A New Analysis of Cosmic Radiation Including the Hard Component*

L. W. NORDHEIM
Duke University, Durham, North Carolina

(Received February 19, 1938)

A representation of all known data on cosmic-ray intensities in their dependence on geomagnetic latitude and on altitude, respectively, depth below sea level, has been obtained on the following assumptions: 1. A soft component consisting of electrons which follow the Bethe-Heitler theory up to the highest energies with a primary distribution E^{-n} (E = energy, n between 3 and 2). 2. A hard component of different nature which in addition to normal ionization is absorbed by a discontinuous process leading also to the production of numerous secondaries with a cross section inversely proportional to E for high energies. The primary distribution in energy is about the same form as for the soft component, and also the absolute numbers of hard and soft primaries are of the same order of magnitude. This picture is furthermore well compatible with the observed secondary and transition effects, especially the Rossi transition curves for showers, and their occurrence below thick layers of heavy material and underground.

I. THE DUALISTIC NATURE OF COSMIC RADIATION

DURING the last years it has become more and more clear that there are two distinct components in the cosmic radiation, the hard and the soft one. They both have electronic charge (positive and negative) but are distinguished by their penetrating power through heavy material, for instance, lead. The soft component is absorbed in a few cm of this material and produces secondaries (showers) in profusion, whilst the hard component can pass through one meter without being reduced by more than about 50 percent. The soft component can be well accounted for by the assumption that it consists of electrons (and photons) which behave according to the theory of radiation.

It is indicated by the following facts that the hard rays cannot possibly consist of electrons of a different, higher energy range for which the theory does not hold any more:

1. The ionization-curvature measurements of Anderson and Neddermeyer[1] and others which seem to show (though it can hardly be taken as definitely proven) the existence of radiating and nonradiating particles of the same curvature.

2. The good representation of many shower phenomena by the "multiplication theory" as worked out by Carlson and Oppenheimer[2] and Bhabha and Heitler[3] which demand the validity of the theoretical formula for radiative effects to very high energies (even in lead) together with the observation that only a small fraction of all ionizing rays produce showers.[4]

3. The absence of equilibrium between the two components as found by Rossi[5] and Auger and others[6] and discussed by Heitler.[7]

4. The absorption curves at high altitudes and different latitudes, as will be discussed in Section II.

In view of this situation the question arises whether it is possible to arrive at a complete description of all the cosmic-ray phenomena by ascribing to the hard component a definite set of properties. The following attempt seems, as a whole, to give a simple picture of the majority of the observed facts without being in contradiction to any particular detail.

It should be realized, however, that such a phenomenological approach can only be of a preliminary nature until a definite theory has been found which connects the high energy phenomena with our other knowledge of the laws of physics. Until this is accomplished it is not possible to prove the uniqueness of a particular picture as at least two functions, *viz.* the primary distribution and the absorption and secondary effects in dependence of energy, can be adjusted

---

* A report on this paper has already been given at the Indianapolis Meeting of the American Physical Society, December 30, 1937.

[1] Anderson and Neddermeyer, Phys. Rev. **50**, 263 (1936); **51**, 884 (1937).

[2] Carlson and Oppenheimer, Phys. Rev. **51**, 220 (1937).

[3] Bhabha and Heitler, Proc. Roy. Soc. **A159**, 432 (1937).

[4] This point is made particularly clear by the extensive investigation of Starr, Phys. Rev. **53**, 6 (1938).

[5] Rossi, *La radiation cosmique* (Hermann, Paris, 1935).

[6] Auger, Ehrenfest and Leprince Ringuet, J. de phys. et rad. **7**, 58 (1936).

[7] Heitler, Proc. Roy. Soc. **A161**, 261 (1937).

to give the desired results. It is, therefore, not surprising that previous attempts have failed so far to converge, and also the present one should not be taken as more than a working hypothesis which might possibly lead to a better understanding of the connections between the observed phenomena.

In the next section we give firstly a discussion of the high altitude measurements in order to be able to separate off the contribution of the soft component.

II. Discussion of the High Altitude Data

The hypothesis of two different components in the cosmic radiation, one of which consists of electrons and photons behaving according to theory, is strongly supported by the intensity *versus* altitude data at different geomagnetic latitudes.

Figure 6 (full lines) contains the observational results as available to the author. Curves I and II have been taken from the ionization curves of Bowen, Millikan and Neher[8] (B.M.N.) at geomagnetic latitudes $\lambda=3°$ and $38°$ converted to vertical incidence by the well-known Gross[9] transformation

$$V(x)=I(x)-x(dI(x)/dx), \tag{1}$$

where x is the depth below the top of the atmosphere (in mass units), I the intensity for all-around incidence (as obtained by an ionization chamber) and $V(x)$ the corresponding vertical intensity. Curve III is the one measured by Pfotzer[10] with a counter telescope at $\lambda=49°$. The units are those used by B.M.N.[8] Pfotzer's data have been expressed in approximately the same units by multiplying his values by a factor 2.7 which gives the correct latitude effect at a pressure of 6 to 7 m H_2O.[11]

The minimum energy for vertical incidence at a latitude λ is given by Stoermer's formula

$$E_{\min}=18\cos^4\lambda\cdot10^9 \text{ ev.} \tag{2}$$

Hence curve I represents the effect of uncharged primary radiation and of charged primaries above about 18 billion electron volts. II includes the charged primaries down to 6.7 and III to 3.1 billion electron volts.

As the percentage of the soft component increases strongly with altitude[6] it is suggestive to compare the observations first with the results which would follow from a suitable primary electron distribution alone. Such a comparison[12] is given in Fig. 1. The full line gives the difference between the curves II and III of Fig. 6 corresponding to the energy range "C" from 3 to 6.7×10^9 ev, and the barbed line the difference between I and II, the energy range "B" from 6.7 to 18×10^9 ev. The dotted curves are calculated from a primary electronic distribution which behaves essentially like $E^{-2.8}$ (E = energy) in the range from 3 to 18×10^9 ev. The details of the calculation are given in the Appendix. The calculated values are the same as for the soft component in Table III for the corresponding energy ranges multiplied by 1.35. For the upper part of the curves the agreement is quite good and certainly within the limits of accuracy of observation and theory. The relative heights of the peaks can, of course, be adjusted by the choice of a corresponding primary distribution so that the only check on theory is given by the shape of the curves which is not very sensitive against a change of the primary distribution within the intervals. One sees that the position of the maxima is given correctly in both energy ranges and that the curves for interval "C" are steeper and narrower than for interval "B." Below about 4 m H_2O both the theoretical curves fall off much

---

[8] Millikan, Neher and Haynes, Phys. Rev. **50**, 992 (1936); Bowen, Millikan and Neher (B.M.N.) Phys. Rev. **52**, 80 (1937).

[9] Gross, Zeits. f. Physik **83**, 217 (1933).

[10] Pfotzer, Zeits. f. Physik **102**, 23, 41 (1936).

[11] This factor gives a latitude effect of $\sim$10 percent at 6.7 m H_2O between $\lambda=49°$ (Pfotzer curve) and $\lambda=38°$ (transformed B.M.N. curve at Fort Sam Houston). It should be emphasized that the data contained in Fig. 6 cannot be considered as too accurate. The transformation (1) magnifies any error of the original measurements, especially when the second term involving a differentiation becomes larger than the first one; this is the case for all $x>4$ m H_2O whilst for smaller x the measurements themselves are necessarily less certain. The assumption of uniform all around incidence on which (1) is based is furthermore not quite correct owing to the complicated nature of the geomagnetic effect, but it seems not to be worth while at the present stage to attempt a more elaborate discussion. Though the points of B.M.N. follow a much smoother curve than those of Pfotzer, curves I and II are not inherently better than III. The difference III–II, of course, is still less certain owing to the empirical conversion of the units.

[12] A similar comparison with nearly the same results has already been made by B.M.N. (reference 8). The argument is repeated here as our conclusions are somewhat different from theirs and in order to obtain data which are needed later.

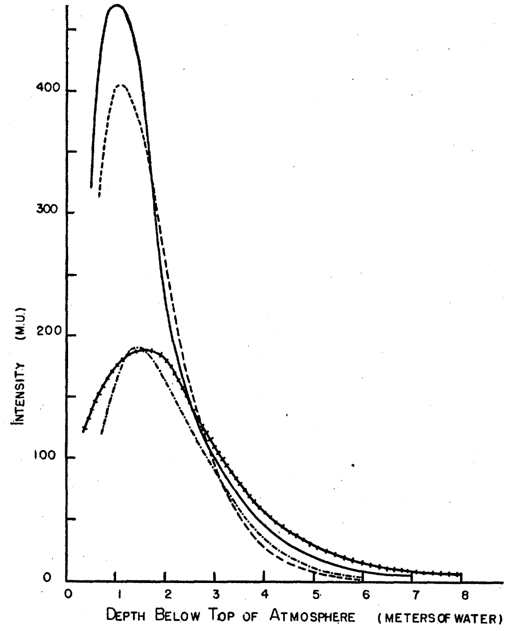

FIG. 1. Field sensitive vertical intensities (dashed, and dot-dashed curves, calculated from primary electron distribution; solid and barbed curves, observed).

more rapidly than the observed ones. According to theory there is no perceptible influence farther down than 6 m H_2O whilst a marked geomagnetic effect is known to reach down to sea level (10 m H_2O) and even somewhat below. This discrepancy cannot be removed, at least not in a simple way, by the assumption of a failure or break-down of the theory. The fit at small depths is equally good and the disagreement for larger depths is equally pronounced for both intervals so that the long tail of the empirical data cannot be ascribed to a break-down at high energies. A "stretching" of the curves along the abscissa (corresponding to a change in the constants of the formulae of radiation theory) to obtain more intensity at lower depths would make the upper regions entirely too broad and shift the maxima too far down. Fig. 1 in conjunction with the observations mentioned in Section I suggests thus strongly a practically quantitative validity of radiation theory up to at least 18×10^9 ev *plus the existence of a separate component of higher penetrating power.*

If we then adopt this interpretation we have, of course, to reduce somewhat the amount of soft radiation in the upper reaches of the atmosphere. We can further conclude that the hard component alone is responsible for the geomagnetic effect at sea level, that it also increases strongly with elevation and that it is still responsible for the greater part of the geomagnetic effect up to about 4 m H_2O.

A few remarks should be added about the third interval "A" pertaining to curve I, Fig. 6, i.e., primary energies over 18×10^9 ev and contributions from uncharged radiation (neutral particles and γ-rays). For this practically every law of absorption at low depths can be obtained by a suitable choice of the primary distribution. The observed absorption coefficient of 0.5 m^{-1} H_2O for the soft component which near sea level is nonfield-sensitive would be given by a primary electronic distribution proportional to $E^{-2.3}$ at high energies, as shown by the author, and a smooth combination of this with an initial $E^{-2.8}$ law for lower energies accounts for practically every property of the soft component.[13, 7] It should be mentioned, however, that according to this view the rest of the soft component near sea level is due to electrons of primary energies of about 10^{12} ev and over. It is quite possible, and even probable that the radiation theory which seems to hold well up to some 10^{10} ev will have to be modified in this region. For instance, the finite mass of the atomic nuclei and their recoil momenta in radiative processes would have to be taken into account. The same final absorption law might therefore be due to a more rapidly decreasing primary distribution together with smaller radiation probabilities at these extremely high energies.

III. GENERAL PROPERTIES OF THE HARD COMPONENT

We consider now a number of facts pertaining to the hard component, which seem to allow some significant conclusions.

1. The apparent absorption of the hard rays is nearly proportional to the mass of the traversed layers, certainly not to Z^2.

2. At sea level about 70 percent of the total radiation belongs to the hard group. The fraction caused by it may even be somewhat higher as it also produces some soft radiation (showers) by

---

[13] Nordheim, Phys. Rev. **51**, 1110 (1937).

secondary processes. The large majority of the more energetic rays over 10^9 ev is certainly hard (i.e., they do not radiate appreciably).

3. The energy distribution at sea level has been measured by Blackett.[14] He finds an about equal number of positives and negatives. Leaving out the particles below 5.10^8 ev the number of which is not very accurately determined and which seem to be mostly electrons, the center of gravity of Blackett's distribution lies around 3×10^9 ev with a slow tailing off at high energies. The particles with energies above 10^{10} ev form about 10 percent, the ones above 20×10^9 ev (not deflected by his magnetic field) about 5 percent of the total. According to 2, we can take these figures as representative for the hard component alone.

4. The geomagnetic effect at sea level shows that 80 to 85 percent of the total, and therefore, (when allowance is made for a field-insensitive contribution due to the soft component) that around 70 percent of the hard rays are not influenced by the magnetic field of the earth. This means, that so much of it must be due to primaries over 18×10^9 ev (or neutral radiation). On the other hand, the geomagnetic effect at sea level extends up to 50° latitude which means that primaries of 3×10^9 ev must already be able, with some probability, to penetrate through the atmosphere.

From 3 and 4 it seems necessary to conclude that also the majority of the hard rays at sea level are not the primaries themselves but of secondary nature.[15] Though the majority of the rays is due to primaries over 18×10^9 ev their average energy at sea level is of the order of 3×10^9 ev. This difference of about 15×10^9 ev cannot be due to a continuous slowing down process as this would be in contradiction to the extension of the geomagnetic effect to 50°; and it would furthermore be difficult to reconcile such a large energy loss with the great penetrating power of the hard component after having come down to sea level. The degeneration of the high primary energies must be effected by a discontinuous process, leading to very extended straggling.

Further indications for the largely secondary nature of the hard rays at sea level are given by the following observations. The east-west effect[15] shows that the majority of the field sensitive rays are due to positive primaries whilst Blackett[14] has found no significant deviation from a half-and-half division. The absorption curves of the hard component in lead interposed between counters seem to be practically the same at all latitudes and also below a thick layer of rock[16] (of about 60 m water equivalent) which means that their energy distribution must be nearly the same under these different conditions. Lastly the existence of a long range secondary radiation has directly been proved by the transition effects for showers under thick layers of heavy material (second maximum of the Rossi curve) and the transition effect air to water or rock as will be discussed in Section IV.

As a last significant fact we take the continuation of the absorption curves to depths below sea level, investigated extensively by Ehmert[17] and Wilson.[18] It is found that the intensity as a function of depth is very well given by simple power laws over very large ranges. For the first 20 m water equivalent below sea level a transition effect[19] apparently takes place. From about 20 m to 250 m (Ehmert's end point) both Ehmert's and Wilson's data follow very closely the function x^{-n} where x is the total mass of the layer above including the atmosphere and the exponent lies between 1.8 and 2 (Ehmert gives 1.87). From 250 m to about 1500 m

---

[14] Blackett, Proc. Roy. Soc. **A159**, 1 (1937).

[15] Also in case that part of the hard rays were due to an uncharged primary radiation they had to be classified, of course, as secondaries. There is, however, no other evidence in favor of such an assumption which would only make the picture more complicated. On the other hand, a considerable fraction of the hard radiation, larger than indicated by the geomagnetic effect at sea level alone, must be due to charged primaries, as the east-west effect persists even at the equator, and also the simple geomagnetic effect increases considerably at moderate altitudes, where the soft component should still not be appreciably field sensitive. For data on the east-west effect compare Johnson, Phys. Rev. **48**, 287 (1935).

[16] Measurements by Morgan and Nielsen, now in progress.

[17] Ehmert, Zeits. f. Physik **106**, 751 (1937).

[18] Wilson, Phys. Rev. **53**, 337 (1938). The author is greatly indebted to Professor Compton and Dr. Wilson for kind communication of their data.

[19] This transition effect could formally be made to disappear by ascribing to the atmosphere a water equivalent of about 15 m instead of the 10 m according to its mass; i.e. with this assumption the $\sim x^{-2}$ law would hold immediately from sea level on. Such an interpretation seems, however, to be unlikely as there is no significant deviation between Ehmert's measurements in water and Wilson's in rock, and it would be difficult to understand, why there should be a considerable difference between air and water, but none between water and rock.

(Wilson's end point) the exponent is somewhat higher (about 2.4) corresponding to a more rapid decrease. Some single points by Barnothy and Forró[20] fall in between the extrapolated $x^{-1.87}$ curve and Wilson's observations.

These results are the more surprising in view of the discontinuous absorption mechanism discussed above. From this one would have expected an exponential type law but the observations do not show the least resemblance to the latter as the apparent absorption coefficient

$$\kappa = -1/V(dV/dx)$$

(V=intensity) changes from 0.1 m^{-1} H_2O near sea level to about 0.003 m^{-1} at a depth of about 1000 m H_2O. Such a slow absorption and apparent hardening of the radiation can, however, result also from a discontinuous absorption mechanism, provided the absorption cross section decreases with energy. The combination of such a behavior with a suitable primary distribution tailing off slowly to high energies might, of course, produce any kind of absorption curve. When we admit, however, that the validity of simple power laws over such large ranges is more than a mere coincidence, we can limit the possibilities so far, as to give at least a model for the observed behavior without too much arbitrariness. This shall be attempted in the next section where various schemes will be discussed.

The absorption law described above constitutes a rather strong argument against a hypothesis put forward by Heisenberg[21] that the cosmic radiation is brought down to the greatest depths by neutrinos. According to his ideas, the neutrinos, generated themselves as secondaries in the atmosphere and higher strata of the surface of the earth, would be distributed over a comparatively narrow energy band, and their absorption by conversion into ionizing tertiaries should consequently follow an exponential law.

IV. THE ABSORPTION LAW FOR THE HARD COMPONENT

We try now to bring the qualitative considerations of Section III—which seem to be well founded—into a quantitative form, which necessarily will be much more uncertain.

The hard rays firstly lose energy by normal ionization, an effect which certainly must be taken into account, when treating the penetration to great depths. At a distance x from the top of the atmosphere the energy of a ray of initial energy E will be

$$E(x) = E_0 - \beta x, \qquad (3)$$

where β is the specific energy loss. β can be considered as a constant since the density of ionization seems to be practically independent of energy.

We furthermore were led to assume an absorption coefficient $\kappa(E)$ dependent on energy. The chance for a primary of energy E_0 to survive down to a depth x will then be given by an attenuation factor

$$A(E_0, x) = \exp\left[-\int_0^x \kappa(E_0 - \beta z)dz\right] = \exp\left[-\int_{E_0-\beta x}^{E_0} \kappa(E)dE/\beta\right] \qquad (4)$$

and the number of rays of energy E from a primary distribution $N(E_0, 0)$ will be

$$N(E, x) = N(E+\beta x, 0) \times \exp\left[-\int_0^x \kappa(E+\beta z)dz\right]. \qquad (5)$$

The energy lost in the absorption processes will reappear through the production of secondary radiation which, as it seems, is very important. We can obtain an estimate of this effect on the basis of the following assumptions.

1. The secondaries have a shorter range than the primaries so that they can be considered to be in equilibrium with the primary beam. This appears to be well justified in our case where we have an absorption coefficient decreasing with increasing energy. (It is, however, absolutely wrong for electrons and photons.) 2. The dissipation of the primary energies is ultimately always effected by ionizing particles all of the same ionizing power β, regardless of their nature. This too seems to be very nearly correct as the fraction of heavy ionizing tracks and of actual nuclear disintegrations in the cosmic radiation seems to be quite small.

---

[20] Barnothy and Forró, Zeits. f. Physik **104**, 744 (1937).

[21] Heisenberg, Zeits. f. Physik **101**, 533 (1936).

With these two assumptions the number $S(E)$ of secondaries in equilibrium with a primary beam of energy E will be simply the energy lost by absorption from the primary bundle over the energy loss of a single ray by ionization, i.e.,

$$S(E)=E\kappa(E)/\beta. \qquad (6)$$

This formula will hold also in case the secondaries are of different nature (hard and soft rays). It is irrelevant whether the secondaries are emitted singly or in bundles. Eq. (6) will also remain correct in case of further multiplication of the secondaries (even with nonionizing links) if only the total conversion of energy takes place in a short enough distance. Of course, it is not possible to give a more detailed discussion of transition effects between different materials without more specific statements regarding the production of secondaries.

With these assumptions the total number of rays penetrating to a depth x becomes

$$\begin{aligned} Z(x)=\int_{E=0}^{\infty} N(E+\beta x)\exp\left[-\int_0^x \kappa(E+\beta z)dz\right] \cdot \\ \times(1+E\kappa(E)/\beta)dE. \qquad (7)\end{aligned}$$

$Z(x)$ is empirically found to be close to x^{-2} (see Section III). Such a behavior can be obtained from (7) in various ways, as there are two adjustable functions ($N(E_0)$ and $\kappa(E)$).

The following cases may be noted.

(a) If the absorption is neglected ($\kappa=0$), i.e., if only ionization is present, a primary distribution $N(E_0)dE_0\sim E^{-n}dE$ will give a $Z(x)\sim x^{-(n-1)}$.

(b) If the energy loss through ionization is neglected ($\beta=0$), a power law for the absorption coefficient $\kappa(E)=\kappa_s E^{-s}$ together with a power law $N(E_0)=E_0^{-n}$ will give asymptotically

$$Z(x)\sim x^{-(n-1)/s}.$$

(c) For the combination of the two effects a *simple* solution is possible only for

$$\kappa=\kappa_1/E, \qquad (8)$$

i.e., an absorption coefficient inversely proportional to energy. This gives for the attenuation factor (4)

$$A=\exp\left[-\int_0^x \kappa_1/(E+\beta z)dz\right]=(E/(E+\beta x))^{\sigma}; \qquad (\sigma=\kappa_1/\beta) \quad (4a)$$

and for $N(E_0)=CE_0^{-n}$

$$Z(x)=\frac{C}{(\beta x)^{n-1}}\,\frac{(\sigma+1)!(n-2)!}{(\sigma+n-1)!}. \qquad (7a)$$

For $n=3$ this would be

$$Z(x)=C/(x^2\beta^2(\kappa_1/\beta+2)). \qquad (7b)$$

The law (8) is practically the only one which combines the previously deduced qualitative features without being too complicated or artificial and seems, therefore, to be very suitable for a discussion of the hard component. But (8) would give rise to very large cross sections at low energies. To avoid this singularity we make the further assumption that the absorption cross section becomes constant below a certain energy β_0, i.e.

$$\kappa(E)=\begin{cases}\kappa_1/\beta_0 & \text{for } E<\beta_0,\\ \kappa_1/E & \text{for } E>\beta_0.\end{cases} \qquad (8a)$$

This law should not be taken too literally. It means simply that below a certain energy β_0 the behavior of the hard rays can be represented by a sort of average absorption coefficient which might well conceal a more complicated dependence on energy while above β_0 a definite tendency to decrease with energy is present. The change in the low energy region does not influence the asymptotic behavior at large depths.

For the practical application of the above hypothesis we define an influence function $I(E_0, x)$ giving the number of rays (primaries and secondaries) at a depth x, due to primaries of initial energy E_0. Using (7) and (8a) we obtain

$$I(E_0x)=\begin{cases} N(E_0,0)((E_0-\beta x)/E_0)^{\sigma}(1+\sigma) & \text{for } \beta_0<E_0-\beta x,\\ N(E_0,0)\left(\dfrac{\beta_0}{E_0}\right)^{\sigma}\exp\left[-\sigma\left(1-\dfrac{E_0-\beta x}{\beta_0}\right)\right]\left\{1+\dfrac{\sigma(E_0-\beta x)}{\beta_0}\right\} & \text{for } E_0-\beta x<\beta_0<E_0,\\ N(E_0,0)\exp\left[-(\kappa_1/\beta_0)x\right]\{1+(\sigma(E_0-\beta x))/\beta_0\} & \text{for } \beta x<E_0<\beta_0.\end{cases} \qquad (9)$$

The total intensity due to primary energies above a given $E_{\min}$ becomes

$$Z(x)=\int_{E_{\min}}^{\infty} I(E,\, x)dE. \tag{10}$$

By proper choice of $E_{\min}$ (compare (2)) we can evaluate also the geomagnetic effect. In (9) β represents the specific energy loss through ionization, β_0 the critical energy in (8a) where the law of absorption changes, and $\sigma=\kappa_1/\beta$ the average number of secondaries in equilibrium with a high energy primary.

V. NUMERICAL REPRESENTATION OF COSMIC-RAY INTENSITIES AT ALL LATITUDES AND ALTITUDES

With the hypotheses about the hard component discussed in the preceding section their primary distribution for medium and high energies is already determined by the absorption curve below sea level as CE^{-n} with n between[22] 2.8 and 3. We extrapolate this distribution also to low energies as the simplest possible assumption. To obtain numerical agreement with the data and units given in Section II we take

$$N(E_0,\, 0)=(10/E_0)^{2.8} \tag{11}$$

(E_0 in billion electron volts). The absorption curve below sea level is then taken care of automatically.[23] To adjust the two available constants in the absorption law (8a) we use the geomagnetic effect at sea level. The following values have finally been assumed

energy loss through ionization
$\beta_0=0.\,3\times 10^9$ ev/m H_2O
critical energy at which absorption law changes $\beta_0=9\times 10^9$ ev
absorption coefficient below β_0; (12)
$\kappa_0=0,\, 5$/m H_2O
absorption coefficient above β_0;
$\kappa(E)=\kappa_1/E=4.\,5/E$ (E in 10^9 ev)
from this the average number σ of secondaries for $E>\beta_0$; $\sigma=\kappa_1/\beta=15$.

Fig. 2 shows the influence curves (9) calculated for the values (11) and (12) for the depths $x=10$ (sea level), 5 and 2 m H_2O. The insert gives the geomagnetic effect at sea level obtained by integrating the influence curve from the corresponding minimum energies (2) on. It has been made[24] to be 20 percent between the equator and the plateau reached at $\lambda=50°$. (A contribution of 2 in our units for the soft component which is independent of latitude at sea level has been added to the total.)

The calculated curve gives very well the type of the observed effect. The smallness of the effect is due to the greater penetrating power and larger ratio of secondaries to primaries of high energy, the pronounced beginning at 50° to a large number of primaries of low energy together with a comparatively high absorption coefficient. The beginning of the plateau at 50° is, of course, due to our choice of β with which just 3×10^9 ev are required for a vertical traversal of the atmosphere. The irregularity at $\lambda=30°$ (from the dip in the influence curve) comes from the break in the functional behavior (8a) of the absorption coefficient. It could be removed, of course, by rounding off (8a). As a matter of fact, such an irregularity in the geomagnetic effect seems actually to exist, though, of course, the interpretation by the change in the absorption coefficient is highly doubtful. We have given no experimental points for comparison as the geomagnetic effect has not been measured very exactly with counters (i.e., vertical incidence). Also the observations with ionization chambers (which give a somewhat smaller value) show large fluctuations due to seasonal and local influences[25] (irregularities in the earth magnetic field).

---

[22] According to the observations of Wilson (reference 18) the absorption exponent changes to slightly higher values below 250 m H_2O. From this follows a somewhat stronger falling off of the energy distribution at extremely high energies. It is not necessary, to take this into consideration for the discussion of the behavior of the cosmic radiation in the atmosphere, as about 60×10^9 ev are required from the ionization losses alone before a ray can penetrate to these depths.

[23] The transition effect air underground (water or rock) mentioned in Section III might be due to any of the following causes or a combination of them: (a) deviation of the primary distribution from (11) at lower energies. A change of the exponent to 2.5 below $\sim 15\times 10^9$ ev would be sufficient. (b) The values of the constants in (8a) in different media might be different. A change in the product $\kappa_1\beta$ (compare (7b)) by about 30 percent would be sufficient. (c) Inadequacy of the assumption of equilibrium between primaries and secondaries, i.e., the secondaries at a given depth would already be created in a higher stratum with greater primary intensity. (d) Change of counting efficiency due to the surroundings by dense material. It seems not possible, at present, to decide between these possibilities.

[24] This corresponds to the figure given by Johnson and Read, Phys. Rev. **51**, 557 (1937).

[25] Compton and Turner, Phys. Rev. **52**, 799 (1937).

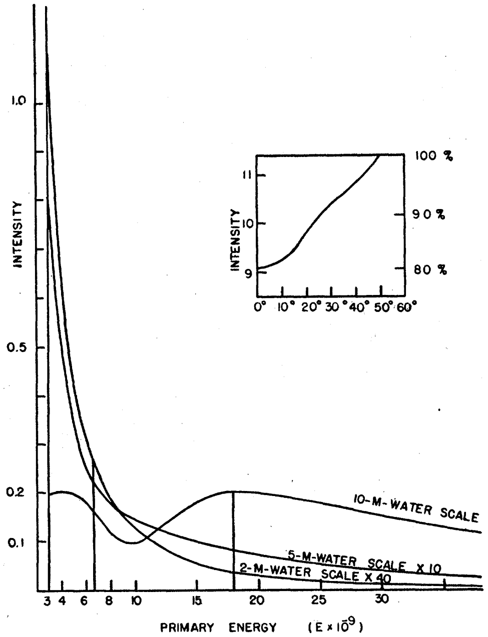

FIG. 2. Influence curves for the hard component. Insert, geomagnetic effect at sea level.

With the constants thus determined the intensity of the hard component can be calculated for all latitudes and altitudes. This has been done by drawing the influence curves (9) for various depths (some of which are shown in Fig. 2) and numerical integration. For the long drawn out high energy tail a suitable transformation has first been made. The obtained values are just of the kind required by the high altitude analysis in Section II.

For the soft component the distribution previously determined from Pfotzer's data[7, 13] were used as described in the appendix. Its form is nearly the same as for the hard component, i.e., an initial $E^{-2.8}$ law, which only falls off more slowly at high energies, the exponent being -2.3 in the region over 10^{11} ev.

The relative numbers of hard and soft primaries can be seen from the first column ($x=0$) of Table III in the appendix. They are of the same order of magnitude with the electrons slightly predominant.

The results of these calculations are shown in Figs. 3 to 6. The first three give the energy ranges "A," "B," "C" of Section II with the contribution of hard component alone, the sum of the hard and soft components as calculated (dotted lines), and the observational data (full lines). It is seen that the hard component too shows a strong increase with altitude especially in the low energy range. A synopsis of the results is given in Fig. 6. The fit is extremely good and certainly within the accuracy of theory and experiment. At very high altitudes the calculated values fall somewhat below the observed ones. An additional soft primary γ-radiation (of energy below 3×10^9 ev) of small intensity would just account for the differences shown in all three curves but as all the data, the experimental as well as the theoretical ones, are least certain in this region, its reality can hardly be taken as proven. The numbers of particles outside the atmosphere before the secondary effects set in are given in Table III. They are much smaller than at the intensity maxima, in qualitative agreement with the recent observations of Curtiss and others.[26]

The data observed in the balloon flights of B.M.N. and Pfotzer are, of course, less accurate at low altitudes as their apparatus are adapted to the high intensities farther up. They can be used only to about 6 to 7 m H_2O. From 5 to 6 m down to sea-level ionization chamber measurements have been given by B.M.N.[27] and Regener and Hoerlin.[28] From a Gross transformation of their curves (which does not give very accurate results in this region) the author finds a ratio of between 3.5 and 4 for the vertical intensities between 10 m and 6.7 m H_2O in good agreement with the calculated figures. (Compare Table III.) On the other hand, Auger and others[6] report a ratio of not more than 2 as measured by counter telescopes. The reason for this disagreement is not clear. It might be due to a change in efficiency of the counter set used. Further precision measurements both with counters and unshielded ionization chambers at small and moderate altitudes would, therefore, be very desirable.

---

[26] Curtiss, Astin, Stockmann, Brown and Korff, Phys. Rev. **53**, 23 (1938). The value of about 1/150 of the maximal ionization reported by these authors seems, however, to be far too low. It is to be noticed that the maximum intensity at our latitudes is, at least, twice as high as near the equator and this difference has to be accounted for by charged primaries. This would require the production of approximately 75 secondaries per primary in about 2 m H_2O which can neither be obtained from radiative multiplication nor any other known effect.

[27] Bowen, Millikan and Neher, Int. Conf. Phys. London 1934.

[28] Regener and Hoerlin, Int. Conf. Physics, London 1934; Hoerlin, Zeits. f. Physik **102**, 652 (1936).

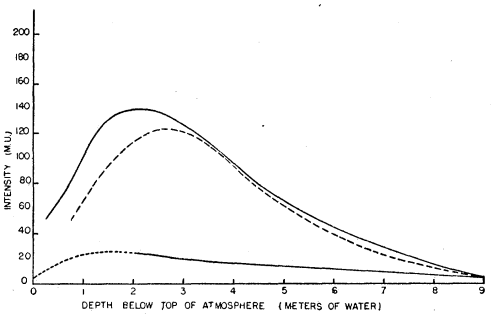

FIG. 3. Vertical intensity due to primaries over 18×10^9 ev. (Range "A.")

A discussion of the measurements obtained with shielded ionization chambers would be possible only with a more detailed knowledge of the secondary processes of the hard component which seem to be of quite an involved nature (see following section). The general results for a shielded chamber in comparison to an unshielded one, i.e., a much less rapid increase of ionization with altitude (due to stronger absorption of the soft component in the shield) and an at least as high latitude effect at moderate altitudes are in qualitative agreement with our picture.[29]

Our formulae give thus a very good representation of the cosmic-ray intensities at all latitudes, altitudes and depths and also of the relative influence of the hard and soft components. They allow us to retain fully the multiplication theory of showers and explain therefore the general parallelism[30] between the intensities of showers and the soft component as already pointed out by Heitler[7] and the author.[13]

Some remarks should be added on how far the assumed constants could be varied without destroying the agreement with experience. With

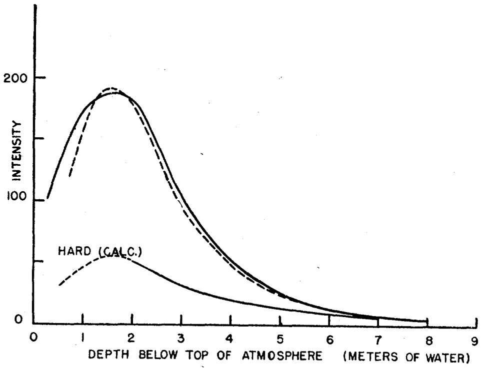

FIG. 4. Vertical intensity due to primaries between 6.7 and 18×10^9 ev. (Range "B.")

the power law (11) for the primary distribution and the geomagnetic effect at sea level the constants are actually fixed within rather narrow limits. The comparatively high value for the absorption constant κ_1 in (8a) (and with that the high value for the average number of secondaries σ) is necessary to account for the smallness of the geomagnetic effect near sea level. The high value for β_0 has then to be taken in order to keep the absorption coefficient for primaries of low and medium energy sufficiently down to give them a chance to penetrate through the atmosphere. With this the amount of hard radiation in the atmosphere is completely determined. Only the constant β for the linear energy loss could be changed in rather wide limits. It has been found that all calculated intensities remain the same within a few percent for all values of β between 0.2 and 0.4×10^9 ev/m H_2O (the assumed value was 0.3) provided the other constants (κ_1, β_0) are kept fixed. ($\sigma=\kappa_1/\beta$ has, of course, to be changed accordingly.)

However, one has to remember that the primary distribution (11) is made necessary by the absorption curve underground only for energies[31] above 15 to 20×10^9 ev. A less pronounced increase in the number of primaries below this energy would lead to smaller values for κ_1, σ and β_0 and a somewhat smaller geomagnetic effect from the hard component at moderate altitudes. It would furthermore offer an explanation for the transition effect air-underground (compare

---

[29] The filtering of part of the soft component will increase the geomagnetic effect below 5 m H_2O. The absorption of the hard component in the shield will tend to decrease the geomagnetic effect. The balance of these two effects might produce an apparent insensitivity of the geomagnetic effect against thickness of shielding as reported by some observers.

[30] The showers produced by the hard component certainly play an insignificant role in the upper atmosphere. At sea level, however, we estimate that between 1/4 and 1/2 of the soft rays and showers are actually generated by the hard component (which are included in our figures for the latter). The small latitude effect for showers as reported recently by Neher and Pickering, Phys. Rev. **52**, 111 (1938) might be well due to this portion. The absence of an exact parallelism of their curve to the one for single rays (if it really exists) cannot be taken as an argument against our view as the shower production of the hard rays might well depend on energy.

[31] The maximum of the influence curve for a depth x occurs at the energy $E_{\max}=\beta\kappa\sigma/n$ (n = exponent in primary distribution) and the greatest contribution comes from energies $>E_{\max}$.

reference 22). These features would facilitate the representation of the observational data.[32] We have, however, refrained from introducing such modifications in order not to make too many hypotheses *ad hoc*.

VI. The Secondary Effects of the Hard Component

We have, so far, not attempted any detailed description of the absorption mechanism of the hard rays. From our general discussion we deduced a number $\sigma = 15$ of secondaries[33] in equilibrium with energetic hard primaries, and it remains to be seen whether such a high value is compatible with our other knowledge. It allows firstly a natural explanation of Blackett's energy spectrum.[14] Practically all the rays below 10 to 15×10^9 ev will be secondaries according to our view and the ratio of rays below and above this is of the order 10 to 1. The greater part of the total intensities at sea level and underground is then carried down by the rays of high energy which are accompanied by a secondary spectrum which does not depend much on the primary energies and is therefore nearly the same everywhere. The low value of the ionization near the top of the atmosphere is also an indication for a high number of secondaries, though only of a qualitative nature.

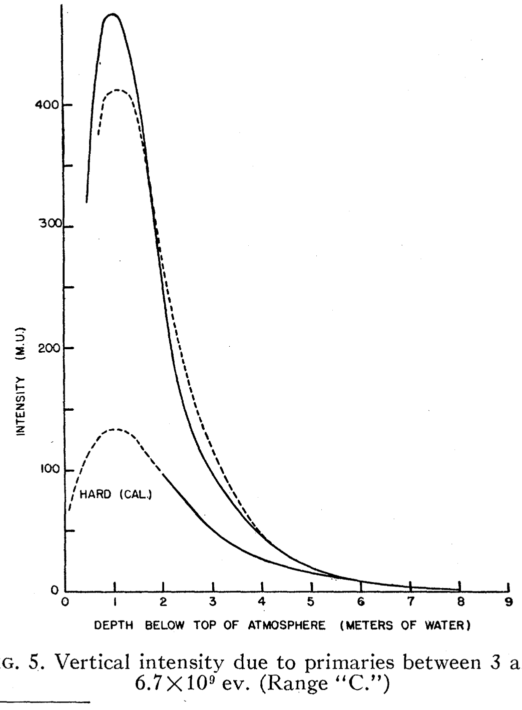

FIG. 5. Vertical intensity due to primaries between 3 and 6.7×10^9 ev. (Range "C.")

As for the chance of a direct observation of the secondary processes we note first that our maximum absorption coefficient (holding for the majority of observable rays) is of the order of 0.5 m^{-1} H_2O. This means that such an event should happen in the average after traversing a layer of about 200 g/cm^2, i.e., for instance, about 20 cm of lead. These processes will be in most cases indistinguishable from electronic processes (forks or showers) and the above figure seems to be low enough to account for the small ratio of showers to single rays observed in cloud chambers.

The hard rays have to be responsible also for the showers observed below thick layers of heavy material, i.e. the tail end of the Rossi curves, and for showers underground. These showers seem to be in general also of electronic nature as shown by

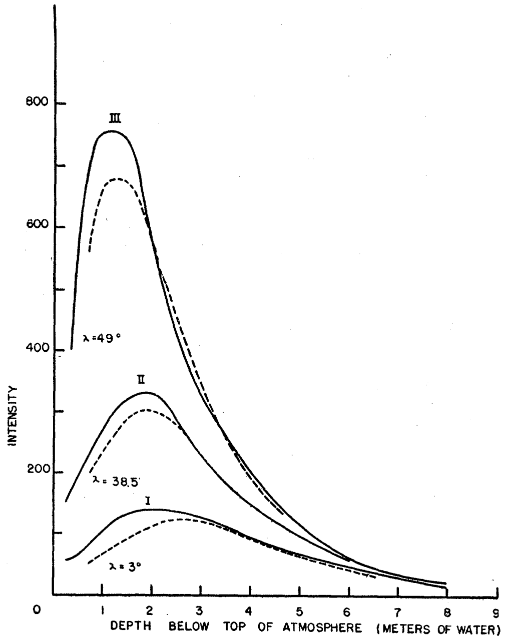

FIG. 6. Vertical intensity in the atmosphere at different latitudes; full lines observed, dotted lines calculated.

---

[32] B.M.N. (Abstract, Phys. Rev. **53**, 329, (1938)) have reported that their newly measured absorption curves at $\lambda = 50°$ and 60° show only a very small difference even at high altitudes. This would indicate definitely a limitation of the primary distribution at lower energies.

[33] This figure has to be taken as entirely provisional. It could well be lowered for the following reasons. (a) change in the primary distribution as discussed at the end of last section. (b) not all absorbed energy goes into production of normally ionizing rays. (c) the average linear energy loss is higher than assumed.

the transition effects between different materials.[34] To explain this it is only necessary to assume that in some of the absorption processes at least one electron or photon can be produced which automatically will appear as a shower if it is energetic enough. Since the number of soft particles in equilibrium with the hard component (as found at some meters underground when the rest of the effect of soft primaries has apparently already been filtered out) seems to be of the order 10 to 20 percent, about the same fraction[35] of the absorbed energy should go into production of soft radiation following from the argument in Section IV.

This picture of a composite effect of a hard secondary radiation of average range of about 200 g/cm^2 and a soft one corresponding to electronic nature, finds considerable support in the recent experiments of Bothe and Schmeiser.[36] They find the "second maximum" of the Rossi transition curve to come out much more markedly[37] for showers of small angular divergence so that its existence is now beyond doubt. Such a maximum can only be due to a transition effect of rays of corresponding range which agrees remarkably well with our absorption coefficient for the less energetic hard rays. On the other hand, the final shower rays, also emerging from the region of this second maximum are highly absorbable, i.e., they are of "soft" nature.

The absorption mechanism of the hard component is, thus, certainly of a rather complex nature. It seems to be premature to attempt a detailed analysis of these secondary effects. We therefore leave it open whether the absorption mechanism consists in the emission of single rays, bundles or even explosion phenomena, whether the emission of hard and soft radiation occurs in the same or different events, and whether the nuclear disintegration processes sometimes observed in the cloud chamber are representative of its nature or belong to a different category.

The chief point of the foregoing considerations was to show the compatibility of our analysis with the observed effects and it seems even that it offers an explanation of many of them in a very natural way.

VII. Conclusions

The picture at which we finally arrive might be summarized as follows: There are two different components, electrons (always accompanied by photons) which behave according to the theory of radiation, and hard rays. The latter are absorbed by a discontinuous mechanism in which both hard rays and soft ones are emitted. Their absorption coefficient is more or less constant up to energies of some billion electron volts and decreases thereafter with energy. The primary distributions of both kinds are very similar[38] and nearly a power law E^{-n} with n between 2 and 3. Also the absolute numbers of hard and soft primaries are nearly the same. As a matter of fact, with only slight deviations from the assumed simple form of the absorption laws or with small changes in the observed intensity curves, even an exact coincidence between the

---

[34] Morgan and Nielsen, Phys. Rev. **52**, 564 (1937) have shown that the effects of thin plates of Pb under thick plates of Fe and *vice versa* conform very well to the predictions of Carlson and Oppenheimer (reference 2). The absence of the first Rossi maximum for showers underground or below thick layers of light material (concrete, ice) is simply an effect of geometry. The multiplication distance in these materials (40 cm in H_2O) is still small enough so that all soft radiation (electrons and photons) will already appear in bundles in contrast to air with a free path of several hundred meters. This difference, therefore, constitutes an argument for and not against the multiplication theory of showers.

[35] According to Ehmert and Wilson (references 17 and 18) the proportion of showers to single rays increases somewhat when going to very great depths. This might be due to a dependence of shower production on energy as the average energy of the rays will increase with depth.

[36] Bothe and Schmeiser, Naturwiss. **25**, 669, 834 (1937). For the proof of existence of a hard secondary radiation compare also Maass, Ann. d. Physik **27**, 507 (1936).

[37] That this second maximum is, in general, so little pronounced must be caused by the fact that the absorption coefficient is apparently very nearly proportional to the mass of the material so that the distribution of the hard rays does not change very much when going to another medium.

[38] This similarity of the two distributions is somewhat surprising as they have been derived from quite different arguments, for the electrons from the high altitude data (shape of the absorption curve and geomagnetic effect) and for the hard component from the absorption measurements underground. The two distributions might even be exactly the same within the limits of error of our determination. This likeness is suggestive perhaps of a common origin. It also seems rather certain that the distribution must fall off rather slowly with energy, i.e. not exponentially but with a power law. E^{-3} would still be possible according to our analysis though we obtain a somewhat better fit with $E^{-2.8}$. As from ordinary processes one would expect rather a Maxwellian kind of distribution, a power law might point to a cosmological origin.

two primary distributions might be effected.[39]

The given figures and tables show that it is possible to obtain on these lines a more or less quantitative description of the intensities of both components at all depths, altitudes and latitudes, and at least a qualitative understanding of the Rossi transition curves.

In judging the success of this attempt, it should be borne in mind that throughout the simplest possible assumptions have been made, and complications such as irregularities in the primary distributions and in the behavior of the absorption coefficient have been avoided. Though the qualitative features seem in most cases to be well founded, the numerical details will in course of time certainly need considerable readjustment, and it might well be that some of the facts which we accepted as fundamental will turn out to be a premature generalization.

With all due reserve as to the final meaning of our speculations a few general remarks might be added. This analysis does not say anything about the nature of the hard particles. We only have to conclude that their mass must be large compared to the mass of an electron as otherwise they would radiate strongly[40] and the energy loss through radiation would increase with energy, in contrast to the behavior of our absorption coefficient. A mass of about a hundred times the electronic mass, as suggested by some recent observations,[41] would be amply sufficient while a protonic nature seems only to be excluded by the ionization curvature measurements of Anderson,[1] Street and Stevenson,[42] and the failure to find slow protons in sufficient amount in the cosmic radiation.[43]

In view of the experimental uncertainty about the hard component two other possible interpretations should be mentioned. It is also compatible with our present knowledge that the primaries belonging to the hard rays are of a different nature than the secondary rays observed near sea level. To be more specific they might be protons to which the absorption coefficient κ_1/E might belong. They, in turn, would generate a new particle to which belongs our average absorption coefficient at lower energies. Or else the hard rays might be created by electrons of high energy in addition to ordinary radiative processes in which case most of them would come from the upper region of the atmosphere where the electronic intensity is highest.[44] From the point of simplicity, however, it seemed to be desirable, to discuss the hard rays as they appear, i.e., as a definite separate primary group.

APPENDIX: NUMERICAL DATA

The calculation of the contribution of the soft component has been carried out in the same way as by Heitler.[7] The total number of electrons (negatives and positives) with an energy larger than E, due to a primary electron of energy E_0 after traversing a layer of thickness l, can be represented by a function $Z(l, y)$, where $y = \log E_0/E$, tabulated in Table I.

Some new values have been added to those given by Heitler, part of which have been calculated, the others having been obtained by logarithmic interpolation. For the atmosphere the unit for l is 280 m standard air = 0.35 m water equivalent. The final energy E has to be chosen so that there is no appreciable multiplication below it, i.e., for air $E = 0.15 \times 10^9$ ev.

Our adopted primary distribution for the soft component is listed in Table II, also in a logarithmic energy scale. The total number of rays at depth l, coming from a certain

---

[39] The number of primaries (column $x = 0$ in Table III) is not very exactly determined by our procedure and might easily be in error by a factor of order 2. In using the Bhabha-Heitler influence function $Z(l, y)$ (compare Table I) the low energy electrons with $E < 0.15 \times 10^9$ ev have been neglected. As the distribution in energy of the soft rays does not change much with altitude (with the exception of the very highest layers) the inclusion of these rays of low energy will approximately give a constant factor to the soft total and the number of primaries should be reduced accordingly. The number of hard rays, on the other hand, depends essentially on the average number of secondaries σ which is also rather uncertain (comp. footnote 33) and a lower value of σ would require a higher number of primaries. Both these effects would work in the direction of equalizing the two primary distributions.

[40] Though the energy loss through radiation cannot contribute much to the absorption coefficient it might possibly be responsible for some of the shower production.

[41] Street and Stevenson, Phys. Rev. **52**, 1003 (1937); Nishima, Takeuchi and Ichimiya, Phys. Rev. **52**, 1198 (1937).

[42] Street and Stevenson, Phys. Rev. **51**, 1005 (1937) (A).

[43] Montgomery, Montgomery, Ramsay and Swann, Phys. Rev. **50**, 403 (1936).

[44] This point of view has recently been strongly advocated by B.M.N., Phys. Rev. **53**, 217 (1938). The arguments used by these authors are, however, very similar to some in this paper and entirely compatible with our analysis. This shows again the impossibility to prove the uniqueness of a phenomenological picture as long as the physical laws regarding the hard component are unknown (note added after completion of the manuscript).

TABLE I. *Influence function for electrons* $Z(l, y)$.

l \ y	3	3.5	4	4.5	5	6	7	8	9	10	11	12	13
2	2.1	2.6	3.4	4.4	5.8	10	17	21	26	31	33	34	
3	2.7	3.6	5.0	6.9	8.9	15	26	43	63	76	89	96	
4	2.4	3.6	5.4	7.9	12	22	38	59	84	140	190	250	
5	2.2	3.2	4.9	7.4	13	25	51	98	160	250	360	550	
6	1.3	2.2	4.0	7.2	13	27	66	120	220	370	620	1000	
7	0.9	1.7	3.2	6.2	12	32	68	140	300	540	830	1400	
8.5	0.43	1.0	2.2	4.7	9.3	29	73	170	400	830	1600	2800	
10	0.22	0.63	1.5	3.5	6.6	24	76	200	500	1150	2200	4100	
11.5	0.1	0.28	0.78	1.9	4.1	18	61	180	540	1250	3000	6000	
13	0.05	0.15	0.40	1.0	2.6	12	46	160	480	1300	3500	8300	
15	0.02	0.06	0.17	0.49	1.2	6.4	27	100	360	1150	3200	9300	
17	0.008	0.02	0.07	0.20	0.53	3.1	17	71	260	910	2800	7800	
19	0.003	0.008	0.03	0.08	0.21	1.5	8.7	40	170	650	2100	6000	17000
22		0.002	0.006	0.019	0.056	0.45	3.0	16	73	310	1050	3000	8100
25			0.001	0.004	0.015	0.14	0.95	5.6	30	140	500	1600	4900
29					0.002	0.023	0.21	1.5	8.5	42	180	700	2300

TABLE II. *Primary electronic distribution* $F(y)dy = F(E_0)dE_0/E_0$.

y	3	3.5	4	4.5	5	6	7	8	9	10	11	12	13
E_0 in 10^9 ev	3	5	8.2	13.5	22	60	165	450	1200	3300	9000	2.5×10^4	6.7×10^4
$F(y)$	250	89	33	12.8	5.0	0.87	0.22	0.062	17×10^{-3}	4.7×10^{-3}	1.3×10^{-3}	3.6×10^{-4}	1×10^{-5}

TABLE III. *Cosmic-ray intensities for the different latitude intervals at various depths.*

DEPTH BELOW TOP OF ATMOSPHERE IN m H_2O		0	2	3.3	5	6.7	10
Interval A ($\lambda = 0°$) $E_0 > 18\ 10^9$ ev	soft	2.2	90	97	49	18	2
	hard	1.9	22.5	18	14	11	7.1
	total	4.1	112.5	115	63	29	9.1
Interval B $6.7 > E_0 > 18\ 10^9$ ev	soft	24	125	62	10	0.5	—
	hard	9.5	53	29	13	7	1.6
	total	33.5	178	91	23	7.5	1.6
Interval C $3 > E_0 > 6.7\ 10^9$ ev	soft	96	93	48	5	0.3	—
	hard	37	95	42	16	6.4	0.7
	total	133	188	90	21	6.7	0.7
Intensity at $\lambda = 49°$	soft	122.2	308	207	64	18.8	2
	hard	48.4	170.5	89	43	24.4	9.4
	total	170.6	478.5	296	107	43.2	11.4

interval of primary energies, is then

$$N(l) = \int_{y_1}^{y_2} F(y) Z(l, y) dy. \tag{13}$$

To the limits of our intervals A, B, C correspond the values

A: $E = 18\times10^9$ ev corresponding to $y = 4.75$,
B: $E = 6.7\times10^9$ ev corresponding to $y = 3.75$,
C: $E = 3\times10^9$ ev corresponding to $y = 3$.

The integrals (13) have been evaluated graphically. Table III shows some of our final results from which the Figs. 3 to 6 have been drawn. It gives the calculated intensities for the soft and hard component separately for the three intervals at various depths below the atmosphere. The values for $x = 0$ give, of course, the number of primaries. A small fraction (perhaps 15 percent) of the figures for the hard group will actually be soft rays, though produced by hard primaries and in equilibrium with them. For instance, 1.4 of the 9.4 for the "hard" component at sea level ($x = 10$) might be electrons, which brings the percentage of highly absorbable rays up to about 30 percent as observed. At 6.7 m H_2O corresponding to an elevation of 3500 m, perhaps 3.5 of the hard intensity of 24.4 might be soft and the percentage of the soft rays is then about 50 percent as it should be. The increase of the total intensity between sea level and 3500 m is from 11.4 to 43.2 that is 1 to 3.8 in agreement with the values deduced from ionization chamber measurements. Thus our formulae give also the correct behavior for the intensities of the hard and soft component in the range between sea level and 3500 m altitude, which is not covered by the data summarized in Figs. 3–6.