The Rate of Selective Thermonuclear Reactions

In the paper of one of us (G.G.)[1] in the present issue, the probable importance of selective temperature effects of thermonuclear reactions for the structure and evolution of stars has been discussed in some detail. It is therefore of interest to obtain quantitative expressions for the rate of energy production in the various reactions involved. The formula for the rate of ordinary thermonuclear penetration followed by radiative capture was first given by Atkinson and Houtermans.[2]

Because of the subsequent development of nuclear physics this formula has to be changed in several respects and we give here the new derivation. Consider a gas consisting of two elements with atomic numbers Z_1 and Z_2 and atomic masses m_1 and m_2. Let x and y be the relative amounts (by weight) of two components, and ρ and T the total density and temperature. The number of collisions between two nuclei of the two different kinds with the collision energy in the interval $[E; E+dE]$ is given by:

$$dN=\frac{4xy\rho\sigma}{(2\pi)^{\frac{1}{2}}m_1m_2m^{\frac{1}{2}}(kT)^{\frac{3}{2}}}e^{-E/kT}EdE, \tag{1}$$

where $m=m_1m_2/(m_1+m_2)$ is the reduced mass and σ the effective cross section. The effective cross section for radiative capture is given by the cross section for penetration multiplied by the ratio of emission probability of γ-rays $\Gamma_\gamma\backsimeq 10^{+14}$ sec.$^{-1}$ (for the energies involved) and the proper frequency $\hbar/mr_0^2(\backsimeq 10^{22}$ sec.$^{-1})$ of a particle oscillating inside the nucleus. Using the penetration formula we write:

$$\sigma\backsimeq\frac{\Lambda^2}{4\pi}\exp\left[\frac{-2\pi e^2m^{\frac{1}{2}}Z_1Z_2}{\hbar(2E)^{\frac{1}{2}}}+\frac{4e(2mZ_1Z_2r_0)^{\frac{1}{2}}}{\hbar}\right]\cdot\frac{\Gamma_\gamma mr_0^2}{\hbar}, \tag{2}$$

where $\Lambda=2\pi\hbar/(2mE)^{\frac{1}{2}}$ is the de Broglie wave-length. Substituting (2) into (1) we get an expression possessing a sharp maximum at $E\backsimeq(\pi e^2m^{\frac{1}{2}}Z_1Z_2kT)^{\frac{2}{3}}/(2^{\frac{1}{2}}\hbar)^{\frac{2}{3}}$ the breadth of which is $\Delta E\backsimeq(8/3kT)^{\frac{1}{2}}(2\pi e^2m^{\frac{1}{2}}Z_1Z_2kT^{\frac{1}{2}})/(2^{\frac{1}{2}}\hbar)^{\frac{1}{2}}$. For the total number of captures per unit mass we get:

$$N\backsimeq\frac{\pi^{5/6}}{3^{\frac{1}{6}}}\frac{e^{\frac{2}{3}}\hbar^{\frac{2}{3}}Z_1^{\frac{1}{3}}Z_2^{\frac{1}{3}}r_0^2\Gamma_\gamma}{m_1m_2m^{\frac{1}{3}}}\times\exp\left[\frac{4e(2mZ_1Z_2r_0)^{\frac{1}{2}}}{\hbar}-3\left(\frac{\pi^2e^4mZ_1^2Z_2^2}{2\hbar^2kT}\right)^{\frac{1}{3}}\right]\frac{xy\rho}{(kT)^{\frac{3}{2}}}. \tag{3}$$

As an example we consider the reaction chain proposed by Weizsäcker (formula (18) in I) though this chain, requiring the stability of ${}_2He^5$, seems to be in contradiction with the mass defect of this nucleus as determined by Williams, Shepherd and Haxby.[3] Using formula (3) with $Z_1=1$; $Z_2=2$; $m_1=m_H$; $m_2=4m_H$; $x=y=\frac{1}{2}$; $\rho=1$ and $r_0=3\cdot10^{-13}$ cm we obtain the number of chain starting reactions for different temperatures. The total energy libration per gram per second, if we accept the energy balance of each complete chain as 18 Mev, is given by curve A in Fig. 1.

In case there is resonance at a certain energy E_r the dispersion formula has to be used. If Γ_p is the probability of particle re-emission we can write:

$$\sigma\backsimeq\frac{\Lambda^2}{4\pi}\frac{\Gamma_p\Gamma_\gamma}{[E-E_r]^2\hbar^{-2}+\frac{1}{4}[\Gamma_p+\Gamma_\gamma]^2}. \tag{4}$$

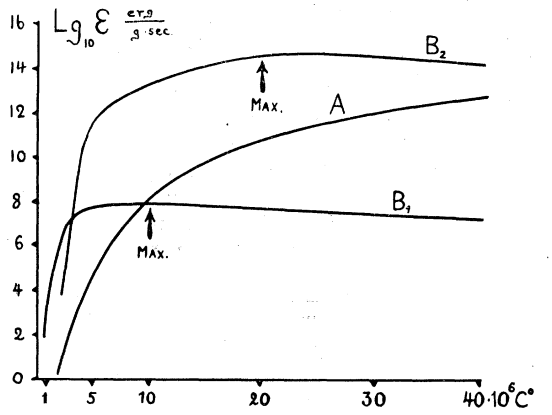

FIG. 1. Energy liberation ϵ in erg per gram per sec. Curve A, energy balance of each complete chain is 18 Mev. Curve B_1 and B_2, energy liberation of Weizsäcker's chain; resonance at 2 Kev and 4 Kev.

If $\Gamma_\gamma\gg\Gamma_p$ we have at the resonance $\sigma\backsimeq\Lambda^2\Gamma_p/\pi\Gamma_\gamma$ and the breadth of the resonance is $\Delta E\backsimeq\hbar\Gamma_\gamma$. Using formula (1) we obtain for the total number of captures per unit mass

$$N\backsimeq 8\left(\frac{\pi}{2}\right)^{\frac{1}{2}}\frac{\hbar^4}{m_1m_2m^{5/2}r_0^2}\times\exp\left[\frac{-2\pi e^2m^{\frac{1}{2}}Z_1Z_2}{\hbar(2E_r)^{\frac{1}{2}}}+\frac{4e(2mZ_1Z_2r_0)^{\frac{1}{2}}}{\hbar}\right]\frac{xy\rho}{(kT)^{\frac{3}{2}}}\cdot e^{-Er/kT}. \tag{5}$$

As an example the energy-liberation of Weizsäcker's chain with the resonance lying at 2 Kev and 4 Kev ($T_r=1$ and $2\cdot10^7$ C°) is shown by curves B_1 and B_2 in Fig. 1. We notice that for $E_r=2$ Kev the resonance is comparatively unimportant, whereas for $E_r=4$ Kev and higher the energy liberation at resonance is about 10^4 times larger than in the ordinary case. The energy liberation due to such resonance falls off slowly at high temperatures and therefore the real model of a star with such source will be somewhere between the "shell source model" discussed in I, and the "ball source model" in which the energy production remains constant within a sphere of a certain radius.

Another type of thermonuclear reaction leading to a sharp maximum and therefore to a shell model can be obtained if one assumes the stability of ${}_4Be^8$ nucleus (formula (19) in I). At low temperatures the rate of energy production will be mainly determined by the reaction ${}_2He^4+{}_2He^4\rightarrow{}_4Be^8+h\nu$. This rate can be calculated from (3) by putting $Z_1=Z_2=2$ and $m_1=m_2=m=4m_H$. At higher temperatures, however, the chainbreaking reaction ${}_4Be^8+h\nu\rightarrow{}_2He^4+{}_2He^4$ with the rate N_2 will compete with the next step in the chain ${}_4Be^8+{}_1H^1\rightarrow{}_5B^9+h\nu$ (rate N_3). The total rate of this complex process will be given by

$$N=\frac{N_1}{1+N_2/N_3} \tag{6}$$

with

$$\frac{N_2}{N_3}\backsimeq\frac{2^{5/3}\pi^{\frac{2}{3}}}{3^{\frac{2}{3}}}\frac{\hbar^3[\Gamma_\gamma r_0]_3}{m_H^{5/2}[\Gamma_\gamma r_0]_2}\exp\left[\frac{8e}{\hbar}((mr_0)_3^{\frac{1}{3}}-(mr_0)_2^{\frac{1}{3}})\right]\times\frac{x\rho}{(kT)^{\frac{3}{2}}}\exp\left[\frac{E_d}{kT}+1.8\cdot\left(\frac{\pi^2e^4m_H}{\hbar^2kT}\right)^{\frac{1}{3}}\right], \tag{7}$$

where E_d is the dissociation energy of $_4Be^8$ into two $_2He^4$ nuclei.

The expression (6) shows a sharp maximum for the temperature at which the thermal dissociation of $_4Be^8$ nuclei into two α-particles starts. We should notice here, however, that the possibility of such reaction requires the stability of $_4Be^8$ which at the moment is rather doubtful because of recent experiments of Kirchner, Laaff and Neuert.[4]

It is also questionable, in view of comparatively low probability of α-particle penetration into another α-particle, whether such a chain will give sufficient energy to secure the existence of the star.

George Washington University,
Washington, D. C.
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G. GAMOW
E. TELLER

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[2] R. d'E. Atkinson and F. G. Houtermans, Zeits. f. Physik **54**, 656 (1929).
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Multiple States in the High Pressure Discharge

In studying the arc discharge in the one atmosphere range of pressure we have for some time accumulated evidence that there exist, in addition to the "normal arc" and a glow discharge, additional glow states. The possibility that these additional states arise from spurious effects, such as magnetic lengthening of the discharge column, has now been eliminated.

We measure simultaneously the total voltage e, electric gradient E, and the diameter D of the discharge column, all as a function of arc current, i, varying between 10 amperes and zero. In addition to the oscillographically recorded e, E, and D, we have taken motion pictures of the discharge at 1000 frames per second.

The findings are as follows: With hydrogen at one atmosphere pressure and pure carbon electrodes, there exist a normal arc state I, a glow state II, and a glow state III. The normal arc state I has a gradient E that varies (linearly on log-log paper) between 82 v/cm at 10 amperes to 280 v/cm at 2 amperes, and is identical with the hydrogen arc studied by Mackay and Ferguson[1] and Langmuir.[2] In the range 2–0.6 amperes glow II appears, with $530 < E < 900$ v/cm. From 0.6 to 0.1 ampere a glow III is found, with $900 < E < 1300$ v/cm. The transition between I and II is thus accompanied by a change in gradient of 240 (v/cm). The transition between II and III shows no change in gradient, but an abrupt change in total arc voltage.

From measurements of the total voltage e, it is found that the transition between I and II is accompanied by a change in e of 96 volts, the transition between II and III by 130 volts.

The photographs show that I is a highly luminous column, homogeneous along the axis, with a current density of approximately 1000 amp. cm^{-2}. The form II is a less intense striated discharge, with a lower current density, and has a well-developed cathode dark space. The new discharge III has the same column as II, but the cathode dark space is replaced by an intense glow from which streamers emanate. It is evident that the cathode fall of II has increased by 130 volts in going to III.

There is incomplete evidence of an additional fourth state in hydrogen when the electrodes are incandescent tungsten. Further incomplete evidence suggests the presence of a total of four states with copper electrodes in nitrogen.

The results thus point conclusively to the presence of a new discharge type in the high pressure discharge, and suggest the existence of still others. The data are too incomplete at present to allow a conclusion as to the mechanism.

Research Laboratory,
General Electric Company,
Schenectady, N. Y.
March 8, 1938.

C. G. SUITS

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Concentration of Radiohalides, and Failure to Observe Gamma-Rays from I^{128}

In connection with experiments in which radioactive iodine, I^{128}, is being used as an indicator in the study of thyroid physiology, it was necessary to determine whether this substance emits gamma-rays. There is disagreement on this point in the literature; Amaldi[1] lists a gamma-ray as present, while Livingston and Bethe[2] do not.

With 150 mC of radon and beryllium, we have prepared sources of I^{128} which emit 10^5 beta-rays per minute, and have been unable to detect with a screen cathode gamma-ray counter any gamma-rays which pass through one-sixteenth of an inch of lead (1.8 g cm^{-2}). This corresponds to less than 0.1 gamma-ray of energy 0.5 Mev, per beta-ray, as determined by comparison with a radium standard.

A modification of the method of concentrating radioactive iodine given by d'Agostino[3] has been used for both iodine and bromine with excellent results. In the case of iodine, a few milligrams of free iodine (e.g., ten) are added to irradiated ethyliodide, and a water solution of the combining weight of sodium bisulphite (e.g., 4.1 mg) is shaken with the ethyliodide until the iodine color disappears. The water layer is separated from the ethyliodide in a separatory funnel, and silver iodide precipitated from it by the addition of an excess of slightly acid silver nitrate solution. Filtration is hastened by heating to boiling before filtering. Almost all the activity can thus be concentrated into a precipitate which may if desired contain as little as one milligram of iodine. The ethyliodide can be used over and over, and the complete separation can be carried out in less than ten minutes.

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ARTHUR ROBERTS
JOHN W. IRVINE, JR.

Harvard Medical School and Mass. Inst. of Tech. (A. R.),
Massachusetts Institute of Technology (J. W. I.),
Cambridge, Massachusetts,
March 1, 1938.

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