over, it is in such a direction as to increase rather than to decrease, the discrepancy between the value of h/e obtained from this experiment and the other fundamental constants.

In conclusion it should be emphasized that the present theory can explain only the newly discovered knee in the isochromat in the immediate vicinity of the threshold—the well-known knee that appears at a hundred volts or so from the threshold must find another explanation; and that the theory is not sufficiently accurate to enable one to predict the presence or absence of this knee in the isochromat obtained from an x-ray tube with a target of a given material.

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The Fine Structure of the Nuclear Ground Level of Li^7

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Calculations of the fine structure of the ground level of the Li^7 nucleus are made with the object of throwing light on the *form* of the interaction energy between pairs of nuclear particles. The requirements of relativistic covariance do not determine the spin-orbit interactions uniquely. Adjustable parameters, which are essentially coefficients of invariant additions to the Hamiltonian, are not fixed by considerations of covariance. The calculations reported show how, with approximate assumptions about the wave function, these parameters should be chosen in order to obtain agreement with the observed fine structure. A general rule simplifying the calculation of the spin-orbit interaction with a closed shell is developed for interactions between pairs of particles. The results are sensitive to the wave function used. The assumption of approximate symmetry of the nuclear Hamiltonian leads to an approximately symmetric coordinate function which is a linear combination of ^1S and ^1D states of the two neutrons in the p shell. For this function relatively large values of the adjustable parameters (~ 3) are needed. If, however, the ^1S state is supposed to be the predominant one, values of the parameters ~ 1 suffice. Present evidence favors the symmetric function and, therefore, the larger values of the parameters. These values do not correspond, even approximately, to the picture of a nuclear particle moving in a scalar field. An additional interaction having the transformation properties of an electromagnetic field is needed to obtain them. The sensitivity of the fine structure to perturbations suggests caution regarding the quantitative side of the calculations. Only qualitative significance can be attached to them.

INTRODUCTION

ACCORDING to Rumbaugh and Hafstad[1] there is an excited state of Li^7 about 400 kv above the normal level. Schuler,[2] Granath,[3] and Rabi[4] find the spin of this nucleus to be $\frac{3}{2}$. The normal state of Li^7 is expected to be ^2P according to the calculations of Feenberg and Wigner.[5] It is, therefore, logical to interpret the level found by Rumbaugh and Hafstad as the $^2P_{1/2}$ part of the ^2P state and the normal level of Li^7 as the $^2P_{3/2}$ condition of the same state. On this hypothesis an estimate was given by Inglis[6] for the magnitude of the expected energy difference between the two levels of the ^2P state, and it was pointed out by him that the spin-orbit coupling arising from the Thomas precession can be expected to be of special importance for nuclei. A similar estimate has been made by Rose and Bethe.[7] In both of these considerations

[1] L. H. Rumbaugh and L. R. Hafstad, Phys. Rev. **50**, 681 (1936).
[2] H. Schuler, Zeits. f. Physik **42**, 487 (1927).
[3] L. P. Granath, Phys. Rev. **42**, 44 (1932).
[4] M. Fox and I. I. Rabi, Phys. Rev. **48**, 746 (1935).
[5] E. Feenberg and E. Wigner, Phys. Rev. **51**, 95 (1937).
[6] D. R. Inglis, Phys. Rev. **50**, 783 (1936).
[7] M. E. Rose and H. A. Bethe, Phys. Rev. **51**, 205 (1936).

the valence particles in the p shell are pictured as moving in an attractive central field which has the transformation properties of a four-dimensional scalar. It is perhaps possible that in nuclei the elementary particles merge together in such a way as to lose their identity and to provide a scalar field of this character. Such a point of view would, however, be rather difficult to apply in a quantitative manner, at the present time, because of its extreme vagueness. Nuclear theories at present are developed using interactions between pairs of particles. Although they are not as yet completely quantitative, the success which they have met makes it unnecessary at present to abandon the simple point of view of interactions between pairs of particles. The possibility of a fundamental change in the description involving a merging of elementary particles is open, of course, but will not be investigated in the present note.

For forces between pairs of elementary particles one cannot make direct use of the Thomas precession or the scalar equation discussed in this connection by Furry.[8] A consideration of possible forms of wave equations for particles with spin determines[9] the interaction energies that may be used in these equations as

$$H^{\mathrm{W}}=\sum \mathbf{B}_{kl}[a^{\mathrm{W}}\boldsymbol{\sigma}_k+(a^{\mathrm{W}}-1)\boldsymbol{\sigma}_l]-\sum{}' J_{kl}, \quad (1)$$

$$\mathbf{B}_{kl}=-(\hbar/4M^2c^2)[\mathbf{p}_k\times\boldsymbol{\nabla}_k J_{kl}], \quad (1')$$

$$H^{\mathrm{H}}=\sum \mathbf{B}_{kl}[a^{\mathrm{H}}\boldsymbol{\sigma}_k+(a^{\mathrm{H}}-1)\boldsymbol{\sigma}_l]P_{kl}^{\mathrm{H}} -\sum{}' J_{kl}P_{kl}^{\mathrm{H}}, \quad (2)$$

$$H^{\mathrm{M}}=\sum \mathbf{A}_{kl}[a^{\mathrm{M}}\boldsymbol{\sigma}_k+(a^{\mathrm{M}}-1)\boldsymbol{\sigma}_l]-\sum{}' J_{kl}P_{kl}^{\mathrm{M}}, \quad (3)$$

$$\mathbf{A}_{kl}=(i/4M^2c^2)[\mathbf{p}_k\times J_{kl}P_{kl}^{\mathrm{M}}\mathbf{p}_k]. \quad (3')$$

The letters W, H, M, here, refer to Wigner, Heisenberg, and Majorana forces. The particles in the nucleus are referred to by subscripts k, l; the potential energy between two particles is $-J(r_{kl})$ for an ordinary force, $-J(r_{kl})P_{kl}$ for an exchange force, with r_{kl} standing for the distance between particles k and l; the momentum operators $\mathbf{p}_k=(\hbar/i)\boldsymbol{\nabla}_k$. $\sum'$ refers to sums over pairs of particles $(k>l)$; $\sum$ indicates sums over all values of k and l except $k=l$ (in other words, $k\gtrless l$). The interaction energies as written above do not include all the terms necessary[10] for invariance to order v^2/c^2. The parts omitted do not give rise to spin-orbit interactions and are of no interest for the present paper. The constants a^{W}, a^{H}, a^{M} are arbitrary and from considerations of invariance it is impossible to determine their value. For particles interacting through the electromagnetic field the value of a^{W} is -1. The picture of each particle producing a scalar field defined in the reference system moving instantaneously with it corresponds on the other hand to $a^{\mathrm{W}}=1$. By comparing calculation with experience one may hope to determine the constants a^{W}, a^{H}, a^{M} and to learn in this way a restricting condition for the interaction energy of the electron-neutrino field. Thus if it were found that $a^{\mathrm{W}}=-1$ the inference would be that the electron-neutrino field interacts with the heavy particles in a manner similar to the field of photons interacting with a charge.

A preliminary calculation of the energy splitting of the ground state of Li^7 has already been made[9] from Eq. (3) with $a^{\mathrm{M}}=1$. This calculation is incomplete because the two neutrons in the p shell were supposed to be coupled into a ${}^1S$ state, and because the influence of H^{W}, H^{H} as well as values of $a^{\mathrm{M}}\neq 1$ were not taken into account. In the present note the calculation will be made directly for a state having complete symmetry in the space coordinate wave function for the three particles in the p shell. On account of the approximate symmetry of the nuclear Hamiltonian, which appears probable in view of the scattering experiments of protons and neutrons in hydrogen,[11] this is expected[12] to be approximately the condition of the two neutrons and the one proton outside the alpha-particle shell in Li^7. On account of the Heisenberg force the symmetry of the space wave function is somewhat spoiled; this effect will be estimated as well.

[8] W. H. Furry, Phys. Rev. **50**, 784 (1936).

[9] G. Breit, Phys. Rev. **51**, 248 (1936).

[10] G. Breit, Phys. Rev. **53**, 153 (1938).

[11] M. A. Tuve, N. P. Heydenburg and L. R. Hafstad, Phys. Rev. **50**, 806 (1936); G. Breit, E. U. Condon and R. D. Present, Phys. Rev. **50**, 825 (1936); C. Cassen and E. U. Condon, Phys. Rev. **50**, 846 (1936); G. Breit and E. Feenberg, Phys. Rev. **50**, 850 (1936).

[12] E. Wigner, Phys. Rev. **51**, 947 (1937); E. Feenberg and M. Phillips, Phys. Rev. **51**, 597 (1937); F. Hund, Zeits. f. Physik **105**, 202 (1937).

General Relations

The interaction energies considered in Eqs. (1), (2), (3) consist of sums referring to pairs of particles. The evaluation of their expectation values is slightly more complicated than that of the expectation values of sums of single particle energies, such as are commonly used in approximate theories of spin-orbit interactions in atomic spectra. Nevertheless in these two cases there is a considerable similarity between the behaviors of that part of the energy which arises through interactions of the valence particles with a closed shell. The way in which this energy depends on the number of particles in the valence shell and on their coupling is the same for all kinds of interactions of the symmetric orbit-vector, spin-vector type and it is, furthermore, the same as for an interaction of the type $\sum \mathbf{A}(i)\boldsymbol{\sigma}_i$. This is a consequence of the spherical symmetry of a closed shell. Thus consider a closed shell with wave functions $u_1, u_2, \cdots, u_n$ and a set of three valence particles. Let u_a be the wave function corresponding to definite values of the orbital and spin magnetic quantum numbers of the states in the shell. Similarly let v_b stand for a wave function with definite orbital and spin magnetic quantum numbers in the incomplete shell. The states under consideration are then represented by the normalized determinants

$$\Psi = |u_1, u_2, \cdots, u_n; v_{b1}, v_{b2}, v_{b3}|,$$

having $n+3$ rows and columns, each row containing the functions with the coordinates of one of the particles. These determinants describe the "strong field" states of the system. The states b_1, b_2, b_3 may, in general, be any of the $2(2L+1)$ strong field states in the incomplete shell. Because of the exclusion principle only the possibilities $b_1 \neq b_2$, $b_1 \neq b_3$, $b_2 \neq b_3$ need be considered. The interaction energy is of the form $H' = \sum_{i>k} H(ik)$ with H_{ik} symmetric in the particles i and k. The matrix elements of H', $(\Psi', H'\Psi)$, are different from zero only if at most two of the states b are different for Ψ and Ψ'. If Ψ and Ψ' differ by two b's the matrix element contains only the functions v and will be said to contribute only to the interaction of the incomplete shell with itself. If Ψ and Ψ' differ by one b, or if they are identical, the matrix element $(\Psi', H'\Psi)$ is a sum containing matrix elements either with two v's or with one u and two v's (which may be identical). For $\Psi = \Psi'$ there is in addition a sum involving only u's. The latter vanishes in the present application since it is equal to the spin-orbit interaction of a closed shell with itself. The terms with one u and one v form the interaction of the valence particles with the closed shell. The terms with two v's give the interactions between the valence particles, which are readily seen to be the same as would be found if one worked with the normalized determinants

$$|v_{b1}v_{b2}v_{b3}|$$

so that the closed shell can be disregarded in this connection.

The interaction of the valence particles with the closed shell gives rise to matrix elements

$$(\Psi, H'\Psi) = \sum_{a=1}^{n} \sum_{b} (u_a(1)v_b(2), H(12)[u_a(1)v_b(2) - u_a(2)v_b(1)]),$$

$$(\Psi', H'\Psi) = \sum_{a=1}^{n} (u_a(1)v_b(2), H(12)[u_a(1)v_b(2) - u_a(2)v_b(1)]).$$

In the latter case the order of the functions v is supposed to be arranged in such a way as to have the last v different Ψ in and Ψ'. Let

$$H_{12} = \mathbf{A}_{12}\boldsymbol{\sigma}_1 + \mathbf{A}_{21}\boldsymbol{\sigma}_2$$

with $\mathbf{A}_{12}$, $\mathbf{A}_{21}$ independent of the $\boldsymbol{\sigma}$'s. For ordinary and Majorana interactions $H(12)$ is of this type. The typical matrix element $(\Psi', H'\Psi)$ is a sum of two parts obtained by inserting $\mathbf{A}_{12}\boldsymbol{\sigma}_1$ and $\mathbf{A}_{21}\boldsymbol{\sigma}_2$, respectively, for $H(12)$. Each of these is again a sum of two parts, one due to $u_a(1)v_b(2)$ and the other due to $-u_a(2)v_b(1)$. There are thus four parts in all. Since the wave functions u and v are products of orbital and spin wave functions the matrix of $\mathbf{A}\boldsymbol{\sigma}$ for each of the terms in the sum over a is the direct product of the vector matrix $\mathbf{A}$ by the vector matrix $\boldsymbol{\sigma}$. Let

$$(u_a(1)v_\beta(2),\ \mathbf{A}u_a(1)v_b(2))=(M_aM_\beta|\mathbf{A}|M_aM_b)\delta(m_am_\beta;\ m_am_b),$$
$$(u_a(1)v_\beta(2),\ \mathbf{A}u_a(2)v_b(1))=[M_aM_\beta|\mathbf{A}|M_aM_b]\delta(m_am_\beta;\ m_bm_a),$$

with M and m standing, respectively, for orbital and spin magnetic quantum numbers. Similarly let

$$(u_a(1)v_\beta(2),\ \boldsymbol{\sigma}u_a(1)v_b(2))=(m_am_\beta|\boldsymbol{\sigma}|m_am_b)\delta(M_aM_\beta;\ M_aM_b),$$
$$(u_a(1)v_\beta(2),\ \boldsymbol{\sigma}u_a(2)v_b(1))=[m_am_\beta|\boldsymbol{\sigma}|m_am_b]\delta(M_oM_\beta;\ M_bM_a).$$

Then

$$\textstyle\sum_a(u_a(1)v_\beta(2),\ \mathbf{A}\boldsymbol{\sigma}u_a(1)v_b(2))=\sum_{M_a}(M_aM_\beta|\mathbf{A}|M_aM_b)\sum_{m_a}(m_am_\beta|\boldsymbol{\sigma}|m_am_b),$$
$$\textstyle\sum_a(u_a(1)v_\beta(2),\ \mathbf{A}\boldsymbol{\sigma}u_a(2)v_b(1))=\sum_{M_a}[M_aM_\beta|\mathbf{A}|M_aM_b]\sum_{m_a}[m_am_\beta|\boldsymbol{\sigma}|m_am_b].$$

Consider $\sum_{M_a}(M_aM_\beta|A^i|M_aM_b)$, where $i=1, 2, 3$ denotes a component of A. Under rotation of the coordinate system, the orbital wave functions corresponding to M_a, transform among themselves by linear formulas which form an irreducible representation of the group of rotations. Similarly the functions of the incomplete shell transform among themselves and the components of A^i do so as well, both according to irreducible representations. The orthogonality relations between the coefficients of the representations of the shell functions introduce $\delta(M_aM_a)$ in the summation over M_a and do not enter the result for the transformation of $\sum_{M_a}(M_aM_\beta|A^i|M_aM_b)$. This quantity thus transforms itself as though the M_a were not present. Since the representations for the A^i and M_b are irreducible, the sum of matrix elements of an A^i is determined as a function of β, b to within a constant factor.[13] It is seen that only the transformation properties of the sums of matrix elements are essential for the above and therefore, the discussion applies equally well to $[M_aM_\beta|\mathbf{A}|M_aM_b]$, the dependence on β, b being the same as before. The linear formulas determining the ratios of the sums of the matrix elements as functions of β, b are the same as for the matrix elements of a one-particle vector operator taken with respect to wave functions corresponding to M_b. Since all these functions correspond to the same operator $\mathbf{L}$, one can simply use the operator $\mathbf{L}$ to determine these ratios since its matrix elements do not vanish. In the same way $\sum_{m_a}(m_am_\beta|\boldsymbol{\sigma}|m_am_b)$ and $\sum_{m_a}(m_am_\beta|\boldsymbol{\sigma}|m_am_b)$ are determined to within a constant factor, the dependence on β and b being again the same in both cases. Hence also the combined dependence of the $\sum_a$ of $\mathbf{A}\boldsymbol{\sigma}$ on M_β, M_b, m_β, m_b is the same for all $\mathbf{A}$. The matrix $(\Psi', H'\Psi)$ is, therefore, also determined to within a constant factor and the ratios of the matrix elements corresponding to different strong field states are thus seen to depend only on the azimuthal quantum number of the incomplete shell as well as the functions v_b entering Ψ, Ψ'. They are the same as though one were calculating the spin orbit interaction of the particles in the incomplete shell in a central field.

The above discussion applies only to those cases in which $\mathbf{A}$ is an operator diagonal in the spin. For the Heisenberg operator an additional consideration is necessary. For identical particles the operator $P^{\mathrm{H}}=-1$. According to Eq. (2) it thus reduces to an ordinary interaction. If the particles are not identical, as in the interaction of an incomplete shell of protons with a complete shell of neutrons, the relations are still true because they are true for the four parts corresponding to combinations of $\mathbf{A}_{12}\boldsymbol{\sigma}_1$, $\mathbf{A}_{21}\boldsymbol{\sigma}_2$ with direct and exchange terms, the proportionality also holds for the Heisenberg operator of Eq. (2).

It thus suffices, in the calculation of an interaction with a complete shell, to determine the diagonal matrix elements of the operator $\sum\mathbf{L}_i\boldsymbol{\sigma}_i$ taken over the incomplete shell for the states of coupling that correspond to fixed total angular momenta. The product of these elements and the ratio of the diagonal element of $\frac{1}{2}\sum H(ik)$ for a single particle outside a complete shell to the value of $\mathbf{L}_1\boldsymbol{\sigma}_1$ for a single particle in the same state gives the change in energy due to the spin orbit interaction.

[13] E. Wigner, *Gruppentheorie* (Braunschweig, 1931).

INTERACTION WITH s SHELL (ALPHA-PARTICLE)

The state of the p shell can be conveniently described as a linear combination of the state in which the two neutrons are coupled into a ^1S state with the state in which they are in a ^1D condition. Thus

$$\psi = U[(+-+)-(-++)], \tag{4}$$

where $U = cU_S + sU_D; \quad c^2+s^2=1. \tag{4.1}$

Here the symbols $(+-+)$ etc. denote the spin function of the three particles, indicating in order, the sign of the projection along the z axis of the spins of the particles 1, 2, 3. Particles 1, 2 are the neutrons while 3 is the proton. The function U contains the space coordinates of 1, 2, 3 and is subject to the normalizing condition

$$(\psi, \psi) = 2(U, U) = 1. \tag{4.2}$$

The functions U_S, U are the orbital functions with the above normalization for S and D states respectively. For the maximum orbital magnetic quantum number one may take

$$U_S = (3/2)(4\pi)^{-\frac{3}{2}} Q_1Q_2Q_3(\mathbf{r}_1\mathbf{r}_2)\xi_3, \tag{4.3}$$

$$U_D = (81/80)^{\frac{1}{2}}(4\pi)^{-\frac{3}{2}} Q_1Q_2Q_3[\xi_1(\mathbf{r}_2\mathbf{r}_3)+\xi_2(\mathbf{r}_3\mathbf{r}_1)-\tfrac{2}{3}\xi_3(\mathbf{r}_1\mathbf{r}_2)] \tag{4.4}$$

with $\xi = x+iy. \tag{4.5}$

Here $Q_i = Q(r_i)$ is the radial function of the ith particle normalized so that

$$\int_0^\infty Q^2(r) r^4 dr = 1. \tag{4.6}$$

The values of ψ corresponding to U_S, U_D will be written as ψ_S, ψ_D. One has

$$(\psi_S, \sum \mathbf{L}_i \boldsymbol{\sigma}_i \psi_S) = 2(U_S, L_3^Z U_S) = 2(U_S, U_S) = 1,$$

$$(\psi_D, \sum \mathbf{L}_i \boldsymbol{\sigma}_i \psi_D) = \frac{L(L+1)+L_3(L_3+1)-(L_1+L_2)[(L_1+L_2)+1]}{2L(L+1)} = -\tfrac{1}{2},$$

$$(\psi_D, \sum \mathbf{L}_i \boldsymbol{\sigma}_i \psi_S) = 0.$$

The last equation is true because $L_3^Z U_S = U_S$ and U_S is orthogonal to U_D. Thus for the general equation (4)

$$(\psi, \sum \mathbf{L}_i \boldsymbol{\sigma}_i \psi) = \tfrac{3}{2}c^2 - \tfrac{1}{2} = P_2(c). \tag{4.7}$$

For $c = 5^{\frac{1}{2}}/3$, $s = \frac{2}{3}$ one obtains a state symmetric in the Cartesian coordinates of the three particles

$$U' = 3(20)^{-\frac{1}{2}}(4\pi)^{-\frac{3}{2}} Q_1Q_2Q_3 \mathfrak{H} \tag{4.8}$$

with $\mathfrak{H} = (\mathbf{r}_1\mathbf{r}_2)\xi_3 + (\mathbf{r}_2\mathbf{r}_3)\xi_1 + (\mathbf{r}_3\mathbf{r}_1)\xi_2. \tag{4.9}$

It is this state that should be close to the actual one according to the considerations of Wigner, Hund, Feenberg and Phillips.[13] It will be noted that according to Eq. (4.7) the splitting of the level due to interaction of the p shell with the alpha-particle is in the ratios of 1 : (1/3) : (−1/2) for the states U_S, U', U_D, respectively. According to the previous estimate[9] the interaction corresponding to $a^M = 1$ was sufficient, for the state U_S, to account for the observed splitting and the principal contribution came from the interaction of the p shell with the alpha-particle. Since for U' this energy is decreased to $\frac{1}{3}$ of its previous value it becomes necessary to consider other values of a^M, a^H, a^W. According to the preceding section the energy splitting due to the interaction with the s shell is obtained simply by calculating it for a single particle in the p shell and multiplying it by the factor, $P_2(c)$. The alpha-particle will be supposed to be a complete shell of two neutrons and two protons, both in s states. The orbital functions of these particles will be called $R(r)$ normalized so that

$$\int_0^\infty R^2r^2dr=1. \tag{5}$$

The energy splitting for a single p proton due to its spin orbit interaction with the s shell of neutrons will be called $(\Delta E)_{s\nu}$ and the splitting due to the interaction with the s shell of protons will be written as $(\Delta E)_{s\pi}$. In all cases $\Delta E=E_{3/2}-E_{\frac{1}{2}}$ where $E_{3/2}$, $E_{\frac{1}{2}}$ are respectively the energies for $^2P_{3/2}$ and $^2P_{\frac{1}{2}}$. The values of ΔE will be listed below in terms of integrals involving Q, R as well as the special values that one obtains for

$$Q=N_Qe^{-\nu r^2/2},\qquad R=N_Re^{-\mu r^2/2} \tag{5.1}$$

$$N_Q^2=8\nu^{5/2}/3\pi^{\frac{1}{2}},\quad N_R^2=4\mu^{\frac{3}{2}}/\pi^{\frac{1}{2}} \tag{5.2}$$

with

$$J=Ae^{-\alpha r^2}. \tag{5.3}$$

The abbreviations

$$\beta^2=\tfrac{1}{4}(\mu+\nu)^2+\alpha(\mu+\nu);\quad \gamma^2=\mu\nu+\alpha(\mu+\nu) \tag{5.4}$$

and

$$\lambda\!\!\!^{-}=\hbar/Mc \tag{5.5}$$

will be used. One finds

$$(\Delta E)_{s\nu}{}^{M1}=-(3/64\pi^2)\lambda\!\!\!^{-2}\int J\{6Q_1Q_2+2r_1Q_1'Q_2+2r_2Q_1Q_2'+r_1r_2\sin^2\theta Q_1'Q_2'\}R_1R_2d\tau_1d\tau_2$$
$$=-(3/4)\lambda\!\!\!^{-2}\nu^{5/2}\mu^{5/2}A(\mu+4\alpha)\beta^{-5} \tag{6}$$

$$(\Delta E)_{s\pi}{}^{M1}-(\Delta E)_{s\nu}{}^{M1}=(3/32\pi^2)\lambda\!\!\!^{-2}\int Jr_2\cos\theta Q_2{}^2R_1R_1'd\tau_1d\tau_2=-(3/2)\lambda\!\!\!^{-2}\nu^{5/2}\mu^{5/2}A\alpha\gamma^{-5} \tag{6.1}$$

where the superscripts M1 indicate that quantities are taken for $a^M=1$. These formulas are in agreement with the previous calculation.[9]

$$(\Delta E)_{s\nu}{}^{M0}=-(3/64\pi^2)\lambda\!\!\!^{-2}\int Jr_1r_2\sin^2\theta Q_1Q_2R_1'R_2'd\tau_1d\tau_2=-(3/4)\lambda\!\!\!^{-2}\nu^{5/2}\mu^{7/2}A\beta^{-5} \tag{6.2}$$

$$(\Delta E)_{s\pi}{}^{M0}-(\Delta E)_{s\nu}{}^{M0}=-[(\Delta E)_{s\pi}{}^{M1}-(\Delta E)_{s\nu}{}^{M1}] \tag{6.3}$$

$$(\Delta E)_{s\nu}{}^{H1}=-(3/128\pi^2)\lambda\!\!\!^{-2}\int R_1Q_2(J'/r)\{(2r_2{}^2-2\mathbf{r}_1\mathbf{r}_2)R_2Q_1$$
$$+[Q_1'R_2/r_1-Q_1R_2'/r_2]r_1{}^2r_2{}^2\sin^2\theta\}d\tau_1d\tau_2=(3/2)\lambda\!\!\!^{-2}\nu^{5/2}\mu^{5/2}\alpha A\beta^{-5} \tag{6.4}$$

$$(\Delta E)_{s\pi}{}^{H1}-(\Delta E)_{s\nu}{}^{H1}=2[(\Delta E)_{s\pi}{}^{M0}-(\Delta E)_{s\nu}{}^{M0}]; \tag{6.5}$$

$$(\Delta E)_{s\nu}{}^{H0}=-(\Delta E)_{s\nu}{}^{H1};\quad (\Delta E)_{s\pi}{}^{H0}=(\Delta E)_{s\nu}{}^{H0}; \tag{6.6}$$

$$(\Delta E)_{s\nu}{}^{W1}=-(\Delta E)_{s\pi}{}^{H1}+(\Delta E)_{s\nu}{}^{H1};\quad (\Delta E)_{s\pi}{}^{W1}-(\Delta E)_{s\nu}{}^{W1}=-(\Delta E)_{s\nu}{}^{H1}; \tag{6.7}$$

$$(\Delta E)_{s\nu}{}^{W0}=0;\quad (\Delta E)_{s\pi}{}^{W0}-(\Delta E)_{s\nu}{}^{W0}=(\Delta E)_{s\nu}{}^{H1}. \tag{6.8}$$

The order of the above equations is such as to give in succession the values of the needed quantities for $a^M=1$, $a^M=0$, $a^H=1$, $a^H=0$, $a^W=1$, $a^W=0$. For $a^M=1$ and $a^M=0$ it is supposed that there are no spin orbit interactions of the Heisenberg and Wigner types and similarly in Eqs. (6.5), (6.6) only the Heisenberg and in Eqs. (6.7), (6.8) only the Wigner spin orbit interactions are considered. An equation such as (6.5) is, of course, not supposed to hold for the actual contributions due to the Heisenberg and Majorana forces but rather for the expressions on the two sides of the equation using the same function $J(r)$ in Eqs. (1), (2). The relations between quantities for different a^M, a^H, a^W are expressed

TABLE I. *Values of* $\Delta E = E_{3/2} - E_{1/2}$ *for the symmetric state due to interactions in the p shell.*

$a^M=1$	$a^M=0$	$a^H=1$	$a^H=0$	$a^W=1$	$a^W=0$
W	$-W$	$-W$	W	$-W$	W

simply in terms of $(\Delta E)_{s\nu}$ and $(\Delta E)_{s\pi} - (\Delta E)_{s\nu}$. For this reason Eqs. (6.3), (6.5), (6.6), (6.7), (6.8) were written in terms of them. For practical applications one needs, however, $(\Delta E)_{s\nu} + (\Delta E)_{s\pi}$ since this gives the result due to the whole alpha-particle.

INTERACTION WITH THE *p* SHELL

For the symmetric state represented by U' of Eq. (4.8) the relations are particularly simple. One obtains a contribution to the energy difference $E_{3/2} - E_{\frac{1}{2}}$ the values in Table I with

$$W = -\frac{3\lambda^2}{16\pi^2}\int [\tfrac{1}{2}(\mathbf{r}_1\mathbf{r}_2)JQ_1^2Q_2^2 + \tfrac{2}{5}r_2(\mathbf{r}_1\mathbf{r}_2)JQ_1^2Q_2Q_2']d\tau_1 d\tau_2$$
$$= +\frac{9\lambda^2}{320\pi^2}\int Q_1^2Q_2^2\frac{J'}{r}[\tfrac{2}{3}(r_1^2+r_2^2)\mathbf{r}_1\mathbf{r}_2 - r_1^2r_2^2 - \tfrac{1}{3}(\mathbf{r}_1\mathbf{r}_2)^2]d\tau_1 d\tau_2. \quad (7)$$

For the special forms of Eqs. (5.1), (5.2), (5.3)

$$W = \lambda^2 A\alpha(1+2\alpha/\nu)^{-7/2}. \quad (7.1)$$

The relations between the values of ΔE given in Table I may be seen from the symmetry of U'. Thus one finds $\Delta E/3 = 4(U', A_{21}{}^Z U')$ for $a^M=1$. The change to $a^M=0$ is made by changing $\mathbf{A}_{kl}$ into $-\mathbf{A}_{lk}$. Since U' is completely symmetric $(U', A_{21}{}^Z U') = (U', A_{12}{}^Z U')$ and therefore ΔE for $a^M=0$ is just the negative of that for $a^M=1$. For $a^H=1$

$$\Delta E = 3\lambda^2 i\left(U', \frac{dJ}{rdr}\left[(y_1-y_2)\frac{\partial}{\partial x_1} - (x_1-x_2)\frac{\partial}{\partial y_1}\right]U'\right)$$
$$= 3\lambda^2 i\left(U', J\left[\frac{\partial^2}{\partial y_2\partial x_1} - \frac{\partial^2}{\partial x_2\partial y_1}\right]U'\right) + 3\lambda^2 i\int J\left[\frac{\partial U'^*}{\partial y_2}\frac{\partial U'}{\partial x_1} - \frac{\partial U'^*}{\partial x_2}\frac{\partial U'}{\partial y_1}\right]d\tau_1 d\tau_2 d\tau_3.$$

The first term of this expression is zero since on interchanging 1 and 2 the square brackets change sign while U' does not. The last term is, on the other hand, just the negative of ΔE for $a^M=1$. The relations between $(\Delta E)^{H0}$ and $(\Delta E)^{H1}$ and between $(\Delta E)^{W0}$ and $(\Delta E)^{W1}$ are the same as between $(\Delta E)^{M0}$ and $(\Delta E)^{M1}$. For

$$a^H=1, \quad (\psi, Q_0{}^H\psi) = (U', [B_{23}{}^Z + B_{32}{}^Z + B_{31}{}^Z + B_{13}{}^Z]U')$$

for

$$a^W=1, \quad (\psi, Q_0{}^W\psi) = 2(U', [B_{31}{}^Z + B_{32}{}^Z]U').$$

On account of the symmetry of U' these expressions are equal.

Using Eqs. (6), Table I and Eq. (7) one obtains the general result for ΔE for the state U' in the case of the most general interaction of the type in which the potential energy is

$$-(1-g-g_1)J^M P^M - gJ^H P^H - g_1 J^W \cdot 1$$

TABLE II. *Values of* $\Delta E/mc^2$.

μ/α	$a^M=-a^H=a^W=1$	$a^M=-a^H=a^W=2$	$a^M=-a^H=a^W=3$
2	−0.32	−0.60	−0.87
1.5	−0.21	−0.41	−0.60
1	−0.11	−0.22	−0.33

with corresponding correction terms in a^{M}, a^{H}, a^{W}. The general result being lengthy, it suffices to write it down for $J^{\mathrm{M}}=J^{\mathrm{H}}=J^{\mathrm{W}}$ and the special forms of Eqs. (5):

$$\begin{aligned}\Delta E=\lambdabar^2 A\{&(1-g-g_1)a^{\mathrm{M}}[-\nu^{5/2}\mu^{5/2}(\mu+4\alpha)/2\beta^5-\nu^{5/2}\mu^{5/2}\alpha/2\gamma^5+\alpha(1+2\alpha/\nu)^{-7/2}]\\&+(1-g-g_1)(1-a^{\mathrm{M}})[-\nu^{5/2}\mu^{7/2}/2\beta^5+\nu^{5/2}\mu^{5/2}\alpha/2\gamma^5-\alpha(1+2\alpha/\nu)^{-7/2}]\\&+ga^{\mathrm{H}}[\nu^{5/2}\mu^{5/2}\alpha/\beta^5+\nu^{5/2}\mu^{5/2}\alpha/\gamma^5-\alpha(1+2\alpha/\nu)^{-7/2}]+g(1-a^{\mathrm{H}})[-\nu^{5/2}\mu^{5/2}\alpha/\beta^5+\alpha(1+2\alpha/\nu)^{-7/2}]\\&+g_1a^{\mathrm{W}}[-2\nu^{5/2}\mu^{5/2}\alpha/\gamma^5-\nu^{5/2}\mu^{5/2}\alpha/2\beta^5-\alpha(1+2\alpha/\nu)^{-7/2}]\\&+g_1(1-a^{\mathrm{W}})[\nu^{5/2}\mu^{5/2}\alpha/2\beta^5+\alpha(1+2\alpha/\nu)^{-7/2}].\qquad(8)\end{aligned}$$

For $\mu=\nu$, $\beta=\gamma$ and the formula simplifies:

$$\begin{aligned}\Delta E=\lambdabar^2 A\alpha\Bigg\{&(1-g-g_1)a^{\mathrm{M}}\left[-\frac{(5\alpha+\mu)\mu^5}{2\alpha\beta^5}+\frac{\mu^7}{\beta^7}\right]+(1-g-g_1)(1-a^{\mathrm{M}})\left[\frac{(\alpha-\mu)\mu^5}{2\alpha\beta^5}-\frac{\mu^7}{\beta^7}\right]+ga^{\mathrm{H}}\left[\frac{2\mu^5}{\beta^5}-\frac{\mu^7}{\beta^7}\right]\\&+g(1-a^{\mathrm{H}})\left[-\frac{\mu^5}{\beta^5}+\frac{\mu^7}{\beta^7}\right]+g_1a^{\mathrm{W}}\left[-\frac{5\mu^5}{2\beta^5}-\frac{\mu^7}{\beta^7}\right]+g_1(1-a^{\mathrm{W}})\left[\frac{\mu^5}{2\beta^5}+\frac{\mu^7}{\beta^7}\right]\Bigg\}.\qquad(8.1)\end{aligned}$$

For $\alpha=16Mmc^2/\hbar^2$, $A=72mc^2$, $\lambdabar^2A\alpha=0.627mc^2$. Substituting numbers into the last equation one obtains for $g=\frac{1}{4}$, $g_1=\frac{1}{4}$

$$\begin{aligned}\mu/\alpha&=2, &\Delta E&=\lambdabar^2A\alpha[-0.066-0.177a^{\mathrm{M}}+0.088a^{\mathrm{H}}-0.177a^{\mathrm{W}}],\\ \mu/\alpha&=1.5, &\Delta E&=\lambdabar^2A\alpha[-0.030-0.129a^{\mathrm{M}}+0.064a^{\mathrm{H}}-0.116a^{\mathrm{W}}].\end{aligned}\qquad(8.2)$$

According to Feenberg and Wigner,[5] μ/α is between 1.6 and 2.0. In order to account for the experimental value $-0.78\ mc^2$ one needs rather high values of a^{M}, a^{H}, a^{W} as is seen from Table II. The result is sensitive to μ/α. For the larger $\mu/\alpha=2$ one needs values of a^{M}, $-a^{\mathrm{H}}$, a^{W} in the vicinity of 3; for $\mu/\alpha=1.5$ these values have to be about 4. The result is not sensitive to reasonably small changes of g and g_1. The presence of an ordinary interaction (g_1) is favorable for getting a large absolute value of ΔE without using excessive values of a^{W}, a^{H}, a^{M}. If A and α are changed so as to remain in agreement with the binding energy of the deuteron the factor $A\alpha$ changes approximately as α^2. Due to this cause alone it is difficult to have a sufficiently large change in ΔE to make an important difference in the results without using values of α in bad contradiction with Feenberg and Knipp's and Feenberg and Share's[14] determination of α. This cause might change $a^{\mathrm{M}}=4$ into $a^{\mathrm{M}}=3$ for $\mu/\alpha=1.5$ since this would require using $\alpha=18\ mc^2/\hbar^2$.

The values of a^{M}, etc. in the neighborhood of 3 appear to be so high that one might discredit them on the grounds that they indicate the theory to be forced, since it appears to give the experimental value as a difference of two large numbers. This, however, is a false argument in several respects. The contributions to $\Delta E/mc^2$ for $\mu/\alpha=2$, for example are $-0.33(1-g-g_1)a^{\mathrm{M}}$, $-0.11(1-g-g_1)(1-a^{\mathrm{M}})$ for the Majorana interaction. For $a^{\mathrm{M}}=3$ these contributions are in the ratio of $-0.99:+0.22$. Although the signs of these contributions are different, one of them is less than $\frac{1}{4}$ of the other. For the Heisenberg interaction the terms in ga^{H} and $g(1-a^{\mathrm{H}})$ are of opposite signs for $a^{\mathrm{H}}=-3$ but one of them is about twice the other; for the Wigner interaction the signs of the contributions of g_1a^{W} and $g_1(1-a^{\mathrm{W}})$ are the same. It should also be remembered that although the absolute value of a^{W} for electromagnetic interactions is small ($a^{\mathrm{W}}=-1$) yet the coefficients that occur in the interaction energy [Eq. (1)] are as in

$$\mathbf{B}_{kl}(-\boldsymbol{\sigma}_k-2\boldsymbol{\sigma}_l),$$

which gives rise to combinations $2\mathbf{p}_2-\mathbf{p}_1$ when terms in one spin vector are collected. The occurrence

[14] E. Feenberg and J. K. Knipp, Phys. Rev. **50**, 253 (1936); E. Feenberg and S. Share, Phys. Rev. **50**, 253 (1936).

of large numbers in the electromagnetic interaction is not much more plausible than that suggested by the experimental fine structure *viz.*

$$\mathbf{B}_{kl}(3\boldsymbol{\sigma}_k+2\boldsymbol{\sigma}_l).$$

Departure from the Most Symmetric Wave Function

When account is taken of the ^1S and the ^1D condition of the neutrons the interaction energy as used for the calculation of the spin orbit splitting does not correspond exactly to the wave function U' on account of the presence of the Heisenberg interaction. In terms of L and K of Feenberg and Wigner one finds for the matrices of the potential energy[15]

$$\begin{array}{cccc} & \text{Majorana} & \text{Heisenberg} & \text{Ordinary} \\ \begin{array}{c}\psi' \\ \psi''\end{array} & \begin{pmatrix} 3L+2K, & 0 \\ 0, & 5K \end{pmatrix} & \begin{pmatrix} 0, & 5^{\frac{1}{2}}K \\ 5^{\frac{1}{2}}K, & -3L/2+2K \end{pmatrix} & \begin{pmatrix} 3L+2K, & 0 \\ 0, & 3L-4K \end{pmatrix} \end{array}$$

Here the rows and columns are referred to the wave functions

$$\psi'=(5^{\frac{1}{2}}/3)\psi_S+(2/3)\psi_D,\quad \psi''=-(2/3)\psi_S+(5^{\frac{1}{2}}/3)\psi_D.$$

The Heisenberg interaction mixes a small amount of ψ'' into ψ'. From the values of K, L obtained by Feenberg and Wigner and by Feenberg and Phillips, the ratio K/L is seen to be so small that K may be neglected in the diagonal elements of the above matrices for purposes of estimating the coefficient of ψ''. This coefficient is according to a simple perturbation calculation

$$\delta\cong\frac{5^{\frac{1}{2}}gK}{3(1-g/2-g_1)L}$$

and correspondingly the change in c of Eqs. (4.1) and (4.7) is $-\frac{2}{3}$ of this. Hence

$$c=\frac{5^{\frac{1}{2}}}{3}-\frac{2(5^{\frac{1}{2}})gK}{9(1-g/2-g_1)L}=0.7454-0.497\frac{gK/L}{1-g/2-g_1}. \tag{9}$$

According to Eq. (9) positive K/L gives smaller c than that corresponding to the most symmetric function ψ'. The wave function thus contains slightly more ^1D than is the case for ψ'. The factor $(3c^2-1)/2$ of Eq. (4.8) is decreased, so that the expected splitting due to the interaction with the alpha-particle is also decreased. For $g=g_1=\frac{1}{4}$ the change in c is -0.021 and $(3c^2-1)/2$ changes due to this from 0.333 to 0.288 or about 14 percent. The uncertainty in the theoretical value of ΔE due to the uncertainty in μ/α is greater than this. There appears to be no point, therefore, in estimating the effect of the departure of c from $5^{\frac{1}{2}}/3$ more closely for the s shell. The spin-orbit interaction of the p shell with itself is smaller than that with the s shell for the symmetric state ψ' as well as the ^1S state of neutrons. It is to be expected that the effect of the small change in c on the contribution to ΔE due to the interaction within the p shell is also slight and it will not be considered in detail here.

An approximate idea of the magnitude of the effect of the p shell can be obtained by calculating the spin orbit interaction in the S condition of neutrons. The interaction with the s shell is taken care of by Eq. (4.7) (6.0), (6.1)$\cdots$(6.8). The interactions within the p shell give

$$\begin{aligned}(\Delta E)^{M1}&=-(3/64\pi^2)\lambda\!\!\!^{-2}\int J_{12}Q_1Q_2(\mathbf{r}_1\mathbf{r}_2)[6Q_1Q_2+2r_1Q_1'Q_2+2r_2Q_1Q_2'+r_1r_2\sin^2\theta Q_1'Q_2']d\tau_1d\tau_2\\ &=-\frac{\lambda\!\!\!^{-2}A\alpha(4\alpha/\nu-3)}{4(1+2\alpha/\nu)^{7/2}}.\end{aligned} \tag{10}$$

[15] Since the main part of the potential energy is $-(1-g-g_1)JP^M-gJP^H-g_1J$, the J of this paper is opposite in sign to that of Feenberg and Wigner. This should be remembered in using their integrals for L and K.

This formula is in agreement with the one previously found.[9]

$$(\Delta E)^{M0} = -(3/64\pi^2)\lambda\!\!\!{}^-{}^2 \int J_{12}Q_1Q_2Q_1'Q_2'r_1^2r_2^2 \cos\theta \sin^2\theta d\tau_1 d\tau_2 = \frac{-5\lambda\!\!\!{}^-{}^2 A\alpha}{4(1+2\alpha/\nu)^{7/2}}, \tag{10.1}$$

$$(\Delta E)^{H1} = -(3/128\pi^2)\lambda\!\!\!{}^-{}^2 \int Q_1^2Q_2^2(J'/r)[-r_1^2r_2^2\sin^2\theta + 2r_1r_2^3\cos\theta - 2r_1^2r_2^2\cos^2\theta]d\tau_1 d\tau_2 = \frac{\lambda\!\!\!{}^-{}^2 A\alpha(\alpha/\nu - 2)}{2(1+2\alpha/\nu)^{7/2}}. \tag{10.2}$$

There is a general relation

$$2(\Delta E)^{H1} = (\Delta E)^{M0} - (\Delta E)^{M1}, \tag{10.3}$$

which follows from the identity

$$\mathbf{A}_{kl} + \mathbf{A}_{lk} = -(\mathbf{B}_{kl} + \mathbf{B}_{lk})P_{kl}^{M}.$$

One has also

$$(\Delta E)^{H0} = -(\Delta E)^{H1}, \tag{10.4}$$

$$(\Delta E)^{W1} = -(3/32\pi^2)\lambda\!\!\!{}^-{}^2 \int Q_1^2Q_2^2r_1^2(J'/r)(r_1r_2\cos\theta - r_3^2)d\tau_1 d\tau_2 = -\frac{\lambda\!\!\!{}^-{}^2 A\alpha(3+\alpha/\nu)}{(1+2\alpha/\nu)^{7/2}}, \tag{10.5}$$

$$(\Delta E)^{W0} = 0. \tag{10.6}$$

Collecting the contributions for the s and p shell one obtains a formula of the type of Eq. (8). For $\mu = \nu$ the equation simplifies and becomes

$$\Delta E = \lambda\!\!\!{}^-{}^2 A\alpha \Bigg\{ a^M(1-g-g_1)\left[-\frac{3\mu^5}{2\beta^5}\left(5+\frac{\mu}{\alpha}\right) - \left(\frac{\alpha}{\mu} - \frac{3}{4}\right)\frac{\mu^7}{\beta^7}\right] + (1-g-g_1)(1-a^M)\left[\frac{3\mu^5}{2\beta^5}\left(1-\frac{\mu}{\alpha}\right) - \frac{5\mu^7}{4\beta^7}\right]$$
$$+ g a^H\left[\frac{6\mu^5}{\beta^5} + \left(\frac{\alpha}{2\mu} - 1\right)\frac{\mu^7}{\beta^7}\right] + g(1-a^H)\left[-\frac{3\mu^5}{\beta^5} - \left(\frac{\alpha}{2\mu} - 1\right)\frac{\mu^7}{\beta^7}\right]$$
$$+ g_1 a^W\left[-\frac{15\mu^5}{2\beta^5} - \left(3+\frac{\alpha}{\mu}\right)\frac{\mu^7}{\beta^7}\right] + g_1(1-a^W)\frac{3\mu^5}{2\beta^5}\Bigg\}. \tag{10.7}$$

The contributions of the p shell are here represented by the last term within each of the square brackets. For $g_1(1-a^W)$ the p shell contributes nothing. For $g = g_1 = \frac{1}{4}$, $A = 72\,mc^2$, $\alpha = 16M\,mc^2/\hbar^2$. and $\mu/\alpha = 2$, Eq. (10.7) gives

$$\Delta E/mc^2 = -0.150 - 0.457a^M + 0.229a^H - 0.298a^W$$

The experimental value is then accounted for without difficulty by letting $a^M = -a^H = a^W = 1$ giving $\Delta E = -1.134\,mc^2$. With somewhat smaller values of μ/α or with different a^M, a^H, a^W the absolute value of ΔE is easily decreased. The contributions to ΔE due to the p shell are relatively insignificant in this case, the main contributions arising from the interaction with the s shell. Thus with the above numbers the ratios of the p shell interaction to that with the s shell are for $a^M = 1$, -0.012; for $a^M = 0$, 0.42; for

$a^{H}=1$, -0.063; for $a^{H}=-1$, -0.093; for $a^{H}=0$, -0.13; for $a^{W}=1$, 0.23, for $a^{W}=0$, 0. The largest of these ratios, 0.42 for $a^{M}=0$, does not enter the result at all if one uses $a^{M}=1$ as has been done above. The net effect of the p shell for the ^1S condition of neutrons is only $-0.011\ mc^2$. For the symmetric state Eq. (8.1) gives values for the interaction with the p shell which are about $-\frac{1}{5}$ of the whole for $\mu/\alpha=2$.

It should be noted that the ^1S condition of neutrons does not represent the actual coupling, even approximately, if the interaction energy is of the form $-(1-g-g_1)JP^{M}-gJ^{H}P^{H}-g_1J^{W}\cdot 1$ with values of the constants such as were used above. The object of calculations with Eq. (10) with such constants is to see how sensitive the splitting is to a change in the wave function for a fixed form of spin-orbit interaction energy.

Experiments of Bothe and Maier-Leibnitz and of Maier-Leibnitz and Maurer[16] indicate the presence of a state of C^{13} at about 0.8 Mev above the ground level. According to Feenberg and Wigner, Feenberg and Phillips as well as Hund[13] the lowest state of C^{13} may be a ^2P arising out of two missing p protons and one missing p neutron similarly to the way in which the ground state of Li^7 arises out of one p proton and two p neutrons. The possibility that the two lowest levels of C^{13} in the level scheme of Bothe and Maier-Leibnitz are to be indentified with this ^2P level is open but it does not fit in a simple way the large ratio of intensities of the 80 cm and 90 cm groups of protons emitted in the reaction $B^{10}+He^4=C^{13}+H^1$. This interpretation appears to be doubtful but if correct it would indicate a fine structure splitting of 0.8 Mev with an error of possibly ± 0.2 Mev. Even though this splitting is larger than that in Li^7 there is no contradiction involved since with spin orbit interactions between pairs of particles the simple relationship between holes and particles does not hold. If the splitting of the normal state of C^{13} is of the order of 0.4 or 0.2 Mev it could remain unobserved in $B^{10}+He^4=C^{13}+H^1$. The 80 cm proton group would have to be interpreted then as being due to a level with a different angular momentum from that of the ground state. This interpretation is apparently not in contradiction with observations of Cockcroft and Lewis[17] on $C^{12}+H^2=C^{13}+H^1$. The ejected protons, according to their Fig. 1 on p. 264, show a number-energy distribution that is not quite symmetric about the maximum, judging by their "protons D+D" curve. The experimental point for the range 11.9 cm and the part of the curve for ranges between 9 and 11 cm are suggestive of fine structure. It would be interesting to see a more detailed experimental curve with more points for ranges between 11 cm and 14 cm with thin carbon targets and a monochromatic proton beam of nearly the same energy used for comparison.

It is a pleasure to record our indebtedness to the Alumni Research Foundation of the University of Wisconsin for its support of this work.

[16] W. Bothe and H. Maier-Leibnitz, Zeits. f. Physik **107**, 513 (1937); H. Maier-Leibnitz and W. Maurer, Zeits. f. Physik **107**, 509 (1937).

[17] J. D. Cockcroft and W. B. Lewis, Proc. Roy. Soc. **154**, 261 (1936).