On the Electromagnetic Properties of Nuclear Systems

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If one supposes that a charge bearing field is responsible for nuclear forces, the charge and current of this field must be considered in describing the electromagnetic properties of nuclei. In this paper we discuss the extent to which these electromagnetic anomalies permit phenomenological description in terms of the charges and magnetic moments of the single heavy particles and the forces acting on them. We are guided in this discussion by the formalism of the electron neutrino field theory and that of the dynaton theory proposed by Yukawa. Apart from the known effect of exchange currents on electric dipole radiation, the most marked differences from the phenomenological treatment appear in the estimates of magnetic dipole radiation and magnetic spin dependent forces. The formalism used is developed in the Appendices. Appendix I gives the proof of Serber's result on the repulsive nature of the field theoretic interaction between neutron and proton in the deuteron. Appendix II considers the electric properties of heavy particles in more detail, and shows that Siegert's theorem, which connects the exchange current with the exchange interaction, is valid for the dynaton theory. Appendix III discusses more completely the magnetic properties of heavy particles.

IT has been suggested that the finer features of the forces between neutrons and protons might be discovered if one were to study processes involving the interaction of these particles with the electromagnetic field, such as the photodisintegration of the deuteron[1] or the radiative capture of neutrons in hydrogen. The assumption has generally been made that it is possible to assign the various electromagnetic properties, such as charge and magnetic moment, to the individual heavy particles in a purely phenomenological manner. Thus the charge and all of the electric moments of the neutron were taken to be zero, while a magnetic moment of $\mu_N \sim -2.0$ n.m. (nuclear magnetons) was assumed. The proton was described by its charge $+e$ and a magnetic moment $\mu_P \sim 2.5$–2.8 n.m. It was then assumed that when two such particles were brought close together, as in the bound states of the deuteron or in heavier nuclei, these properties would remain the same as for the isolated particles; it was also assumed that radiative transitions could be calculated by the use of the electromagnetic moments characteristic of the single heavy particles, independently of the states of the nucleus involved. Again, it was thought that the total nuclear magnetic moment could be calculated additively from contributions of the constituent single particles. There is, of course, no *a priori* reason why these things should not be so. One has, however, been led to believe that a field theory of some sort might be found which would afford a description not only of the nuclear forces, but also of the magnetic properties of the single heavy particles. With such a view, one would be quite prepared to expect a "distortion" of these properties by the presence of other heavy particles. In this paper, we shall discuss a number of these questions in order to see in how far such field theories permit of a phenomenological development of nuclear physics in terms of the properties of particles with forces acting between them.

No consistent and satisfactory example of a field theory of nuclear forces exists at present. Several attempts[2] at one were made using modifications of the electron-neutrino field which was introduced by Fermi[3] to describe beta-decay. In order to obtain strong short range forces, and to have a convergent theory, it is here necessary to use a coupling energy between the heavy and light particles which contains a

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[1] G. Breit and E. U. Condon, Phys. Rev. **49**, 904 (1936).

[2] A. Nordsieck, Phys. Rev. **46**, 234 (1934); I. Tamm, Nature **133**, 981 (1934); D. Iwanenko, ibid.; C. F. v. Weizsäcker, Zeits. f. Physik **102**, 572 (1936); M. Fierz, Zeits. f. Physik **104**, 553 (1937); J. Solomon, J. de phys. et rad. **8**, 179 (1937); N. Kemmer, Phys. Rev. **52**, 906 (1937); H. Primakoff, Phys. Rev. **51**, 990 (1937).

[3] E. Fermi, Zeits. f. Physik **88**, 161 (1934).

finite distance operator,[4] and this makes it impossible to set up a quantum theory of the field which is covariant, Hamiltonian and gauge invariant. The presence of a finite distance operator is particularly inconvenient for our purposes, because the differential conservation laws of charge and current are rendered invalid, thus destroying the consistency of the electromagnetic calculations. If one allows convergence factors (as is necessary to obtain finite results), the interaction between a neutron and a proton in a symmetric state is found to be repulsive,[5] in disagreement with the known stability of the triplet state of the deuteron (see Appendix I). Then also, since like particle forces arise only in a higher order than do unlike particle forces, they turn out to be far too small to agree with the experiments on proton-proton scattering. The theory can, however, be made to assign reasonable magnetic moments to the heavy particles without too much artificiality.

Another attempt at a field theory of the interaction of nuclear particles was made by Yukawa.[6] He introduced spinless particles obeying Bose-Einstein statistics, possessing a mass m between the masses of electron and proton and a charge $\pm e$, the charge of the proton. These "dynatons" were assumed to obey the scalar relativistic wave equation which has been investigated by Pauli and Weisskopf.[7] Evidence has recently been found[8] for the presence in cosmic radiation of particles with masses of this order of magnitude. This field theory is more nearly convergent than the electron-neutrino field theory, and for its interpretation, no finite distance operators are needed. However, the difficulties of the repulsive neutron-proton forces and the small like particle forces remain; furthermore, one is driven to very artificial devices in order to obtain magnetic moments for the heavy particles from the light particle field.

More promising is the recent proposal of Teller[9] in which like and unlike particle forces arise in the same order. This suggestion derives nuclear forces from a neutral electron-positron field, coupled to the heavy particles by an energy large compared to the kinetic energy of the pairs, and involving a finite distance operator. On this theory, even the dynamical problem of the heavy particles cannot in general be formulated phenomenologically, i.e., without the explicit consideration of the light particles; this will be so *a fortiori* for the radiative properties of the system.

Because of the very many difficulties involved in the formulation of such field theories, we shall not regard their predictions too seriously. At best they can indicate in what situations caution may be necessary in the phenomenological description of nuclear processes. We shall begin with a qualitative discussion of the various questions raised, occasionally quoting the results of field theoretic calculations, some details of which may be found by the interested reader in the Appendices.

Let us first discuss the electromagnetic properties of a single heavy particle, say a neutron, according to such a field theory. On either field theory one may introduce a dimensionless parameter g to measure the strength of the coupling between heavy and light particles. The neutron-proton forces arise in order g^2, and we will always adjust the value of g to give forces of the observed order of magnitude. The existence of this coupling means that there is a certain probability of finding the neutron dissociated into a proton and a negative light particle. If we restrict our consideration to the expectation values of the charge and current for states of the neutron involving periods long compared to the relaxation time $\tau \sim a/c$ of the light particle cloud, where a is the range of nuclear forces, the charge density is spherically symmetric and all of its electric moments about the heavy particle vanish. The expressions for charge density and total charge may be found in Appendix II. As long as the coupling used does not involve the heavy particle spin, the low frequency components of

[4] By a finite distance operator or convergence factor in a coupling, it is meant that values of the light particle wave functions at finite distances from the heavy particles are involved in the interaction energy.

[5] R. Serber, Phys. Rev. **53**, 211 (1938).

[6] H. Yukawa, Proc. Phys.-Math. Soc. Japan **17**, 48 (1935); J. R. Oppenheimer and R. Serber, Phys. Rev. **51**, 884 (1937).

[7] W. Pauli and V. Weisskopf, Helv. Phys. Acta **7**, 709 (1934).

[8] S. H. Neddermeyer and C. D. Anderson, Phys. Rev. **51**, 884 (1937); J. C. Street and E. C. Stevenson, Phys. Rev. **52**, 1003 (1937); D. R. Corson and R. B. Brode, Phys. Rev. **53**, 215 (1938).

[9] We are indebted to Professor Teller and Professor Oppenheimer for opportunity to see the paper of Critchfield and Teller prior to publication.

the current density also vanish, and therefore fail to give any supplementary magnetic moments to the heavy particles.

If now we consider frequencies ν which although still small compared to the reciprocal of the relaxation time, a/c, are not negligible, the spherical symmetry of the charge distribution will be destroyed and the neutron may emit electric quadrupole radiation. The quadrupole moment will be small. For in the first place, the total charge involved is small, of the order $e\hbar/(Mca)$; this is true even on the dynaton theory, as only dynatons of relatively low energy and consequent long relaxation time can contribute to the distortion of the charge distribution. In the second place, the separation of positive and negative charge in the neutron will be of the order $a(\nu a/c)$, and the ratio of this distance to the wave-length of the radiation is $(a/\lambda)^2$ which will be small unless one considers radiation of frequency comparable to that necessary to produce photodisintegration of the neutron.

It is true that in a nucleus, the forces may be great enough to induce dipoles of the order $a(e\hbar/Mca)\sim e\hbar/(Mc)$. Even these, however, for wave-lengths large compared to a will give completely negligible corrections to the radiative moments. Far more important are the distortions of the charge distribution induced by the presence of neighboring heavy particles.

Let us turn now to the question of the origin of the magnetic moments of the heavy particles. As is well known, the Dirac equation does not describe these correctly. One can, of course, modify this equation by the inclusion of the so-called neutrino terms[10]

$$\eta\sigma_{\mu\nu}F_{\mu\nu}\psi,$$

where ψ is the wave function of the particle, $F_{\mu\nu}$ is the electromagnetic field tensor, $\sigma_{\mu\nu}$ is the six vector spin matrix, and η is a constant which determines the value of the extra magnetic moment. The consequences of this for the singlet-triplet splitting in the deuteron are discussed below. Another possibility, first pointed out by Wick,[11] is that the light particle cloud which surrounds a heavy particle may have a resultant magnetic moment. For this to be possible, it is clearly necessary to couple the magnetic moment of the light particles with the spin of the neutron or proton. In the electron-neutrino field theory, this can easily be done, and one can use such a coupling to calculate the charge and current density about a heavy particle. The current density of course no longer vanishes, although it is divergence-free, and if the heavy particle is at rest, there is a current distribution that is symmetric about its spin axis. By suitably choosing the coupling, the neutron and proton may be given magnetic moments quite independent of one another; in particular the moments may be equal in magnitude and opposite in sign. (See Appendix III.) On the other hand, the scalar dynaton which we have considered till now cannot be coupled to the spin of the heavy particles so as to give them a magnetic moment, unless one introduces a finite distance operator. This is so because the scalar dynaton has no spin magnetic moment, and for there to be an orbital magnetic moment,[12] the dynatons would have to be created at a finite distance from the heavy particle. However, one can introduce in addition to the scalar dynaton another kind of dynaton possessing unit spin and described by a six vector wave function.[5, 13] Alternatively, one may think of a single particle existing in both scalar and six vector states. Both types of dynatons are coupled to the heavy particles and to one another through the electromagnetic field so as to give them an intrinsic magnetic moment of arbitrary magnitude and sign. (Appendix III.) This coupling scheme gives mixed Heisenberg and Majorana forces between neutron and proton, and when the constants are properly adjusted, gives the anomalous magnetic moment of the proton. On this theory, the neutron is given the same magnetic moment as the proton in contradiction with some direct experimental evidence which indicates that μ_N and μ_P have opposite sign. Because, however, of the interference terms which appear, the magnetic moment of the

[10] W. Pauli, *Handbuch der Physik*, Vol. 24/1, p. 233.

[11] G. C. Wick, Accad. Lincei **21**, 170 (1935).

[12] H. Frölich and W. Heitler, Nature **141**, 37 (1938).

[13] H. J. Bhabha, Nature **141**, 117 (1938), has proposed a formalism involving a four vector dynaton; however, in the nonrelativistic limit for the heavy particles, this is completely indistinguishable from the scalar dynaton formalism of Yukawa. Similar considerations apply to the suggestion of N. Kemmer, Nature **141**, 116 (1938).

deuteron could still be brought into agreement with the observed ~ 0.85 n.m. The equality of the field theoretically calculated anomalous magnetic moments for neutron and proton is a consequence of the fact that positive and negative dynatons are given the *same* magnetic moment in spite of their opposite charges; this in turn follows from the fact that the magnetic coupling terms of Eq. (17) in the Lagrangian involve a second rank tensor, unaltered by mirroring.

We now turn to a consideration of the deuteron, where we have to do with two heavy particles which are not very far apart in comparison to the range of nuclear forces. If we deduce the nuclear forces from a charge bearing field theory, there will be currents between the two heavy particles. These exchange currents will give rise to radiation and must be taken into consideration when calculating the probability of electric dipole radiation; i.e., it will not suffice to know only the currents that are due to the motion of the heavy particles. For the interaction of the system with the radiation field, in the limit of wave-lengths large compared to the size of the deuteron, is determined by

$$-(\mathbf{A}/c)\cdot\int \mathbf{s}(x)d\tau,$$

where $\mathbf{A}$ is the vector potential of the radiation field, and $\mathbf{s}(x)$ is the current density. There are contributions to $\mathbf{s}(x)$ from both heavy and light particles, and one might suppose that a detailed knowledge of the light particle current would be necessary to determine this interaction energy. However, as Siegert[14] has shown, one may avoid this by use of the conservation laws of charge and current. By their help, the above interaction energy may be transformed into the form

$$-(\mathbf{A}/c)\cdot\int \mathbf{x}\partial\rho(x)/\partial t d\tau = -(\mathbf{A}\cdot\partial\mathbf{D}/\partial t)/c, \quad (1)$$

where $\mathbf{D}$ is the dipole moment of the system. The advantage of this form is that one would expect the matrix elements of the light particle charge density to vanish identically, since particles of both signs contribute to the exchange current. This point may be verified on either electron-neutrino or dynaton field theory. Then the dipole moment of the system $\mathbf{D}=e\mathbf{r}/2$, where $\mathbf{r}=\mathbf{P}-\mathbf{N}$, and the required $\partial\mathbf{D}/\partial t$ can be calculated from the equations of motion of the heavy particle, namely:

$$\partial\mathbf{D}/\partial t = \frac{i}{\hbar}(H\mathbf{D}-\mathbf{D}H). \quad (2)$$

The Hamiltonian H contains the kinetic energy, and an exchange potential which (unlike an ordinary potential) does not commute with $\mathbf{r}$. One finds

$$\partial\mathbf{D}/\partial t = e\left\{\frac{\hbar}{Mi}\,\text{grad}_r + \frac{\mathbf{r}}{i\hbar}J(r)\right\}, \quad (3)$$

where the first term, as is usual, comes from the kinetic energy term $-(\hbar^2/M)\Delta_r$ of H, while the new term with $\mathbf{r}J(r)$ arises from the exchange potential $J(r)P$, where P is an operator which exchanges neutron and proton. One may further transform the interaction energy

$$-(\partial\mathbf{D}/\partial t\cdot\mathbf{A})/c = -1/c\{\mathbf{D}\cdot\mathcal{E}+\partial/\partial t(\mathbf{D}\cdot\mathbf{A})\}, \quad (4)$$

where the last term will not contribute to radiative transitions. This provides a justification for the procedure of Breit and Condon,[1] who calculated the probability of the photodisintegration of the deuteron by the use of only the dipole moment operator $\mathbf{D}$ of the system. Naturally, the exchange currents still play a role, only this is now hidden in the effect of the exchange forces on the wave functions that are used in the calculation of the matrix elements. Investigation in which the scalar dynaton theory is used checks this result of Siegert in detail. The connection between exchange force and exchange current is given by the equality[15]

$$\int \mathbf{s}(x)d\tau = \mathbf{S}(r) = -ie\mathbf{r}J(r). \quad (5)$$

However, since the above relation between force and current depends on the validity of the conservation laws, one would not expect to be able to derive it on the basis of the electron-neutrino theory, and in fact, one cannot so derive it.

[14] A. J. F. Siegert, Phys. Rev. **52**, 787 (1937). We have enjoyed several helpful discussions with Dr. Siegert, and wish to thank him for them.

[15] We use "relativistic units" in what follows: $\hbar$ and c are set equal to unity.

We now consider the questions of the magnetic moment and magnetic dipole radiation in the deuteron. If a heavy-light particle coupling is used which gives isolated neutrons and protons a magnetic moment, we would expect extra terms to appear in the interaction of a deuteron with an electromagnetic field because of the magnetic moment of the exchange current. The six vector scalar dynaton, which gives a magnetic moment of the same sign and magnitude for neutron and proton, gives a magnetic moment operator for the deuteron of

$$\boldsymbol{\mu}_{\text{deuteron}} = g\gamma\eta(M/16\pi em)(1-2e^{-mr})(\boldsymbol{\sigma}_N+\boldsymbol{\sigma}_P). \quad (6)$$

The interference term with e^{-mr} appears with a negative sign, which comes from the action of the exclusion principle on the heavy particles, and is related to the repulsive nature of the neutron-proton forces. With reasonable values of the constants, one can fit both the observed proton and deuteron magnetic moments. However, since $\boldsymbol{\sigma}_N+\boldsymbol{\sigma}_P$ is a constant of the motion, one could get magnetic dipole radiation from a transition between states of the deuteron only from an intrinsic magnetic moment of the proton; if this were about one n.m., it would be insufficient to account for the radiative capture of slow neutrons in paraffin. In the form of the electron-neutrino theory for which neutrons and protons have equal and opposite magnetic moments, there is no net extra magnetic moment for the deuteron, as the operator for the interaction energy with a magnetic field $\mathcal{H}$ contains the spin matrices of the heavy particles only through $(\boldsymbol{\sigma}_N-\boldsymbol{\sigma}_P)\cdot\mathcal{H}$ and $[\boldsymbol{\sigma}_N\times\boldsymbol{\sigma}_P\cdot\mathcal{H}]$. Such terms will however give rise to extra magnetic dipole radiation, and for the capture of slow neutrons, the corrections to the probability are quite appreciable.

We must ask if it will be possible to retain a phenomenological description of magnetic dipole transitions involving the exchange force in a manner similar to the one Siegert found suitable for the electric dipole radiation. The answer seems to be in the negative. In the first place, it is possible in our field theory because of the space dependence of the interference magnetic moment operator, to obtain magnetic dipole radiation in the collision of a neutron and a proton even in order g^2e, while the possibility of this radiation in the usual phenomenological discussion depends both on the existence of magnetic moments (order g^2e) and the presence of Heisenberg exchange forces (order g^2). Second, if one does go to fourth order in g, the contributions of the exchange currents really do appear in addition to those of the currents about each heavy particle. Finally, the terms involving $[\boldsymbol{\sigma}_N\times\boldsymbol{\sigma}_P\cdot\mathcal{H}]$ are quite unlike any that appear in the phenomenological treatment.

Hence it seems likely that processes involving the coupling of magnetic moments with the electromagnetic field should not be trusted when information about the position of the singlet level of the deuteron is desired. While it is known from scattering experiments that this level is close to zero potential, it may not be safe to infer from data on the probability of magnetic dipole capture alone that the level is a virtual one.[16] The possibility of complications of a similar nature may be expected and should be kept in mind when looking for regularities in the magnetic moments and magnetic multipole radiation of heavier nuclei.

Spin-Dependent Forces Between Elementary Particles

If one assumes that the force between neutron and proton is purely of Majorana type, and treats the heavy particles nonrelativistically, the singlet and triplet levels in the deuteron coincide;[17] from experiment, on the other hand, the singlet level is known to lie some two million volts higher. Bethe and Bacher[18] and Casimir[19] have estimated the splitting of the S term due to the magnetic interaction which is present if one assigns phenomenologically described magnetic moments to the particles, and although their

[16] This point will probably be settled definitely by a study of the scattering of slow neutrons by ortho- and para-hydrogen. (J. Schwinger and E. Teller, Phys. Rev. **52**, 286 (1937).) Even if the level should prove to be virtual, there seems to be some discrepancy between the observed and theoretical life times of neutrons in hydrogen, and this may be due to the phenomenon discussed above.

[17] We shall not consider here the spin-spin forces which arise from the many possible relativistic extensions of the two-body interaction, or even those which would arise in our calculations from a retention of the higher terms in the heavy particle velocities. See G. Breit, Phys. Rev. **51**, 248 (1937); **53**, 153 (1938). We are indebted to Professor Breit for several interesting communications dealing with these questions.

[18] H. A. Bethe and R. F. Bacher, Rev. Mod. Phys. **8**, 82 (1936).

[19] H. Casimir, Physica **3**, 936 (1936).

results differ by a factor of a thousand, both are too small (100 ev, 10^5 ev) to fit the experiments, and hence seem to require the admixture of some Heisenberg type interaction in the spin dependence of $J(r)$. In view of the above discrepancy, however, it seems of interest to examine the splitting predicted by one of the field theories which we have been considering.

Bethe and Bacher assumed a spin-spin coupling for neutron and proton

$$U_0=\mu_N\mu_P(\boldsymbol{\sigma}_N\cdot\boldsymbol{\sigma}_P/r^3-3\boldsymbol{\sigma}_N\cdot\mathbf{r}\boldsymbol{\sigma}_P\cdot\mathbf{r}/r^5) \quad (7)$$

based on the classical interaction of two point dipoles. Because of its angular dependence, this gives a splitting of the ground S state of the deuteron only in second order. It is obvious, however, that Eq. (7) is too singular at $r=0$ for unambiguous use in a perturbation calculation.

On the other hand, in his two particle equations, Casimir took the coupling energy appropriate for steady currents

$$-\int\frac{\mathbf{s}(N)\cdot\mathbf{s}(P)}{r}d\tau \quad (8)$$

and for the s's inserted the polarization currents corresponding to the observed magnetic moments, thus

$$\mathbf{s}(N)=\mu_N \operatorname{curl}\{\phi^*(N)\boldsymbol{\sigma}_N\phi(N)\} \quad (9)$$

and similarly for the proton. After partial integrations (in which surface terms also contribute), one finds an interaction containing besides the terms (7), a part independent of orbital angles and containing a delta-function of the separation $\mathbf{r}$ of the particles

$$U_s=U_0-\frac{4\pi}{3}\mu_N\mu_P\boldsymbol{\sigma}_N\cdot\boldsymbol{\sigma}_P\delta(\mathbf{r}) \quad (10)$$

so that a splitting of an S state results in first order.

However, with a phenomenological description of the magnetic moments, it is not possible unambiguously to deduce a magnetic interaction energy for the neutron and proton. Thus the sign and magnitude of the delta-function term in Eq. (10) would be changed[20] if one were to couple the single heavy particles to the electromagnetic field by their magnetic moments, taking for the interaction energy

$$-\{\boldsymbol{\mu}_N\cdot\mathcal{H}(N)+\boldsymbol{\mu}_P\cdot\mathcal{H}(P)\}, \quad (11)$$

where $\mathcal{H}$ is the magnetic field, instead of a current coupling

$$-\{\mathbf{s}(N)\cdot\mathbf{A}(N)+\mathbf{s}(P)\cdot\mathbf{A}(P)\}, \quad (12)$$

which can be used when div $\mathbf{s}=0$ to derive Eq. (8). Of course, with the view that the magnetic moments of the single heavy particles arise largely from the light particle currents which surround them, one would tend to rule out the coupling (11).

Now it may be that a part of the proton's magnetic moment may be treated in analogy to the magnetic moment of an electron. For this part of μ_P, Eq. (8) is applicable, and gives a splitting analogous to that derived in hyperfine structure calculations. According to our model, however, the remaining magnetic moments arise from the light particle field, and the theory of this should be used to calculate the magnetic splitting.

According to an electron neutrino field model which gives neutron and proton equal and opposite magnetic moments,[21] a magnetic spin-spin coupling between neutron and proton arises in order g^4e^2, i.e., in a sixth-order perturbation calculation, and while one can, in fact, show that some of the terms in the calculation may be put into the form of Eqs. (8) and (9) with magnetic moments characteristic of the single heavy particles, there are also contributions from terms which correspond to an exchange of light particles between the two heavy particles. These in effect change the coefficient of the delta function in Eq. (10) by an amount of order unity. This will lead to a magnetic interaction which is in no simple way connected to the observed magnetic dipole moments; hence from this point of view, neither of the previously used magnetic

[20] G. Breit and F. W. Doermann, Phys. Rev. 36, 1732 (1930).

[21] We abstract here from the complication that when the particular electron-neutrino coupling of Eq. (19) which gives neutron and proton oppositely equal magnetic moments is used, the cross terms in the coupling lead to Heisenberg forces. For in the above, we are particularly interested in magnetic spin-spin forces; furthermore, one could avoid the Heisenberg forces by a more general electron-neutrino coupling.

interactions is applicable. From an examination of typical terms in the field theoretic calculation when $r\sim 0$, it seems likely that there will be a coupling which is finite at $r=0$, and which also gives a first order splitting of S states from the anomalous part of the proton's magnetic moment and is probably not different in order of magnitude from Casimir's result.

In view of the considerable role of exchange in the magnetic interaction of neutron and proton, one may ask whether any appreciable changes are to be expected in the formula for the magnetic interaction of heavy particles and electrons, which is used in calculations of hyperfine structure and magnetic scattering of neutrons,[22] etc. In fact, there seems to be some evidence for this in the discrepancy between the values given by Stern[23] and by Rabi[24] for the magnetic moment of the proton. As Young[25] has pointed out, the differences in the apparent magnetic moment $\mu_P=2.46\pm 0.08$ and 2.85 ± 0.15 could be accounted for by an additional spin-dependent proton-electron coupling. If this discrepancy is real, it will suggest strongly that a theory of the electron-neutrino type is preferable to one of the dynaton type, for while with the latter, one would not expect any important new spin-dependent electron-proton forces, the electron-neutrino field theory which we have been investigating does lead in order g^2e^2 to such an additional interaction (arising from an exchange of the original electron and that emitted by the heavy particle.)[26] This leads one to expect deviations from the usual magnetic interaction of order $\sim e^2\sim 1/137$. We do not, however, regard the formalism as sufficiently reliable to justify a more accurate estimate of these deviations.

Appendix I

Interaction energy of a neutron and a proton

For the purposes of this section, we need consider only the scalar dynaton theory with the coupling energy

$$H_d=g\int[\phi_P^*(x)\beta\phi_N(x)]\phi_D(x)d\mathbf{x}+\text{conjugate}. \quad (13)$$

Here, ϕ_N, ϕ_P, ϕ_D are the neutron, proton, and dynaton wave functions, respectively, β is the Dirac matrix, and g is a dimensionless parameter measuring the strength of the coupling. "Relativistic units," in which $\hbar$ and c are set equal to unity, are used throughout. ϕ_D is assumed to satisfy the scalar relativistic wave equation investigated by Pauli and Weisskopf.[7] We use non-relativistic theory for the heavy particles, so that ϕ_N and ϕ_P are two component functions and the β connecting them may be set equal to the unit matrix.

Using the method of quantized waves,[27] we put

$$\phi_N(x)=\sum_\alpha A_\alpha u_\alpha(x),\quad \phi_P(x)=\sum_\beta B_\beta v_\beta(x),$$

where the u's and v's are complete orthonormal sets of state functions, and the A's and B's operate on the occupation numbers of neutrons and protons, respectively, in the Schrödinger functional and obey Fermi commutation rules. For the dynatons we follow Pauli and Weisskopf, and use plane waves normalized in the $\mathbf{k}$ scale

$$\phi_D(x)=\int(2\pi)^{-3}(2E_k)^{-\frac{1}{2}}(a_k-b_k^*)\exp(i\mathbf{kx})d\mathbf{x},$$

where $E_k=(k^2+m^2)^{\frac{1}{2}}$ is the dynaton energy, m the dynaton mass, and the a's and b's operate on the positive and negative dynaton occupation numbers, respectively, and obey Bose commutation rules. We adopt the convention that unstarred operators destroy particles, and starred operators (complex conjugates of the unstarred ones) create them. The coupling (13) then becomes

$$H_d=g\sum_{\alpha,\beta}\int\int(2\pi)^{-3}(2E_k)^{-\frac{1}{2}}v_\beta^*(x)u_\alpha(x)$$
$$\times\exp(i\mathbf{kx})B_\beta^*A_\alpha(a_k-b_k^*)d\mathbf{x}d\mathbf{k}+\text{conjugate}.$$

Here, the first term corresponds to a neutron changing into a proton with either the emission of a negative or the absorption of a positive

[22] F. Bloch, Phys. Rev. **51**, 994 (1937).
[23] I. Estermann, O. C. Simpson and O. Stern, Phys. Rev. **52**, 535 (1937).
[24] J. M. B. Kellogg, I. I. Rabi and J. R. Zacharias, Phys. Rev. **50**, 472 (1936).
[25] L. A. Young, Phys. Rev. **52**, 138 (1937).
[26] There is also a spin-spin interaction even in order g^2, but this must be assumed small due to the fact that the coupling of low energy electrons and heavy particles is known to be small from the observed slowness of β-decay.
[27] Reference 10, page 198.

dynaton; the complex conjugate term corresponds to a proton changing into a neutron with either the emission of a positive or the absorption of a negative dynaton. Because of the anticommuting properties of the A's and of the B's, it is necessary to keep their order straight.

The matrix element of the interaction energy between a neutron and a proton is the second order perturbation energy computed from the transitions in which either (a) the proton emits a positive dynaton and changes into a neutron, the dynaton subsequently being absorbed by one of the neutrons; or (b) the neutron emits a negative dynaton and changes into a proton, the dynaton subsequently being absorbed by one of the protons. These two types of transition give the same formal result; we need consider only (a), and multiply the final answer by two. If the initial neutron and proton states are u_α and v_β, and their final states are u_η and v_γ, respectively, then the interaction energy is

$$-g^2\int d\mathbf{k}/(16\pi^3E_k^2)\bigg\{\int v_\gamma^*(x)u_\alpha(x)\exp(i\mathbf{kx})d\mathbf{x}\times\int u_\eta^*(x')v_\beta(x')\exp(-i\mathbf{kx}')d\mathbf{x}'(1-\delta_{\alpha\eta})Q+\sum_\xi\int v_\gamma^*(x)u_\xi(x)\exp(i\mathbf{kx})d\mathbf{x}\times\int u_\xi^*(x')v_\beta(x')\exp(-i\mathbf{kx}')d\mathbf{x}'\delta_{\alpha\eta}R\bigg\}.$$

Here,

$$Q\equiv\Psi^*(1N_\eta,1P_\gamma)B_\gamma^*A_\alpha A_\eta^*B_\beta a_k a_k^*\Psi(1N_\alpha,1P_\beta)=\Psi^*(1N_\eta,1P_\gamma)B_\gamma^*B_\beta(\delta_{\alpha\eta}-A_\eta^*A_\alpha)\times(1+a_k^*a_k)\Psi(1N_\alpha,1P_\beta)=\delta_{\alpha\eta}-1,$$

$$R\equiv\Psi^*(1N_\alpha,1P_\gamma)B_\gamma^*A_\xi A_\xi^*B_\beta a_k a_k^*\Psi(1N_\alpha,1P_\beta)=\Psi^*(1N_\alpha,1P_\gamma)B_\gamma^*B_\beta(1-A_\xi^*A_\xi)\times(1+a_k^*a_k)\Psi(1N_\alpha,1P_\beta)=1-\delta_{\alpha\xi},$$

on making use of the commutation rules for the various operators; $\Psi(1N_\alpha, 1P_\beta)$ and $\Psi(1N_\eta, 1P_\gamma)$ are the Schrödinger functionals for the initial and final states respectively. The expression for the interaction energy thus becomes

$$+g^2\int d\mathbf{k}/(16\pi^3E_k^2)\bigg\{\int v_\gamma^*(x)u_\alpha(x)\exp(i\mathbf{kx})d\mathbf{x}\times\int u_\eta^*(x')v_\beta(x')\exp(-i\mathbf{kx}')d\mathbf{x}'-\delta_{\alpha\eta}\sum_\xi\int v_\gamma^*(x)u_\xi(x)\exp(i\mathbf{kx})d\mathbf{x}\times\int u_\xi^*(x')v_\beta(x')\exp(-i\mathbf{kx}')d\mathbf{x}'\bigg\}.$$

Considering the second term of this expression, we can perform the sum over ξ by making use of the closure theorem, and thus obtain $-\delta_{\alpha\eta}\delta_{\beta\gamma}g^2\int d\mathbf{k}/(16\pi^3E_k^2)$; this is the negatively infinite "proper" energy of a proton in state v_β, and appears only in the diagonal elements, as it should. The first term in the above expression is readily evaluated and is

$$+\int\int u_\eta^*(x')v_\gamma^*(x)(g^2e^{-mr}/8\pi r)\times u_\alpha(x)v_\beta(x')d\mathbf{x}d\mathbf{x}',\quad \mathbf{r}\equiv\mathbf{x}-\mathbf{x}'.$$

This expression for the interaction energy has been given correctly by Serber[5] and by Yukawa and Sakata.[28] Other investigators, apparently, have not examined the structure of the term Q above carefully enough to realize that it has the effect of making the sign of the interaction energy just the opposite of what would be expected intuitively. The appearance of this sign derives from the fact that the exclusion principle is valid for the heavy particles, and is independent of the statistics obeyed by the light particles in the field. Thus the operator for the total interaction energy is

$$J(r)=+(g^2e^{-mr}/4\pi r)P^H, \tag{14}$$

where P^H is the Heisenberg exchange operator; this is repulsive for a symmetric state. $J(r)$ is of the right order of magnitude if the phase integral $\int[MJ(r)]^{\frac{1}{2}}dr\sim1$, where M is the mass of the neutron or proton. This makes $g\sim1$.

For comparison, it is interesting to consider what form the Q and R above would have if neutrons and protons were to obey Bose statistics. Using Bose commutation rules for the

[28] H. Yukawa and S. Sakata, Proc. Phys.-Math. Soc. Japan, 19, 1084 (1937).

A's and B's, we obtain

$$Q' = \delta_{\alpha\eta} + 1, \quad R' = 1 + \delta_{\alpha\xi},$$

which result in the same proper energy as above, but change the sign of the interaction energy (14).

An exactly analogous calculation can be carried through with the electron-neutrino theory, in which the simplest coupling is used

$$H_a = g \int [\phi_N^*(x)\beta\phi_P(x)] \times [\phi_n^*(x)\beta\phi_e(x)] d\mathbf{x} + \text{conjugate}, \quad (15)$$

where ϕ_e and ϕ_n are the electron and neutrino wave functions that satisfy the ordinary Dirac equation (with and without charge, respectively). In order to have a convergent theory, however, it is necessary to multiply electron and neutrino wave functions by a cut-off factor or finite distance operator of the general form $e^{-a|E|}$, where E is the light particle energy. The resulting interaction energy has the same sign and exchange character as (14); its range is $\sim a$, and its magnitude is $\sim g^2/a^5$. The phase integral condition on the magnitude of the interaction gives $g^2 \sim a^3/M$.

Appendix II

Electric properties and Siegert's theorem

The expectation values of the charge and current density about a stationary neutron on the dynaton theory can be found by computing the diagonal matrix elements of the corresponding operators, which are given by Pauli and Weisskopf.[7] The calculations are straightforward, and result in a zero current density, and a charge density given by

$$\rho(r) = (eg^2 m/16\pi^3 r^2) K_1(mr) e^{-mr},$$

where r is the distance from the neutron, and K_1 is the Bessel function of imaginary argument.[29] The total charge corresponding to this charge density is infinite; but the expectation value of the charge on the heavy particle is also infinite, and it is easy to show that the two charges cancel term by term. On the electron-neutrino theory, with the coupling (15), the current density again vanishes. The expression for the charge density is quite complicated; however, its asymptotic form is

$$\rho(r) \sim -(eg^2 a/r^8) \log (r/a), \quad r \gg a,$$

and the mean radius of the charge cloud is $\sim a$, where a is the parameter in the finite distance operator introduced in Appendix I. The presence of this parameter also makes the total charge in the field finite and $\sim eg^2/a^4 \sim e/Ma \sim e/13$.

When a neutron and a proton are brought close together, as in a deuteron, the expectation value of any quantity can always be expressed as the sum of a "proper" part and an "interference" part. The proper part is just the sum of the expectation values for the quantity in question for the isolated neutron and proton; the interference part arises from the emission of light charged particles by one heavy particle and their absorption by the other, and hence depends on the distance between the heavy particles and vanishes rapidly as this distance becomes large. The charge density on either of our field theories is readily calculated, and it turns out that the interference part of its expectation value vanishes, since the neutron is just as likely to be emitting a negative particle as the proton is to be emitting a positive particle; the proper charge clouds about the heavy particles remain as before and give nothing new. The proper current density, on the other hand, vanishes, while the interference part of the current is appreciable. The calculation of this "exchange current" is analogous to that of the interaction energy (see Appendix I); the matrix elements of the total current operator are computed between Schrödinger functionals perturbed by either of the couplings (13) or (15). With the dynaton theory, the total current for a transition in which the initial neutron and proton states are u_α and v_β, and their final states are u_η and v_γ, respectively, is

$$-eg^2 \int \mathbf{k} d\mathbf{k}/(16\pi^3 E_k^4) \int v_\gamma^*(x) u_\alpha(x) e^{i\mathbf{k}}\, d\mathbf{x} \times \int u_\eta^*(x') v_\beta(x') e^{-i\mathbf{k}\mathbf{x}'} d\mathbf{x}'.$$

[29] Whittaker and Watson, *Modern Analysis*, fourth edition (1935), p. 373.

This can be simplified to

$$-\int\int u_\eta^*(x')v_\gamma^*(x)$$
$$\times(ieg^2\mathbf{r}e^{-mr}/4\pi r)u_\alpha(x)v_\beta(x')d\mathbf{x}d\mathbf{x}'.$$

Thus the operator for the total current is

$$\mathbf{S}(r)=-(ieg^2\mathbf{r}e^{-mr}/4\pi r)P^H.$$

Comparison of this with the expression (2) for the interaction energy operator checks the relation

$$\mathbf{S}(r)=-ie\mathbf{r}J(r), \quad (16)$$

which is given by Siegert.[14]

Appendix III

Magnetic properties of neutron and deuteron

The scalar dynaton possesses no spin, and hence no magnetic moment. However, it is possible to introduce terms into the Hamiltonian so that it gives a magnetic interaction with the radiation field:

$$H_v=g\int[\phi_P{}^*\beta\phi_N]\phi_D d\mathbf{x}+\gamma\int[\phi_P{}^*\sigma_{\mu\nu}\phi_N]\phi_{\mu\nu}d\mathbf{x}$$
$$+\eta\int\phi_D{}^*F_{\mu\nu}\phi_{\mu\nu}d\mathbf{x}+\text{conjugate}. \quad (17)$$

The first term here was present in Eq. (13) and represents the coupling between heavy particles and scalar dynaton; the second term gives the coupling between heavy particles and six-vector dynaton $\phi_{\mu\nu}$, each component of which separately satisfies the Pauli-Weisskopf equation; and the third term connects the two kinds of dynatons with the electromagnetic field tensor $F_{\mu\nu}$. The dynaton magnetic moment operator is thus

$$\eta(\phi_D{}^*\phi_{\mu\nu}+\phi_{\mu\nu}{}^*\phi_D),$$

besides this there will be an imaginary electric moment associated with the dynaton, which will, however, be of the order of the relativistic correction to the heavy particle motion, since it involves the space-like components of the Dirac α_μ matrices. Thus the dynaton cloud about a stationary heavy particle is capable of interacting with the magnetic vector of the radiation field in a way that is connected to the spin vector of the heavy particle. Although the total charge in the dynaton cloud about a single neutron is infinite, the total magnetic moment is not; this is because the dynaton charge density operator is $ie(\phi_D{}^*\partial\phi_D/\partial t-\partial\phi_D{}^*/\partial t\phi_D)$, which contains one less E_k in the denominator than the magnetic moment operator and thus causes the expression for the total charge to diverge at high dynaton momenta. The diagonal matrix element of the magnetic moment operator for a stationary neutron is readily calculated, and is to lowest order in the various coupling parameters $\boldsymbol{\mu}_N=(g\gamma\eta/32\pi m)(2M/e)\boldsymbol{\sigma}_N$ nuclear magnetons. This is of the right order of magnitude for the neutron moment if $g\gamma\eta$ is ~ 1. The magnetic moment of a proton at rest (aside from any intrinsic moment) is also given by the above expression, with the same sign. The appearance of both neutron and proton with the same sign of magnetic moment derives basically from the fact that the coupling with the electromagnetic field is through the field tensor $F_{\mu\nu}$, and not through the vector potential. Thus, interchanging positive and negative dynatons in (17) (i.e., replacing ϕ_D by $\phi_D{}^*$ and $\phi_{\mu\nu}$ by $\phi_{\mu\nu}{}^*$) and also interchanging neutron and proton, the magnetic field coupling terms as well as the light-heavy particle coupling terms remain unchanged. This gives positive and negative dynatons, and hence neutrons and protons, the same magnetic moment.

It is interesting to note that the γ terms in (17) lead to a mixed Majorana-Heisenberg interaction between a neutron and a proton

$$J(r)=(\gamma^2e^{-mr}/4\pi r)(2P^M-P^H),$$

which is again repulsive for a symmetric state. Thus by properly choosing g and γ, the unlike particle interaction can be made to have the right magnitude and exchange character, but still not the right sign.

On the electron-neutrino theory, any magnetic moment that might appear would come from the relativistic electron current, and thus the introduction of a new kind of particle is unnecessary. However, since the current density about a stationary neutron resulting from the coupling (15) vanishes, it is necessary to modify it so that a current appears that is coupled to the spin axis of the neutron. The coupling term

$$H_b = g\int [\phi_N{}^*\beta\phi_e][\phi_n{}^*\beta\phi_P]d\mathbf{x} + \text{conjugate} \quad (18)$$

(which leads to a Majorana interaction between a neutron and a proton) serves to give the neutron a magnetic moment that is approximately

$$\boldsymbol{\mu}_N \sim -(g^2e/a^3)(2M/e)\boldsymbol{\sigma}_N \text{ nuclear magnetons,}$$

but gives the proton none since it clearly contains no connection between proton and electron. However, (18) can be modified so as to make it symmetric between neutron and proton:

$$H_c = H_b + \left\{ g'e^{i\theta}\int [\phi_N{}^*\beta\delta\tilde{\phi}_n{}^*] \times [\tilde{\phi}_P\beta\delta\phi_e]d\mathbf{x} + \text{conjugate} \right\}, \quad (19)$$

where $\tilde{\phi}$ is the transpose of ϕ, δ is Fermi's matrix and θ is any real number. When the two parameters g and g' are chosen equal, the magnetic moments of neutron and proton are equal and opposite in sign, except for a possible intrinsic magnetic moment of the proton.

When the two-body situation is considered, the total magnetic moment appears as the sum of the proper moments of the isolated particles, and an interference term which depends on their distance apart. With the dynaton theory, the matrix element of this interference term between initial neutron and proton states u_α and v_β, and final states u_η and v_γ, respectively, is

$$-\int\int u_\eta{}^*(x')v_\gamma{}^*(x)(g\gamma\eta M/8\pi em)e^{-mr}(\boldsymbol{\sigma}_N + \boldsymbol{\sigma}_P) \times u_\alpha(x)v_\beta(x')d\mathbf{x}d\mathbf{x}' \text{ nuclear magnetons.}$$

Thus the total magnetic moment operator, including the proper moments of the isolated particles, is

$$\boldsymbol{\mu}_{\text{deuteron}} = (g\gamma\eta M/16\pi em) \times (1-2e^{-mr})(\boldsymbol{\sigma}_N + \boldsymbol{\sigma}_P) \text{ nuclear magnetons.}$$

Since this is proportional to $\boldsymbol{\sigma}_N + \boldsymbol{\sigma}_P$, which is a constant of the motion, it cannot give rise to radiative magnetic dipole transitions.

The electron-neutrino theory with coupling H_b of Eq. (18) leads to an interference magnetic moment operator for the deuteron

$$\boldsymbol{\mu}_{\text{deuteron}} = (eg^2/4)\boldsymbol{\sigma}_N \cdot F(r),$$

where

$$F(r) = \sum_{s,\,t}[e^{i(\mathbf{p}_s+\mathbf{p}_t)\mathbf{r}}/E_s(E_s+E_t)^2],$$

where the sum is over intermediate electron states s and anti-neutrino states t. $F(r)$ falls off rapidly for large r, and is $\sim g^2/a^3$ for $r=0$. We see that μ_{deuteron} has the form of an effective extra magnetic moment μ_N' for the neutron. If the neutron and proton were brought together $(r=0)$, μ_N' would be of the order of μ_N for an isolated neutron. If we use the coupling (19), which is symmetric between neutron and proton, the proton will also have an extra moment $\mu_P' = -\mu_N'$. In addition, however, the cross terms of H_c lead to a coupling of the deuteron with the magnetic field

$$-(eg^2/4)\{(\boldsymbol{\sigma}_N - \boldsymbol{\sigma}_P)\cdot\mathcal{H}\cos\theta + [\boldsymbol{\sigma}_N \times \boldsymbol{\sigma}_P]\cdot\mathcal{H}\sin\theta\}F(r).$$

We see that, on the electron-neutrino theory with H_c, the extra magnetic moment operator for the deuteron has only off-diagonal elements, and thus can contribute only to magnetic dipole radiation and not to the net magnetic moment of the deuteron in either singlet or triplet states. In the limit $\alpha_t a \to 0$, $|\alpha_s| \ll \alpha_t$ (where the α's are the wave numbers corresponding to triplet and singlet states of the deuteron), one finds corrections to the transition rate of order $\alpha_t a$; for larger $\alpha_t a \to 1$, the corrections can become of order unity. (Actually $\alpha_t a \sim 0.6$.)

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