potential barrier, and in this case the primary *d-p* reaction is relatively improbable compared to a primary *d-n* reaction, since it is no longer favored by a greater penetrability. Therefore the *d-pn* reaction (which would be hard to observe) will be of rather minor importance.

The question of *d-p*α reactions is rather harder to decide because in this case the energy evolution cannot be determined so accurately. However, it can be said with certainty that the *d-p*α reaction can only have an appreciable probability with a given nucleus if slow neutrons give an *n*-α reaction with the same nucleus. For we have shown above that the product nucleus of the *d-p* reaction will, in general, not have sufficient energy to emit a neutron; it has therefore less excitation energy than the compound nucleus formed by adding a slow neutron to the target nucleus. If that latter compound nucleus emits γ-rays rather than α-particles, i.e., if the capture of slow neutrons is more probable than a *n*-α reaction, the same will be *a fortiori* true of the final nucleus formed in the *d-p* reaction, because the probability of α-emission decreases rapidly with decreasing excitation energy. *n*-α reactions with slow neutrons and heavy nuclei have only been observed for Th and U (reference 2, Table LX); therefore we may expect that only these extremely heavy elements give *d-p*α reactions to any appreciable extent. (A *small* yield of the *d-p*α reaction will, of course, always be obtained; it may be calculated from the penetrability of the potential barrier for α-particles if the energy evolution in the reaction is known). This seems to make unlikely the reaction *Au-d-p*α which was reported by Cork and Thornton[17] and was previously considered probable by the author (reference 5, p. 205).

The rarity of *d-p*α reactions may also be understood if we consider that the *d-p* reaction produces a nucleus with too many neutrons which will naturally have no tendency to lose further charge by emission of an α-particle. This is in contrast to the *d-n* reaction which produces a nucleus of too high charge so that a subsequent α-emission seems favorable.

Our considerations show that *d-p* reactions with deuteron energies below the potential barrier should rarely lead to any cascade disintegration.

[17] Cork and Thornton, Phys. Rev. **51**, 59 (1937).

JANUARY 1, 1938 PHYSICAL REVIEW VOLUME 53

A New Method for Investigating Atomic Electron Velocities

A. L. HUGHES AND MARVIN M. MANN, JR.
Wayman Crow Laboratory of Physics, Washington University, St. Louis

(Received November 13, 1937)

When electrons of sufficient speed pass through helium under conditions favorable to single scattering, the electrons scattered through a suitable angle fall into two distinct classes, those scattered elastically and those scattered inelastically. The former have been scattered by nuclei and the latter by atomic electrons. Because the atomic electrons are in random motion, those electrons which have been scattered by them through a definite angle have a distribution of energies, the most probable energy being that corresponding to a collision with an atomic electron at rest. Jauncey has shown that when a fast electron of energy V_0 collides with an atomic electron having a component velocity u in a certain direction, the electron will have energy given by $V=V_0\cos^2\theta+u(2mV_0/e)^{\frac{1}{2}}\sin\theta$, where θ is the angle of scattering. It can be shown to follow from this relation that the *distribution of energy* among the scattered electrons is identical with the *distribution of component velocities* among the atomic electrons. Moreover, since the last mentioned distribution is closely related to, and identical in shape with, the profile of the Compton modified band in x-ray scattering, measurements of the energy distribution of the scattered electrons will give an experimental determination of the profile of the band. Wave mechanical computations lead to a definite shape for this profile which can now be tested by experiments on electron scattering. A beam of electrons, with energies between 1000 and 4000 volts, was directed into helium at a low pressure and the distribution of energies of electrons scattered at 34.2° measured. It was found that the experimental results gave a profile for the Compton modified band in excellent agreement with the profiles calculated by Hicks and in good agreement with those calculated by Kirkpatrick, Ross and Ritland. Values for the probability of the various component velocities of the atomic electrons are tabulated.

Introduction

THIS paper gives an account of a new method for measuring the distribution of velocities of the electrons in an atom. It is based on the remarkable similarity between the scattering of photons by electrons and the scattering of electrons by electrons, and is therefore intimately related to the Compton effect. Ever since Jauncey gave the essential theory in 1924–25 it has been known that the profile of the modified line in the Compton effect gives, in principle, direct information as to the distribution of velocities among the electrons in the atoms of the scattering material. Although the theory is simple and straightforward, it has been quite difficult to secure reliable experimental x-ray data to which to apply the theory. Only in the last few years have reliable experimental data been secured for solids, and only in the last two years have successful measurements been made on gases. The corresponding measurements in electron scattering are, in certain respects at least, more easily made. (1) The probability of scattering of electrons by electrons is far greater than that of photons by electrons. (This is illustrated by the fact that Debye-Hull-Scherrer photographs require exposures running into hours, while the corresponding electron diffraction photographs are obtainable in seconds.) (2) The width of the band representing the distribution of energy among the inelastically scattered electrons relative to their mean energy is ten to twenty times greater than the width of the Compton band relative to the wave-length of its center. Since both these widths lead to values for the atomic electron velocities, the values determined by electron scattering measurements should be the more accurate. It may well be that electron scattering measurements will surpass x-ray scattering measurements as a means of studying atomic electron velocities.

Theory

X-ray scattering

When a monochromatic beam of x-rays is scattered by matter, there is found in the scattered radiation, some radiation of longer wave-length than the primary (the "modified" radiation) as well as radiation of the same wave-length as that of the primary (the "unmodified" radiation). The presence of the modified radiation, quite unexplainable on the classical theory of x-ray scattering, was accounted for by a new point of view due to A. H. Compton. He postulated that the radiation is made up of photons to which the conservation laws apply. On carrying through the calculation, it is found that, when a photon is scattered in a definite direction from its original direction by collision with an electron, originally at rest, the energy of the photon is reduced by a definite amount (depending on the original energy of the photon and on the scattering angle θ) which manifests itself by a change in wave-length (depending only on θ and not at all on the original wave-length). The wave-length of the modified radiation, λ', is given by

$$\lambda' = \lambda + (2h/mc) \sin^2 (\theta/2) \tag{1}$$

where λ is the wave-length of the original radiation, h is Planck's constant, m is the mass of the electron and c is the velocity of light.

According to the theory as outlined in the foregoing paragraph, the modified radiation would be just as monochromatic as the primary radiation. However it was noticed, even in the early days of the Compton effect, that the modified radiation was more diffuse than either the primary radiation or the unmodified radiation. Jauncey[1] proposed a quantitative explanation of the diffuseness of the modified radiation. The essential point in his theory is this. The electrons in the atom with which the photons collide are not at rest, but, on the contrary, are in random motion, so that the energy with which the photon comes away from a collision in a direction θ depends on the magnitude and direction of motion of the electron at the moment of impact. The wave-length of the modified radiation differs from λ', defined by Eq. (1) by an amount λ'' which is given by

$$\lambda'' = u(2\lambda/c) \sin (\theta/2), \tag{2}$$

where u is the component of the initial velocity of the atomic electron along the bisector of the angle between the direction of motion of the primary radiation and the reverse of that of

[1] G. E. M. Jauncey, Phil. Mag. **49**, 427 (1925); Phys. Rev. **25**, 723 (1925); Phys. Rev. **46**, 667 (1934).

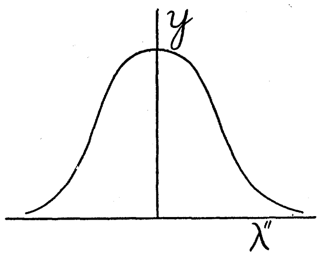

FIG. 1. Profile of the Compton modified band.

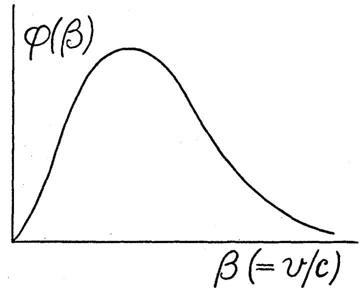

FIG. 2. Distribution of resultant speeds of atomic electrons.

the scattered radiation. Thus, corresponding to each value of the component of velocity u, which the electron had just before impact, there will be modified radiation whose wavelength is displaced from the value λ' by the amount λ''. Since, in general, u may have any value whose probability diminishes monotonically on both sides from $u=0$, we should expect to find the modified radiation to be a band instead of a line. Conversely, a measurement of the distribution of intensity in the modified band, gives information as to the velocity of the electrons in the atom. The importance of this was first realized by Jauncey, who showed how the then accepted values of the velocities of the electrons in the Bohr circular orbit theory and in the Bohr-Sommerfeld elliptical orbit theory could be used to predict the distribution of intensity across the Compton modified band. Comparison with experiment would then afford a decisive test of these theories of the atom insofar as they predict atomic electron velocities.

When Jauncey first proposed his theory of the modified radiation, the most recent atomic theory available was the Bohr-Sommerfeld theory of elliptical orbits, and it was perfectly natural to discuss the application of his theory to this model. It is entirely incorrect to imply, as DuMond[2] has done, that Jauncey's theory became obsolescent with the replacement of the Bohr-Sommerfeld atom by the wave mechanics atom. Jauncey's theory is quite independent of any particular atomic model; on the contrary, its purpose is to provide a test of whatever distribution of velocities the prevailing atomic theory predicts, whether this theory be the Bohr-Sommerfeld theory, the wave mechanics theory, or the post wave mechanics theory when it comes. In fact, the essence of Jauncey's theory is that it provides a method for finding the distribution of velocities of the atomic electrons from observations on the distribution of intensity in the Compton modified band. DuMond also comments on Jauncey's theory adversely because Jauncey places "an unfortunate emphasis" on the energy of the electron rather than on the momentum, and again because he stresses position rather than momentum. This criticism seems to be without much point, for in the atoms discussed by Jauncey there is surely a one to one correspondence between electron energy probability and electron momentum probability. Any experimental approach which directly measures one of these probabilities provides a test of the theory and indirectly of its predictions as to the other probability. Quite apart from these considerations, it is quite evident from the vector diagrams, used by Jauncey in his original papers to illustrate his derivations, that he was relating wave-lengths in the modified band directly to the velocities of the atomic electrons.

A brief comment on the methods by which the experimental data may be used to give the distribution of speeds among the atomic electrons is in order at this point. If the distribution of intensity across the Compton modified band be represented by Fig. 1, then the distribution of atomic electron speeds (i.e., resultant velocities assuming of course that they are isotropic as to direction) is given as $\phi(\beta)$ in Fig. 2 by means of the relation

$$\phi(\beta)=K\lambda dy/d\lambda. \tag{3}$$

This relation in this explicit form is due to DuMond. However, substantially the same idea was used by Jauncey in 1925 in the reverse

[2] Jesse W. M. DuMond, Rev. Mod. Phys. 5, 1 (1933); Phys. Rev. 52, 419 (1937).

order. He assumed certain speed distributions among the atomic electrons and calculated the shape of the Compton modified band. A far more elegant and general method of handling the experimental data was developed later by Jauncey.[3] He noticed that the linear relation between λ'' and u in Eq. (2) immediately led to the following result. If $f(u)du$ is the number of electrons with velocity components between u and $u+du$, then the number of scattered photons with wave-lengths between λ'' and $\lambda''+d\lambda''$ (note that λ'' is measured from the center of the Compton modified band) is $f(\lambda''c/2\lambda \sin(\theta/2))d\lambda''$, or $f(\lambda''\times\text{const.})d\lambda''$ when λ and θ are given. In other words, this states that the *shape of the Compton modified band is identical with the shape of the component velocity distribution function* of the atomic electrons producing the band. Thus Fig. 1, which is obtained directly from experiment, gives at once the shape of the component velocity distribution function which can then be compared with curves obtained theoretically. It is no longer necessary to go through the laborious process of computing the resultant velocities to test a theory.

Electron scattering

It will be shown that the inelastic scattering of electrons by atoms may be identified with the scattering of the electrons by one or other of the atomic electrons, and that the cooperation of two or more scattering centers (be the other center the nucleus or another atomic electron) plays a negligible part if the impinging electrons have sufficient speed. When an electron approaches a center of force, repelling or attracting according to the inverse square law, the resulting deflection, θ, is determined by what, in simple orbit theory, is called the "collision parameter." For the case of an electron of mass m and speed v moving towards an isolated nucleus of charge Ze,

TABLE I. *Collision parameters.*

V	p_n	p_e
1,000 volts	0.047×10^{-8} cm	0.021×10^{-8} cm
2,000	0.023	0.0105
5,000	0.0094	0.0045
10,000	0.0047	0.0021

[3] G. E. M. Jauncey, Phys. Rev. **46**, 667 (1934).

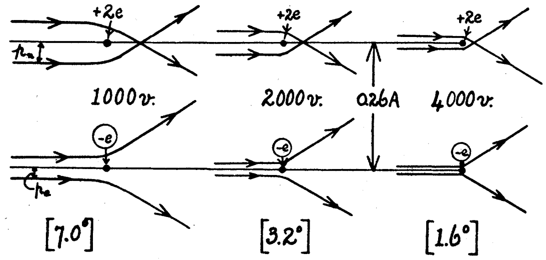

FIG. 3. Collision parameters for the scattering of an electron by a nucleus and by an atomic electron, calculated for a 34.2° deflection.

the relation between the collision parameter p_n and the deflection θ is

$$p_n=(Ze^2/mv^2)\cot(\theta/2)=(300Ze/2V)\cot(\theta/2) \quad (4)$$

when the energy of the impinging electron is V electron volts. For the case of an electron moving towards another electron (initially at rest) the relation between the collision parameter p_e and θ is

$$p_e=(2e^2/mv^2)\cot\theta=(300e/V)\cot\theta. \quad (5)$$

For the specific case experimentally investigated in this paper, *viz.*, a deflection of electrons through 34° on collision with helium atoms, Table I gives values of p_n and p_e for various electron energies. In Fig. 3 we compare the collision parameters with the most probable distance between the helium nucleus and an atomic electron, which is 0.26×10^{-8} cm. It is evident from this diagram that we are justified in regarding the scattering of electrons through a moderately large angle (e.g., 34°) by an atom of helium as being due *either* to the nucleus *or* to one of the atomic electrons, whenever the collision parameters are sufficiently small in comparison with the atomic electron-to-nucleus distance.[4] We may therefore regard helium gas as providing a mixture of nuclei and atomic electrons which scatter independently of one another, when the impinging electrons have enough speed.

[4] It is probably superfluous to mention that the atomic electron-to-nucleus line is not always perpendicular to the line of approach of the impinging electron, as in Fig. 3. On relatively rare occasions the nucleus and one atomic electron may be so orientated that both contribute deflections of the same order of magnitude to give the observed deflection. The contention is merely that the faster the impinging electron the smaller are the collision parameters and the less often do we have any departure from what may conveniently be called "single center scattering."

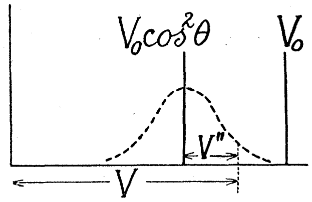

FIG. 4. Division of scattered electrons into an elastically scattered group and an inelastically scattered group.

The atom as a whole plays no part in the scattering of individual electrons. All that the atom, as such, does is to provide the distribution of velocities among the electrons which do the scattering.

Scattering by a nucleus is distinguished from scattering by an atomic electron by the fact that in the first case the collision is "elastic," that is the electron loses no energy, while in the second case the collision is "inelastic," that is, the electron loses some of its energy to the atomic electron during the scattering process. If the atomic electron is initially at rest, the energy which the impinging electron retains is

$$V = V_0 \cos^2 \theta = V_0 - V_0 \sin^2 \theta. \tag{6}$$

This is clearly analogous to Eq. (1), which gives the wave-length of the photon after colliding with an electron. We should expect to find the electrons scattered through a selected angle by a gas such as helium to be divided into two distinct groups, those which have been scattered by the nuclei and which have lost no energy, and those which have been scattered by the atomic electrons and which, if the atomic electrons were initially at rest, would all have one and the same energy, $V = V_0 \cos^2 \theta$. If the atomic electrons are moving at random with various speeds the electrons scattered inelastically will have a distribution of energies centered around $V_0 \cos^2 \theta$ (shown as a dotted line in Fig. 4).

The next step is to show how an experimental determination of the distribution of energy among the inelastically scattered electrons may be used to find the distribution of velocities among the atomic electrons. The argument is due to Jauncey[5] and runs closely parallel to his theory of photon scattering. An electron moving with a velocity v_0, in the direction shown in

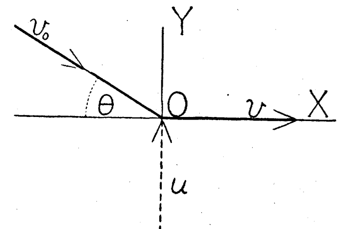

FIG. 5. Collision between a fast electron and an atomic electron.

Fig. 5 is deviated through an angle θ by a collision with an atomic electron at O, so that after collision it has a velocity v. The atomic electron may be moving with any velocity and in any direction. Jauncey finds that the *energy* of the impinging electron after collision is

$$V = V_0 \cos^2 \theta + u(2mV_0/e)^{\frac{1}{2}} \sin \theta - u^2(m/2eV_0) \tan^2 \theta, \tag{7}$$

where u is the *component of velocity* of the atomic electron in a direction which, to a first approximation, coincides with OY (Fig. 5).

On carrying the analysis to a second approximation it is found that the direction of the component of velocity to be used in Eq. (7) makes a small angle δ with OY such that

$$V \cos^2 \delta = V_0 \cos^2 (\theta + \delta).$$

We may illustrate the application of this to a particular case, for which $V_0 = 2000$ volts, and $V_0 \cos^2 34° = 1377$ volts. The values of δ corresponding to V (or to V'') are as shown in Table II. While this result is of interest, it has no direct bearing on our investigation on the scattering by helium, for the distribution of velocities is isotropic and the component velocity has the same value irrespective of direction. However, in principle though possibly not in practice, it would be necessary to consider it in the application of the method of electron scattering to the study of the probably asymmetrical distribution of electron velocities in the surfaces of solids.

Neglecting the third term in Eq. (7) we may rewrite it as

$$V'' = V - V_0 \cos^2 \theta = u(2mV_0/e)^{\frac{1}{2}} \sin \theta, \tag{8}$$

TABLE II. *Location of component velocity vector.*

V	V''	δ
1177 volts	−200 volts	+6° 21′
1277	−100	+3° 03′
1377	0	0°
1477	+100	−3° 33′
1577	+200	−7° 31′

[5] G. E. M. Jauncey, Phys. Rev. **50**, 326 (1936).

where V'' is the surplus energy which the scattered electron acquires on collision with an atomic electron having a component velocity u, over what it would have been had the component velocity been zero. (The "surplus" may be positive or negative.) Attention is called to the striking parallelism between Eq. (8) for electron scattering and Eq. (2) for photon scattering. The significant implication discovered by Jauncey for the linear relation between λ'' and u in Eq. (2) applies to Eq. (8) in precisely the same way. According to Eq. (8), an atomic electron with a component velocity u, gives rise to an inelastically scattered electron, moving in a direction θ, with energy V, or what amounts to the same thing with energy differing from $V_0 \cos^2 \theta$, the value for a collision with an atomic electron at rest, by an amount V''. Let the number of atomic electrons with component velocities between u and $u+du$ be $f(u)du$, and let these give rise to $F(V'')dV''$ inelastically scattered electrons (in the direction θ). Then

$$f(u)du = F(V'')dV'' \times \text{const.} \tag{9}$$

Because of the *linear* relations between V'' and u in Eq. (8), we can write $dV'' = du(2mV_0/e)^{\frac{1}{2}} \sin \theta$, and so change Eq. (9) to

$$f(u) = F(V'') \times \text{const.} \tag{10}$$

where the quantity $(2mV_0/e)^{\frac{1}{2}} \sin \theta$ is absorbed in the constant, since we can treat V_0 and θ as constants. The importance and usefulness of this relation can hardly be overestimated. It tells us that the shape of the experimentally measured *distribution of energies* among the inelastically scattered electrons, *viz.*, $F(V'')$, gives simultaneously the shape of $f(u'')$, the *distribution of component velocities* of the atomic electrons. Any theory of the atom which gives a definite prediction as to the distribution of component velocities among its atomic electrons, can be tested by direct comparison with the experimentally obtained distribution of energies among the inelastically scattered electrons.

Approximations

Certain approximations have been made in deriving the formulas which are to be applied to the experimental data. These will now be discussed with special reference to the particular angle of scattering and electron energies used in the experimental work.

Although a rigorous investigation of the effect of disregarding relativity has not been made, it seems safe to conclude that the fractional change which would be introduced into any formula because of relativity would be of the order of v^2/c^2, or less. Even with the fastest electrons used in these experiments (4000 volts), v^2/c^2 amounts to only 0.016, and may therefore be neglected.

In the derivation of Eq. (6), the atomic electrons were regarded as free, i.e., the binding energy of the atom was not taken into account. The effect of taking the binding energy into account may be estimated by considering the special case in which the atomic electron is at rest. We shall assume that the energy and momentum imparted to the atom may be neglected and that the energy for ionization is supplied by the impinging electron. We then have

$$mv_0^2/2 = mv^2/2 + mw^2/2 + E \tag{11}$$

for the energy balance, and

$$mv_0 = mv \cos \theta + mw \cos \phi, \tag{12}$$

$$0 = mv \sin \theta - mw \sin \phi, \tag{13}$$

for the momentum. Here v_0, v, and w are the velocities of the impinging electron before and after the collision and of the recoil electron after collision, E is the energy of binding (corresponding to the ionization potential), and θ and ϕ are the directions in which the two electrons move after collision. Except for the appearance of E in the energy equation, these equations are identical with those from which Eq. (6) was derived. An approximate solution is

$$V = V_0 \cos^2 \theta - E. \tag{14}$$

This means that the effect of taking the binding energy into account is to shift the center of the inelastic band towards the origin, i.e., away from the elastic band, by an amount E. A shift in the position of the center of the Compton modified band has been attributed to the effect of binding by DuMond, Ross, Kappeler, and Burkhardt, though the method of taking it into account differs from that used here. For 2000 volt electrons impinging on helium atoms (for which E is 25 electron volts) the effect of allowing for E is to shift the center of the band of inelastically scattered electrons from 1368 to 1343 volts. A shift of this amount could be produced by an error in the scattering angle of 0.8°, without bringing E in at all, so that to establish, with certainty, the effect of binding requires the angle of scattering to be known to better than about 0.2°. This is not easy for one has to evaluate correctly the effect of the slight spread (±0.3°) in the angle of scattering, due to the finite width of the slits defining the electron beam entering the analyzer, in addition to determining the angle exactly.

The assumption that "single center scattering" occurs, that is, that the scattering of a sufficiently fast electron is the result of its interaction with a single center (a nucleus or one atomic electron) is an approximation which needs to be examined. The angles in parentheses in Fig. 3 are those through which the attraction of the nucleus would deflect an electron passing close to the atomic electron which is assumed to be 0.26×10^{-8} cm away. The observed deflection, *viz.*, 34°, is really due to the joint action of the atomic electron and the nucleus (and also the other atomic electron which we shall neglect). If we may assume, merely as a rough approximation, that the observed deflection is the sum of what the nucleus and the atomic electron would produce if acting separately, then the actual deflection of a 4000 volt electron by an atomic electron would be 34°±1.6°, i.e., 35.6° and 32.4° (two values because the atomic electron may be on either side of the nucleus). If, in addition, it be assumed that the energy change is associated with only that part of the deflection attributed to the atomic electron, we shall have two values, $V_0 \cos^2 35.6°$ and $V_0 \cos^2 32.4°$ for the energy after collision. In this extremely simplified picture, in which the atomic electron is at rest and located either as shown in Fig. 3, or on the opposite side of the nucleus, we should expect to find inelastically scattered electrons with energies differing by ±3.8 percent from that found on the assumption that the atomic electron acts strictly alone. As an alternative mode of approach, we may assume that the departure from ideal "single center scattering" in the case of inelastic scattering is of the same order of magnitude as for elastic scattering. It will be recalled that, in elastic scattering, a quantity $(Z-F)$, where Z is the atomic number and F the atom form factor, enters in place of Z, which would appear by itself if nothing but the nucleus were to be considered. For $\theta=34°$, and 4000 volt electrons impinging on helium atoms, F is 1.3 percent of Z. Conversely we may assume that the effect of the nucleus and the other atomic electron in modifying the deflection produced by the atomic electron near which the incoming electron passes, is of the same order of magnitude. It is not difficult to see in a general way, that this will result in replacing inelastically scattered electrons of one and the same energy, *viz.*, $V_0 \cos^2 \theta$ by a group whose half-width will be of the order of the first or second power of the 1.3 percent. Both lines of argument lead to the result that 4000 volt electrons, scattered at 34°, by atomic electrons *at rest* will have energies distributed over a band whose half-width is of the order of a few percent of the energy corresponding to the center of the band. When, however, the atomic electrons are in motion, as they must be in actual atoms, this motion of itself results in the energy of the inelastically scattered electrons being spread out over a band of considerable width, so that the effect of the departure from ideal "single center scattering," considered in this paragraph, on the *shape* of the band may be considered almost negligibly small.

The error resulting from neglecting the term in u^2 in Eq. (7) will now be considered. On writing $V''=V-V_0\cos^2\theta$, and solving Eq. (7) for u, we obtain

$$u=(V_0e/2m)^{\frac{1}{2}}(\cos^2\theta/\sin\theta)\{V''/V_0\cos^2\theta+(\tfrac{1}{4})(V''/V\cos^2\theta)^2\},\quad(15)$$

or

$$u=Ax(1+x/4),\quad(16)$$

where A and x are $(V_0e/2m)^{\frac{1}{2}}(\cos^2\theta/\sin\theta)$ and $V''/V_0\cos^2\theta$, respectively. The number of atomic electrons with *component velocities* between u and $u+du$ is $f(u)du$. These will scatter a certain number, $F(V'')dV''$ of the impinging electrons through an angle θ into an *energy* range dV'' at V''. (V'', being defined as $V-V_0\cos^2\theta$, is the excess energy which the scattered electron has as a result of the atomic electron having a com-

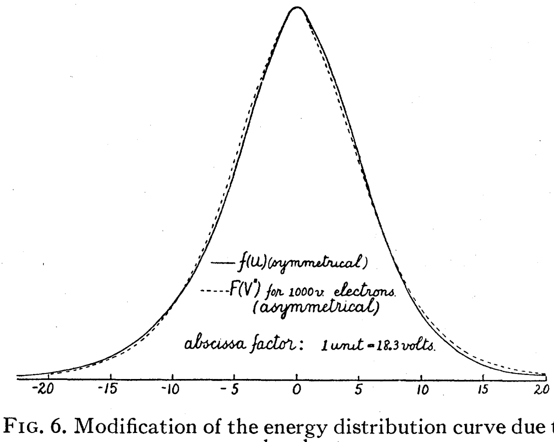

FIG. 6. Modification of the energy distribution curve due to a second-order term.

ponent velocity u over the value $V_0 \cos^2 \theta$ which it would have for the case $u=0$.) We can write

$$f(u)du = \text{const. } F(V'')dV''$$
$$= \text{const. } V_0 \cos^2 \theta \cdot F(x \cdot V_0 \cos^2 \theta)dx,$$

when V_0 and θ are held constant. Since Eq. (16) yields

$$du = A(1+x/2)dx$$

we have

$$f(u) = F(x \cdot V_0 \cos^2 \theta)(1+x/2)$$
$$= F(V'')/(1+x/2), \quad (17)$$

where constant quantities are ignored. To obtain $f(u)$ from $F(V'')$ which is given directly by experiment, we must divide each ordinate by $(1+x/2)$. However the new curve gives $f(u)$ in terms of x as abscissa and we need it in terms of u. The u value for each x can be obtained by writing $u = Ax(1+x/4)$ which gives a nonuniform scale in u. To get a uniform scale in u, all that is necessary is to take each point on the new curve, $F(V'')/(1+x/2)$, and displace it horizontally by an amount $(1+x/4)$. Thus to get $f(u)$ in terms of u uniformly spaced along the abscissa, we first contract each ordinate of $F(V'')$ in the ratio $1 : (1+x/2)$ and then expand each abscissa in the ratio $1 : (1+x/4)$. We shall illustrate the effect of neglecting the term in u^2 in the case where it should be most noticeable in our experiments, i.e., for the slowest electrons, of 1000 volts energy. We shall assume $f(u)$ to have the shape shown in Fig. 6, a shape given by wave mechanics and shown to be substantially accurate by our experiments. Since $f(u)$ represents the distribution of component velocities in the atom it must be symmetrical about the origin. We have seen that if we neglect the u^2 term, the shape of the $F(V'')$ curve giving the distribution of energies among the scattered electrons is identical with that of $f(u)$. But if we take account of the u^2 term, we can get $F(V'')$ from $f(u)$ by reversing the procedure outlined in the previous paragraph. The result is that $F(V'')$ is no longer identical in shape with $f(u)$ but has a shape like that of $f(u)$ slightly warped, as shown in Fig. 6. The predicted asymmetry in $F(V'')$ is probably too small to be detected experimentally at present.[6]

The errors introduced by the various approximations discussed above are of the order of the experimental uncertainties in the present measurements and may therefore be disregarded. However, they are not negligibly small. If the precision of measurement could be improved by a factor of five or ten, it would probably be necessary to allow for them in comparing theory with experiment.[7]

All sources of error in the accuracy of the simple formula, Eq. (10), except the one due to the neglect of relativity, become less the greater the energy of the impinging electron. Unfortunately, the *number* of electrons scattered diminishes as the square of their initial energy, so that the scattered electron currents are too small to measure accurately when the electron energy is increased beyond a certain value. With our present apparatus and methods of measurement this point is reached when the electron energy exceeds about 5000 volts.

Energy distribution curves: theoretical

We have seen that, on Jauncey's theory, $f(u)$ is identical in shape with $F(V'')$, the experi-

[6] This kind of asymmetry should, in principle, be found in the shape of the Compton band. Professor Jauncey has shown how it leads to a slight displacement in the position of the maximum from that given by the simple theory. A displacement has been observed experimentally by Ross and Kirkpatrick and by DuMond and Kirkpatrick, who however have not taken the effect discussed here into account in their explanation of the displacement.

[7] While the error introduced by the neglect of the u^2 term can be taken into account quantitatively, that arising from the lack of perfect "single center scattering" can only be estimated qualitatively. This indicates that, even if the precision were improved, it might not be possible to check the first effect quantitatively, because it would always be more or less masked by the second effect. In principle such a check could be made in x-ray scattering insofar as perfect "single center scattering" prevails.

mentally determined distribution of energies among the inelastically scattered electrons about the center of the band. The shape of $f(u)$ is given at once merely by re-labeling the abscissas in terms of u instead of V'', the conversion being effected by means of Eq. (8). The curve so obtained gives us what may be regarded as an experimentally determined distribution of atomic electron component velocities, which may, if one desires, be compared with curves predicted by theory. However the results of the two available theoretical investigations give, not $f(u)$, but $f(\lambda)$, the closely related profile of the Compton modified band. It is necessary to show how such profiles can be related to our electron energy distributions. On eliminating u between Eqs. (2) and (8), we get

$$V''=(\lambda''/\lambda)\cos(\theta/2)(2mV_0/e)^{\frac{1}{2}} \tag{18}$$

$$=1011(\lambda''/\lambda)\cos(\theta/2)V_0^{\frac{1}{2}} \tag{19}$$

when V'' and V_0 are in volts. By means of this equation we can change from abscissas in λ'', used in plotting the profile of the Compton band, to abscissas in V'' used in giving the results of electron scattering experiments.

Kirkpatrick, Ross and Ritland,[8] using screening data from Slater, have calculated the profiles of the Compton modified bands for the first eighteen elements. They give the profiles, $f(\lambda'')$, for various values of λ'' (in XU) for the case of $\lambda=695$ XU and $\theta=90°$. To get the values of λ'' appropriate to scattering at 34.2°, the angle used in our experiments, a consideration of Eq. (2) shows that the values of λ'' used by these authors in $f(\lambda'')$ for 90° must all be multiplied by $\sin(34.2°/2)/\sin(90°/2)$. Hence to get the values of V'' for 34.2° electron scattering from the values of λ'' for 90° x-ray scattering, we must use

$$\begin{aligned}V''&=1011(\lambda''/695)\cos(34.2°/2)V_0\\&\quad\times\sin(34.2°/2)/\sin(90°/2)\\&=0.5782\lambda''V_0^{\frac{1}{2}}.\end{aligned} \tag{20}$$

Hicks[9] has calculated the shape of the Compton modified band for helium and hydrogen by methods which are considered to be more satisfactory in principle than those using screening data. He gives his values in terms of a non-dimensional quantity, β, which is defined by DuMond's equation

$$\lambda''=\beta(\lambda'^2+\lambda^2-2\lambda'\lambda\cos\theta)^{\frac{1}{2}}. \tag{21}$$

β is also u/c, the ratio between the component electron velocity and the velocity of light. To a close approximation Eq. (21) may be written

$$\lambda''=\beta 2\lambda\sin(\theta/2), \tag{22}$$

which is equivalent to Eq. (2).

To compare Hicks' values with those given by Kirkpatrick, Ross and Ritland for the special case calculated by the latter, we put $\lambda=695$ XU and $\theta=90°$ into Eqs. (1) and (21), and get

$$\lambda''=999.4\times\beta\ (\mathrm{XU}). \tag{23}$$

(The approximate formula, Eq. (22), gives a result only 1.7 percent less.) Since the values of β used by Hicks in his graph are 2, 4, 6, $\cdots\times10^{-3}$, Eq. (23) gives the corresponding sequence $\lambda''=2, 4, 6, \cdots$ in XU. Table III gives (1) the values (denoted by K.R.R.) of $f(\lambda'')$ calculated by Kirkpatrick, Ross, and Ritland for $\lambda=695$ XU and $\theta=90°$, (2) those (denoted by "H-4") computed from Hicks' data for the same λ and θ as outlined above, and (3) the corresponding abscissas in V'' for electron scattering through 34.2° of electrons whose initial energy is V_0 electron volts. Thus, by plotting $f(\lambda'')$ against V'' for the values of V_0 used in our experiments, we

TABLE III. *X-ray and electron scattering: theoretical.*

X-ray Scattering $\lambda=695$ XU, $\theta=90°$			Electron Scattering for Electron Energy V_0 and $\theta=34.2°$
λ''	$f(\lambda'')$ K.R.R.	$f(\lambda'')$ H-4	V''
0 XU	60.0	60.0	0
1	56.3	58.7	$0.578\times V_0^{\frac{1}{2}}$ volts
2	52.2	54.5	1.156
3	47.0	48.5	1.735
4	41.0	41.6	2.313
5	34.8	34.6	2.890
6	28.2	28.2	3.469
7	23.0	22.5	4.047
8	18.1	17.9	4.626
9	14.1	14.2	5.204
10	11.4	11.2	5.782
12	6.95	7.04	6.938
14	4.03	4.52	8.095
16	2.46		9.250
18	1.34		10.407
20	0.90	1.34	11.563

[8] P. Kirkpatrick, P. A. Ross and H. O. Ritland, Phys. Rev. **50**, 928 (1936).
[9] B. Hicks, Phys. Rev. **52**, 436 (1937).

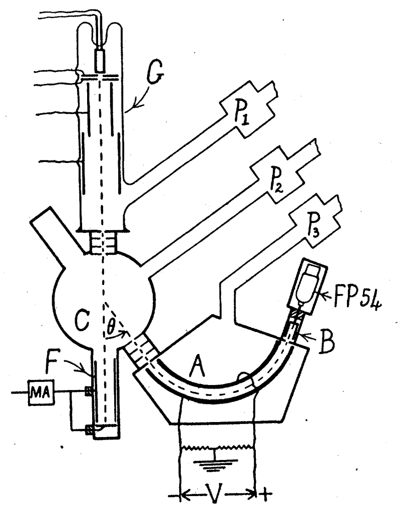

FIG. 7. Apparatus.

can compare the form of the curve predicted by theory with that found by experiment.

EXPERIMENTAL

Apparatus

The apparatus consists essentially of three parts, an electron gun G, a collision chamber C, and an electrostatic analyzer, A (Fig. 7). Several types of electron gun were tried. The one finally used consists of an indirectly heated cylindrical cathode emitting electrons from the flat end which is perpendicular to the axis of the tube.[10] The focusing arrangement consists of two plates, each with a circular hole, and two cylindrical anodes, arranged co-axially. By carefully choosing the voltages applied to the plates and anodes, it is possible, with 4000 volts potential on the second anode, to obtain electron currents up to 150 microamperes through 1.5 mm holes into the collision chamber. With helium at about 0.005 mm pressure in the collision chamber, the path of the electron beam is clearly seen as a narrow pencil of light, not more than about 2 mm in diameter, and showing no visible divergence. The total electron current passing across the chamber into the Faraday cage F is measured by a microammeter. The bottom of the Faraday cage can be moved aside by means of a ground glass joint (not shown) so as to allow the electron beam to strike a fluorescent screen. This is helpful in making adjustments of the voltages to give the best possible beam. A set of four slits, spaced over a length of 52 mm, serves to define the direction of the scattered electron beam entering the analyzer. The scattering angle, i.e., the angle between this direction and that of the primary beam from the gun is 34.2°. Owing to the finite width of the slits, the electrons which are accepted by the slit system have been scattered through $34.2° \pm 0.3°$. The electrostatic analyzer has been described before,[11] and so merely the dimensions of this particular one will be given. The radii of the deflecting plates are 5.50 cm and 6.50 cm. The entrance slit is 0.3 mm by 5 mm, and the exit slit (i.e., into the collector B) 0.47 mm by 8 mm. The collector is connected to an FP-54 tube which is mounted in a vacuum and as close to it as possible. The associated circuit is that described by DuBridge and Brown.[12] A resistor of 5×10^{11} ohms is used to shunt the control grid of the tube to the filament. The output is measured by means of a Leeds and Northrup, type R, galvanometer. The sensitivity of the system is 50,000 divisions per volt, so that with the resistor referred to above, 1 division deflection corresponds to 4×10^{-17} amp. The high sensitivity was necessary because even the largest inelastic current obtained under the most favorable conditions did not exceed 5×10^{-15} amp.

While it is necessary to have helium at a pressure of about 0.008 mm in the collision chamber to scatter enough electrons to measure, it is very necessary to maintain as high a vacuum as possible in the electron gun and analyzer. The gun is separated from the collision chamber by four discs, each with a hole 1.5 mm in diameter through which the electrons pass into the collision chamber. The analyzer is likewise separated from the collision chamber by a set of four slits, the two outermost, 0.3 mm by 5 mm, defining the scattered electron beam and the two innermost, somewhat larger, helping to retard the flow of gas. When the electron gun, the collision chamber, and the analyzer are pumped out by separate pumps and when gas is

[10] We take pleasure in acknowledging our thanks to Dr. R. G. Hergenrother for supplying us with the electron gun used in most of the measurements.

[11] A. L. Hughes and J. H. McMillen, Phys. Rev. **34**, 291 (1929); **39**, 585 (1932).

[12] L. A. DuBridge and Hart Brown, Rev. Sci. Inst. **4**, 532 (1933).

allowed to leak directly into the collision chamber, it is possible, because of the restricted communicating passages between the various regions, to obtain a vacuum in the gun and analyzer one hundred times better than that in the collision chamber. Three two-stage oil diffusion pumps, containing Apiezon oil, are connected respectively to the electron gun, the collision chamber, and the analyzer, by means of short tubes, 3 cm wide. Each pump has a speed of 9.5 liters per second, measured at its intake. Cooled traps, located between the pumps and the parts to which they are connected, prevent oil vapors diffusing into the apparatus. The three two-stage pumps are backed by a common one-stage Apiezon oil pump, which in turn is backed by a fast mechanical pump. To control the pressure in the collision chamber, a specially designed stopcock, whose opening can be changed from about 1 cm^2 down to zero, is placed between the collision chamber and its pump. Helium, carefully purified by passing through two charcoal tubes in liquid air, is stored in a sylphon reservoir, whose maximum volume is about 2 liters. The helium passes into the collision chamber through a very fine capillary glass tube and then through a well outgassed charcoal tube immersed in liquid air. The driving pressure can be varied over a limited range by altering the size of the sylphon reservoir. With all pumps in action it is possible to obtain a pressure below 5×10^{-5} mm in both gun and analyzer while maintaining a useful working pressure of about 5×10^{-3} mm in the collision chamber.

The high voltages necessary to operate the electron gun are supplied by suitable power packs. At first the voltage for the deflecting plates in the analyzer was obtained from a power pack, but as this proved to be rather unsteady it was replaced by a stack of heavy duty Burgess "B" batteries and a set of 30 flashlight batteries so that any voltage up to 1200 volts could be obtained in steps of 1.5 volts. The steadiness of the deflecting voltage is excellent.

Results

Next we shall describe the procedure followed in taking observations. The first thing is to obtain the best possible vacuum conditions,

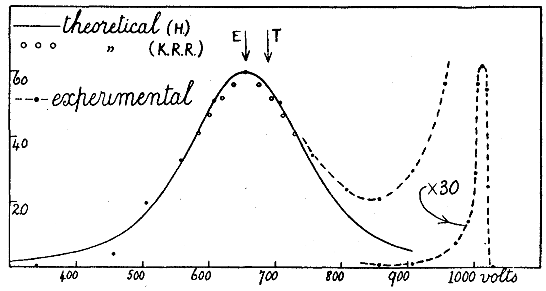

FIG. 8. Energy distribution of 1010 volt electrons scattered at 34.2°.

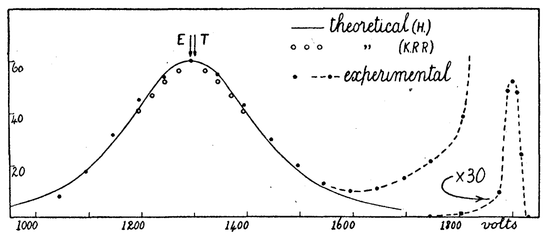

FIG. 9. Energy distribution of 1900 volt electrons scattered at 34.2°.

which may take several hours pumping if the apparatus has been opened to the atmosphere. Then the cathode of the electron gun is heated and the voltages adjusted to give a large electron current through the collision chamber. After conditions have become steady the helium is allowed to flow in, the rate being controlled by changing the size of the sylphon reservoir and the aperture of the stopcock used as a choke. The electron current to the Faraday cage B is measured as the deflecting voltage in the analyzer is altered in small steps over the whole range over which scattered electrons are observed. The maximum of the elastic peak is located and the energy of the electrons is given by the value of the deflecting voltage, multiplied by a factor depending on the geometry of the analyzer. This is found to check well with the value of the voltage accelerating the electrons into the collision chamber, as of course it should. It is found that it is better to complete the exploration of the inelastic peak before making any measurements on the elastic peak. The relatively large currents found at the elastic peak tend to produce a temporary unsteadiness during which it is very difficult to measure the very small inelastic currents. It may be that a large current disturbs the conditions in the FP-54 tube and its associated circuit for an appreciable length of time, or

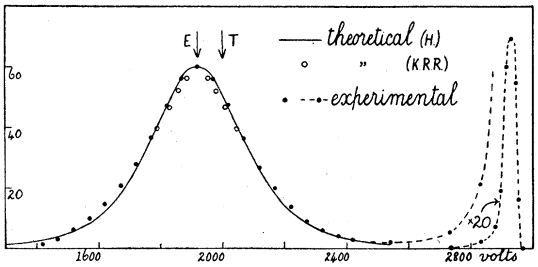

FIG. 10. Energy distribution of 2930 volt electrons scattered at 34.2°.

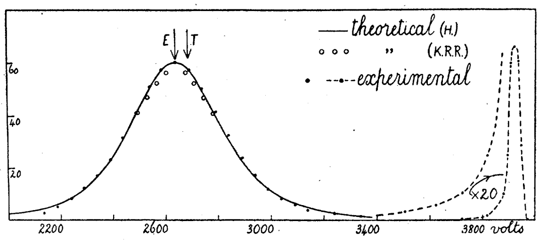

FIG. 11. Energy distribution of 3922 volt electrons scattered at 34.2°.

it may be that, in locating the elastic peak, relatively large electron currents are brushed over one or other of the deflecting plates giving rise to a pseudo contact potential which dies away slowly and irregularly. Whatever may be the explanation, the difficulty is avoided by completing the exploration of the inelastic peak before beginning measurements on the elastic peak.

The experimental independent variable is the deflecting voltage applied to the curved plates in the analyzer. We use, however, in Figs. 8–11, as abscissas for the distribution of energy curves, not the deflecting voltage, but the corresponding voltage which measures the actual energy possessed by the scattered electrons. To get the curve giving the true distribution of energy among the inelastically scattered electrons, it is necessary to divide the value of the electron current obtained at each deflecting voltage, by that voltage. This is because the range dV, of energies admitted by the analyzer, increases in proportion to V.[13] Our results for the scattering of electrons with energies close to 1000, 2000, 3000 and 4000 volts, respectively, are shown in Figs. 8–11. The dots in these diagrams are mean experimental values and were obtained in the following way. With the primary electron energy adjusted to a value close to one of the four listed above, the scattered electron current was measured as the deflecting voltage was raised in small steps from the lowest to the highest value at which measurable currents were obtained. This constituted one "run." Several such runs, requiring about three hours each, were made for approximately the same primary electron energy. The scale of ordinates for each run was adjusted so that the maximum of the inelastic band was always 60 units, and the curves were then shifted laterally (when necessary) by small amounts so as to have them centered as nearly as possible about the same voltage value, which was taken to be the center of the distribution curve. From the individual curves, so superposed, it was possible to draw a mean curve, and the dots shown in Figs. 8–11 were taken off the mean curves for the different primary energies. The number of dots shown in the figures referred to represents a much larger number of experimental measurements (e.g., the 23 dots on the inelastic portion of the 2930 volt curve were obtained by combining four separate runs involving 77 separate measurements).

[13] Similarly when an electron velocity spectrum is obtained by means of a magnetic field, each reading must be divided by the value of the magnetic field, H, at which it was obtained, to give a true distribution of velocities. The need for this step (division by V or H) has been overlooked by many investigators who have taken the measured electron currents as a function of the deflecting voltage to be the true distribution of energies (or velocities). For a very complete discussion of the matter, see a paper by R. Kollath, Ann. d. Physik **27**, 721 (1936).

Since this investigation has to do with the profile of the Compton modified band as determined from inelastic electron scattering, the location and shape of the elastic peak is not of primary importance. However, it is desirable to record how the mean elastic curves plotted in Figs. 8–11 were obtained. The elastic peak was plotted separately for each run and its height, width, and distance from the inelastic maximum was measured. Then a composite elastic peak having the average height and the average width, was placed at the average separation from the inelastic peak. Thus the elastic peaks in Figs. 8–11 were constructed and located. The theoretical resolution, determined by the dimensions of the analyzer, is $dV/V=0.015$, while the experimental resolution, given by the width of the peak at one-fifth of its height is about 0.02. (One-fifth is taken arbitrarily because below this the elastic peak broadens asymmetrically.)

Theoretically, the inelastic maximum should be found at $V=V_0\cos^2\theta$. Actually it is always found at a slightly smaller value (see Table IV and also E and T in Figs. 8–11). The irregularity in the differences suggest some inconstant experimental condition. It is possible that temporary pseudo contact potentials are set up when the strong beam

TABLE IV. *Location of the maximum of the inelastic band.*

Elastic Peak V_0	Inelastic Peak Theor.	Inelastic Peak Exp.	Difference
3922*v*	2683*v*	2640	43
2930	2002	1920	83
1900	1302	1295	7
1010	690	658	32

of electrons is swept across the deflecting plates in the process of locating the elastic peak with the result that the deflecting potentials necessary to bring the peak on the exit slit will be affected. Had the differences in the last column of Table IV been constant and of the order of 25 volts, one would have been tempted to attribute it to the effect of binding of the electrons in the helium atom, as outlined above in the section on "Approximations."

Discussion

The main purpose of this investigation is to determine the distribution of component velocities of the atomic electrons in the helium atom, which, as we have seen, gives simultaneously the profile of the Compton modified band. Investigations by Kirkpatrick, Ross and Ritland,[8] and by Hicks,[9] have led to theoretical predictions as to the shape of the profile of the band. To check these theoretical profiles against our experimental results it is necessary to convert the profiles ($f(\lambda'')$ plotted against λ'') into curves giving the theoretical distribution of energies among the inelastically scattered electrons by replacing abscissas in λ'' by abscissas in V''. The conversion factors are given in Table III. In Figs. 8–11, the continuous lines give the curves so calculated from Hicks' data. (Hicks gives two curves He-2 and He-4 which differ slightly. As the latter is believed to be the more nearly correct on theoretical grounds, we use it to compare with our results.) The open circles give the values calculated from the data of Kirkpatrick, Ross and Ritland in the region where they differ perceptibly from those of Hicks. It is clear that our experimental points fit the theoretical curves remarkably well, and that the fit improves with increasing electron energy. It may therefore be concluded that the theory as to the nature of inelastic scattering of fast electrons is satisfactory. This being so, the next step is to invert the procedure, and assume the theory to be true and use as many experimental measurements as possible to determine the most probable shape of the distribution of component velocities of the electrons in the helium atom. We shall arbitrarily exclude the measurements made with 1010 and 1900 volt electrons since theory suggests that the data with the faster electrons will lead to more accurate results, and since we have plenty of measurements with 2930 and 3922 volt electrons to give dependable mean values. We get the distribution of component velocities among the atomic electrons from the distribution of energies among the inelastically scattered electrons by changing the abscissas in the latter from V'' to λ'' by means of Eq. (20) which gives us

$$\lambda'' = V'' \div (0.5782\, V_0^{\frac{1}{2}}), \tag{24}$$

where λ'' is computed for $\lambda = 695$ XU and $\theta = 90°$ (the case for which Kirkpatrick, Ross and Ritland calculated their profiles) and V'' is for electrons of energy V_0 scattered at 34.2°. Eq. (23) then allows us to write

$$\beta(=u/c)\times 10^3 = \lambda'' = V'' \div (0.5782\, V_0^{\frac{1}{2}}), \tag{25}$$

which gives us the abscissas in terms of u, the component velocity of the atomic electron. By means of this equation, the ordinates of the dots representing experimental points in Figs. 10 and 11 for various values of V'' are re-plotted against β, and the best mean curve is drawn through the four sets of dots.[14] (There are four sets because we use the two halves of both the 2930 and 3922 volt curves.) From this mean curve we pick off the dots shown in Fig. 12. In Fig. 12 we also have the two curves computed theoretically by Hicks (who considers the "H-4" curve to be the more accurate) and the curve computed by Kirkpatrick, Ross and Ritland, which is represented by a broken line. (The last curve agrees so well with the "H-4" curve over the middle part that it is difficult to show the small differences between them because of the necessary finite thickness of the line in the drawing.) It is clear that our experimentally determined values are in excellent agreement with the "H-4" curve, and also with that due to Kirkpatrick, Ross and Ritland, except in the region below about $\beta = 3\times 10^{-3}$. It appears that the K.R.R. curve is not sufficiently

[14] To secure the best possible mean curves, the individual curves were drawn carefully on a large scale. This was necessary because they were all close to each other.

rounded at the maximum to account for the experimental results.[15] At the other end, the last two experimental points are decidedly below the theoretical curves. It may be that this represents a real disagreement, but we must remember that the electron currents from which these points are derived are of the order of 1 or 2×10^{-16} amp., values which are difficult to measure with any degree of accuracy. Finally it should be mentioned that the experimental dots plotted in Fig. 12 are based on seven distinct runs in which a total of 134 separate measurements of electron currents were made. In view of the closeness of fit between the mean experimental points and the "H-4" theoretical curve it is desirable to supplement Fig. 12 by Table V in which the numerical values are listed.

It is clear that the scattered electrons are divided into two well marked groups, those scattered elastically and those scattered inelastically. The division becomes more definite the higher the energy of the electrons before scattering. Even at our highest voltage, the elastic peak has a "foot" on its low voltage side, indicating the presence of electrons which have lost a small amount of energy. (The "feet" are shown on two scales in Figs. 8–11.) An obvious explanation is that the foot is due to the lack of perfect "single center scattering" as defined earlier in the paper. It is easy to see, by allowing the atomic electron in Fig. 3 to take any position on a sphere around the nucleus, that occasionally a deflection observed at the selected angle will be due to a deflection by the nucleus followed (or preceded) by a small but not negligible deflection by an atomic electron. The more unequal the contribution of one or other scattering center to the resulting deflection, the more frequent is this kind of scattering. We may assume that, when the deflection is due chiefly to the nucleus, the smaller the deviation attributed to the atomic electron, the less energy is taken from the impinging electron. Thus the shape of the "feet" in Figs. 8–11 is qualitatively accounted for. The approximation to ideal "single center scattering" improves as the square of the energy of the impinging electron. This consideration by itself suggests that we should carry out experiments with faster electrons than we have used. However, as the number of electrons scattered diminishes as the square of their energy, a point is reached when the scattered electron currents are too small to be measured accurately.

TABLE V. *Theoretical and experimental profiles for the modified band, and the equivalent component velocity distribution of the atomic electrons.*

λ''	u	$f(u)$ (K.R.R.)	$f(u)$ (H-2)	$f(u)$ (H-4)	$f(u)$ Experimental
0 XU	0×10^7 cm/sec	60.0	60.0	60.0	60.0
1	3	56.3	58.9	58.7	58.5
2	6	52.2	55.6	54.5	54.6
3	9	47.0	50.6	48.5	48.9
4	12	41.0	44.6	41.6	41.9
5	15	34.8	38.2	34.6	34.9
6	18	28.2	32.0	28.2	28.6
7	21	23.0	26.2	22.5	23.3
8	24	18.1	21.2	17.9	19.0
9	27	14.1	16.9	14.2	15.2
10	30	11.4	13.4	11.2	11.8
12	36	6.95	8.3	7.03	6.8
14	42	4.03	5.2	4.53	3.5
16	48	2.46	(3.5)	(3.0)	1.4

The last four columns could also be labeled $f(\lambda'')$ the profile of the Compton modified band for $\lambda=695$ XU and $\theta=90°$.

[15] Lack of perfect resolution would of course flatten the experimental curves near the maximum, but it is considered that this explanation is not sufficient to account for the difference between the experimental values and the K.R.R. curve.

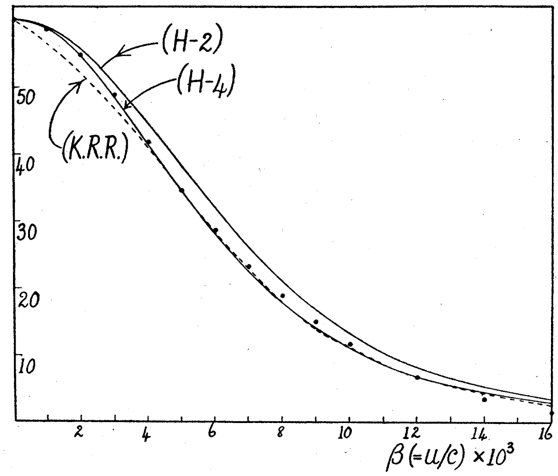

FIG. 12. Theoretical and experimental distributions of component velocities of atomic electrons in helium. Continuous lines: theoretical curves due to Hicks. Broken line: theoretical curve due to Kirkpatrick, Ross, and Ritland. Dots: experimental values. Abscissas: each unit $=\beta\times10^3=u/(3\times10^7)=\lambda''$ in XU (for $\lambda=695$ XU and $\theta=90°$).

It is planned to use this new technique to the study of the distribution of velocities of the atomic electrons in the atoms of various gases. An attempt will be made to see if it can be applied to the analogous problem in solids, where we may be prepared to find anisotropy in the distributions, particularly in certain crystals.

We take great pleasure in acknowledging our indebtedness to Drs. Kirkpatrick and Hicks who very kindly let us have the numerical data from which their published curves were drawn, thereby allowing us to make a better comparison between theory and experiment. We also desire to express our appreciation of the great interest shown by Professor G. E. M. Jauncey in this problem.