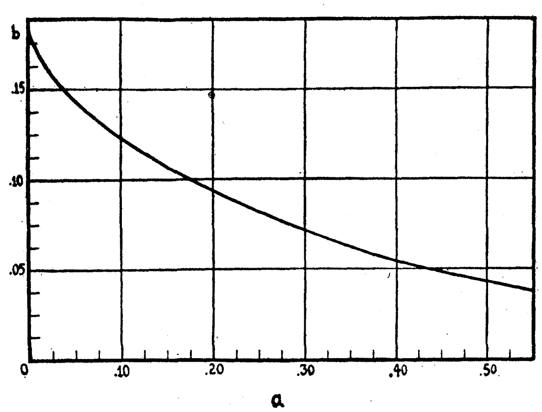

FIG. 3. Dimensions of ring shim for homogeneous magnetic field. Thickness of ring b, width a. The unit of length is half the magnetic gap.

To obtain the field in terms of x we have

$$x = U - U_0 - \int_U^\infty [f(U') - 1] dU', \quad (24)$$

where $f(U)$ is the reciprocal of the expression given on the right-hand side of (23). The constant U_0 is given by

$$U_0 = -a + 2/\pi \log 1/t_2 + \int_0^{t_2} (2/\pi + t d\zeta/dt) dt/t, \quad (25)$$

the integration being taken along the surface of the magnet where t is real. This constant is only slightly geometry dependent and its value is in general about 0.2.

The field obtained in this way is given in Fig. 4 (upper curve) for a special case. In the same figure the field on the median plane for the case of plane parallel pole faces is given. It is seen that the homogeneity of the field is indeed very much improved. The curves also show that the approach to the asymptotic behavior (cf. (17), (24)) of the field is rather rapid. By retention of only the first term in the inhomogeneity ($\sim e^{-2\pi U}$) the error made at $x=1.0$ is only 3 percent of the total inhomogeneity. Thus for larger x we may write

$$U = x + U_0 \quad (26)$$

(cf. (24)) and $\quad V = -z. \quad (26a)$

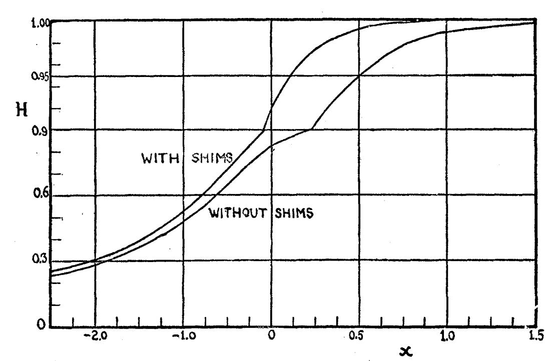

FIG. 4. Magnetic field on the median plane near the edge of the magnets. The unit of field strength is the difference of potential between the magnets divided by the magnetic gap. The field labeled "with shims" applies in the case of a ring shim of width $a=0.095$ and thickness $b=0.125$ in terms of half the magnetic gap as the unit of length. The field labeled "without shims" pertains to the case of plane parallel pole faces. Note the change in scale at $H=0.90$ and at $x=0$.

Then we obtain for the magnetic field in this region

$$H_z = 1 - \tfrac{1}{2} t_1^2 e^{-2\pi(x+U_0)} \cos 2\pi z,$$
$$H_x = \tfrac{1}{2} t_1^2 e^{-2\pi(x+U_0)} \sin 2\pi z. \quad (27)$$

For the uncorrected field of plane parallel pole faces the field inhomogeneity is one-tenth percent at $x=1.8$. For the field with ring shims in the case $b=0.125$, $a=0.095$ we find from Fig. 4 that this same inhomogeneity occurs at $x=0.80$. Thus if the exit slit is placed at this distance (0.4 the magnetic gap) from the edge, the magnetic field over the entire region of motion of the ions will be homogeneous within the required degree of accuracy.

Finally, we may return to a consideration of the assumptions made at the beginning of this section. First of all the assumption of low reluctance of the iron will in general be fulfilled rather well. Of course, it is sufficient if only the cyclotron lids and not the large magnets themselves be of low reluctance, high permeability iron. Secondly, the assumption that the shims be placed inside the chamber against the lids rather than in the air gap need not impose any restriction on the applicability of the results obtained here. Since the shims used to make the field homogeneous may be inserted at the time of construction it should perhaps be not inconvenient actually to place them inside the chamber.

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Some Experiments on the Magnetic Properties of Free Neutrons

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(Received February 5, 1938)

The behavior of partly polarized beams of slow neutrons as regards their precession on passing through homogeneous magnetic fields has been investigated. From the experiments it is concluded that the neutron has a magnetic moment not far from $2\times 1/1840$ Bohr magneton and that the sign is negative. Further, the precession of neutrons inside magnetized iron was investigated; it was found that the field accounting for the observed rate of precession is more than 100 times the actual field strength H and actually of the order of magnitude of the magnetic induction B.

1. Introduction

That a neutron should have a magnetic moment at all, seems somewhat surprising, on account of its being electrically neutral. On the other hand, from the magnetic moments of the proton[1, 2] (2.5 to 2.8 n.m., 1 n.m. = 1 nuclear magneton = 1/1840 Bohr magneton); and the deuteron[2, 3] (0.85 n.m.), a magnetic moment μ_n of the neutron, of about 2 n.m., can be deduced;[4] the sign of μ_n should be negative, that is, the relative position of spin and magnetic moment should be the same as in the (negative) electron. A tentative explanation of this moment, based on the Fermi theory of beta-decay, has been offered by Wick.[5]

A way of measuring, at least roughly, the magnetic moment of free neutrons has been pointed out by Bloch.[6] He showed that the magnetic interaction between neutrons and electrons must have a measurable influence on the scattering of slow neutrons by magnetic atoms or ions (provided the neutron has a magnetic moment of the order of 2 n.m.). Of special interest is the scattering of neutrons from a ferromagnetic substance in

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