On the Nuclear Moments of Indium*

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The atomic beam method of zero moments has been applied to measure the nuclear spin, the h.f.s. $\Delta\nu$ of the normal $^2P_{1/2}$ state and the absolute magnetic moment of In 115. The spin was found to be 9/2 and the $\Delta\nu=0.381$ cm^{-1} in agreement with spectroscopic results. The resolving power which was attained was sufficiently high to allow a measurement of the fine structure of the zero moment peaks. From the separation of the two $m=-3$ peaks and the diamagnetic susceptibility, we calculate the moment of indium 115 to be 6.40 ± 0.20 nuclear magnetons. From the theory of this effect which is given below, it is clear that this value does not depend on any assumption with regard to the interaction between the nuclear spin and the electron configuration. Peaks arising from the metastable $^2P_{3/2}$ state, lying 2212.6 cm^{-1} above the $^2P_{1/2}$ state were found. The intensity of these peaks is in good agreement with what is expected from quantum statistics. Their exact location will permit an evaluation of the nuclear quadrupole moment.

OUR knowledge of the nuclear magnetic moments of all nuclei other than the isotopes of hydrogen has been derived solely from a study of the hyperfine structure of atomic levels. This is as true of the atomic beam methods as of the optical methods. The experimental result has always been the numerical value of some hyperfine splitting of an atomic energy state. The nuclear moment is derived on the assumption that the interaction between the nuclear angular momentum vector and the extranuclear results from an intrinsic magnetic moment of the nucleus and, in addition, some quadrupole moment. The evaluation of these moments involves a calculation of the energy of interaction of the electronic configuration with the nucleus.[1] Although the combined results reveal the existence of nuclear magnetic moment to be quite certain, its evaluation is very difficult. When results for the hyperfine structure of different levels exist, the spread in values is seldom less than 20 percent. In addition, there remains a possibility that other modes of interaction not contemplated in the theory may contribute to the hyperfine separation.[2]

In this paper there will be presented some experimental results obtained with the method of "zero moments"[3] applied to atomic beams of indium under conditions of very high resolution. From the results it is possible to calculate the nuclear moment of indium directly, i.e., the assumptions regarding the nature of the interaction between the nucleus and the electronic configuration do not influence the result. We have in addition redetermined the h.f.s. separation of the normal $^2P_{1/2}$ state of In 115, as well as the angular momentum of this nucleus.

At the high temperatures necessary to produce sufficient beam intensities for these expierments, about 19 percent of the indium atoms in the oven should be in the $^2P_{3/2}$ state which lies 2212.6 wave numbers above the normal $^2P_{1/2}$ state. Since the beam is collision-free, and the radiation transition $^2P_{3/2}-{}^2P_{1/2}$ has an expected lifetime[4] of about one second, these atoms should be present in the beam to the same extent as in the oven.

We have observed "zero moment" peaks arising from these metastable atoms. From their intensity we can verify the predictions of the quantum statistics, and their location yields information about the hyperfine structure of this state. This is the first atomic beam experiment in which it has been possible to locate zero moment peaks from a metastable state or one in which the electronic angular momentum, J, is other

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[1] The work in the field is summarized by Bethe and Bacher, Rev. Mod. Phys. **8**, 82 (1936).

[2] Breit, Phys. Rev. **51**, 248 (1937); Young, Phys. Rev. **52**, 138 (1937).

[3] Cohen, Phys. Rev. **46**, 713 (1934).

[4] The radiation emitted is mostly pure magnetic dipole radiation.

than $\frac{1}{2}$. The higher J value makes it possible to observe departures from the interval rule and consequently to study nuclear quadrupole moment.

The application of the method of zero moments to the study of absolute moments depends on the refinement of the theory of the behavior of an atom in an external magnetic field.

The magnetic moment of an atom with electronic angular momentum $J=\frac{1}{2}$ and nuclear spin i in a magnetic field H is, in units of the Bohr magneton μ_0,

$$\mu_m=-g_i m \mp \frac{x+(2m/2i+1)}{2[1+(4mx/2i+1)+x^2]^{\frac{1}{2}}}(g_J-g_i). \quad (1)$$

(The derivation is given in the appendix.) In this formula, m is the total magnetic quantum number, g_i and g_J are the usual g factors, defined as the negative of the ratio of the magnetic moment expressed in Bohr magnetons to the angular momentum expressed in units of $h/2\pi$, and x is defined by

$$x=\frac{(g_J-g_i)\mu_0 H}{hc\Delta\nu}, \quad (2)$$

where $\Delta\nu$ is the experimental separation in wave numbers between the hyperfine structure levels $F=i+\frac{1}{2}$ and $F=i-\frac{1}{2}$. The basis of the older zero moment experiment was the less refined formula of Breit and Rabi[5] which is obtained from Eq. (1) and Eq. (2) by setting $g_i=0$. In this approximation, it is evident that for negative values of m there are certain values of x for which the value of μ_m is zero in two of the $2(2i+1)$ magnetic levels. These values of x are at $-2m/(2i+1)$, $m=-\frac{1}{2}, -\frac{3}{2}, \cdots -(i-\frac{1}{2})$ if the spin is integral, and $-1, -2, \cdots -(i-\frac{1}{2})$ if the spin is a half-integer.

If a beam of atoms in a state $J=\frac{1}{2}$ with nuclear spin i passes through an inhomogeneous magnetic field, it will suffer deflection according to the law of force $F_y=\mu_m \partial H/\partial y$. A plot of the beam intensity which reaches the detector fixed at the center of the beam against the field or magnet current shows initially a rapidly decreasing intensity, since $\partial H/\partial y$ is proportional to H and the increasing deflecting force deviates more and more atoms out of the beam. When the value of the field reaches the region where x is at the lowest value of $-2m/(2i+1)$, the intensity increases, comes to a peak at this point, and then decreases to repeat (Fig. 3) the same course at each of the critical values of x. The zero moment peaks have a maximum intensity under ideal conditions equal to $1/(2i+1)$ of that of the original beam intensity.

The new feature of this experiment is the fact that the resolution is so high that the more exact Eqs. (1) and (2) must be applied. These equations show that the atoms in each of the two states with the same negative m value have their zero μ_m at two slightly different values of x and therefore of magnetic field H. The difference is such that to a high degree of approximation

$$\frac{\Delta H}{H}=\frac{2i+1}{i}\frac{\mu_i}{\mu_J}\left\{1-\left(\frac{2m}{2i+1}\right)^2\right\}^{\frac{1}{2}}, \quad (3)$$

where ΔH is the interval between the two zero moment values of the field and H is the average of the two values, μ_i and μ_J are respectively nuclear and extranuclear moments. The fraction of the atom in each of these two states is $1/(4i+2)$.

It is clear that Eq. (3) involves the nuclear moment directly and does not involve at all the nature of the interaction between the nuclear and extranuclear angular momentum which causes the hyperfine structure of the atomic level. Experimentally, the problem reduces to one of resolution, which means that the deflecting power of the apparatus must be sufficiently great that atoms with moments only slightly different from zero are deflected completely away from the detector.

Indium possesses many features which make it suitable for a first attempt at an accurate measure of absolute nuclear moment. Firstly, the moment indicated from its h.f.s. is large; secondly, the fact that the normal state is $^2P_{1/2}$ with the consequent value of $\mu_J=\frac{1}{3}\mu_0$ makes the ratio $\Delta H/H$ of Eq. (3) a much more favorable one than with the alkalis. Furthermore, there was the hope that with sufficient resolution one could observe the peaks arising from the isotope In^{113}. A preliminary investigation showed that it

[5] Breit and Rabi, Phys. Rev. 38, 2082 (1931).

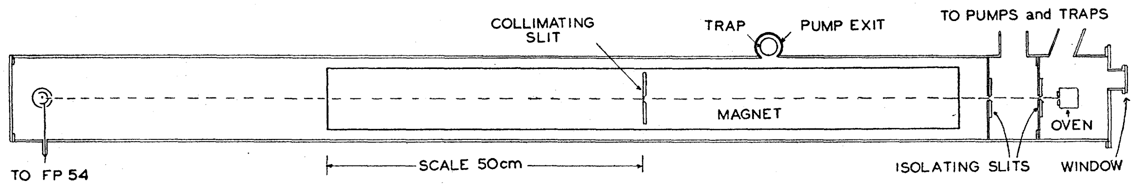

FIG. 1. Diagram of the apparatus.

is possible to detect indium with an oxide coated tungsten wire prepared in a manner[6] similar to that used for Li, even though the ionization potential is much higher (5.76 volts).

The requirements of high resolution were met by making the beam long and narrow. The total beam length was 161 cm; the defining slits were set at about 0.01 mm and the path in the deflecting field was 1 meter long. Since $\Delta\nu$ for the $^2P_{1/2}$ state of indium is 0.38 cm^{-1} and the value of g_J only $\frac{2}{3}$ high fields of the order of 10,000 gauss must be produced to reach the appropriate value of x (Eq. (2)). This circumstance made it necessary to use an iron magnet to produce the deflecting field, rather than a system of wires carrying current, which has been the previous practice.

APPARATUS

A top view of the apparatus is given in Fig. 1. It consists of a long brass tube 5″ inside diameter and $\frac{1}{8}$″ wall thickness, separated by partitions into three chambers, each pumped separately. Two narrow separating slits mounted on the partition and movable by means of screws which pass through the side of the tube permit the passage of the beam, and yet provide sufficient flow resistance to isolate effectively the oven chamber, where the vacuum is poor, from the main beam chamber.

The source is an oven which consists of a molybdenum block, with a well drilled diagonally down, in which the indium is placed. Molybdenum slit jaws with about 0.01 mm separation cover the well. The slit jaws have to be removed to reload. This design was found to be superior to the type with a separate opening for loading, because it is easier to make one exit tight to indium vapor than two. A set of tungsten heating spirals, insulated by quartz tube and grouped around the slit, serve to heat the oven. The oven itself is mounted on three tungsten pegs set in a slide. An invar screw on the side of the tube allows lateral motion of the oven while the experiment is in progress. The power supply to the oven is about 800 watts to attain a temperature of 1500°K. The oven chamber is thoroughly water-cooled.

The magnet is of a very accurate and convenient construction. A cross section is shown in Fig. 2. It is essentially a tube 1 meter long with a longitudinal gap, and is completely inside the beam chamber. It is made in two parts of Armco iron and is of the form shown in the shaded area of Fig. 2. The form of the gap is such that the boundaries are the equipotentials corresponding to the field produced by two parallel wires carrying current in opposite directions and 6.2 mm apart. The advantages of such a field have already been described.[7] The internal windings consist of four copper tubes insulated by mica and the external windings were the four segments of a hollow copper cylinder, each provided with a copper tube soldered to it. The windings are thus completely water-cooled. The magnet is

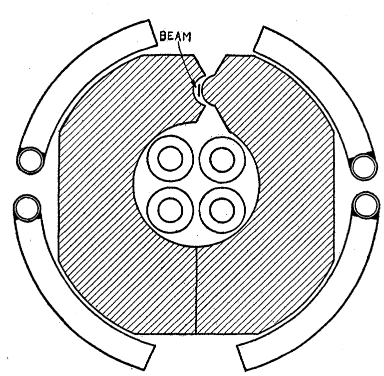

FIG. 2. Cross section of magnet and windings. The beam runs through the curved gap. This diagram is $\frac{2}{3}$ full size.

[6] Manley and Millman, Phys. Rev. **51**, 19 (1937).

[7] Rabi, Kellogg and Zacharias, Phys. Rev. **46**, 157 (1934).

excited by a bank of 2-volt storage cells connected in parallel. A field of 10,000 gauss is attained with 250 amperes. The current was measured by means of a calibrated shunt and a Leeds and Northrup type K potentiometer. It was necessary to be able to vary the current in steps of 0.01 percent and continuously in one direction in order to remain on the same hysteresis loop. The rheostat was of a simple slide wire type and could vary the current from 0 to 250 amperes. The value of the field as a function of the current was measured roughly by means of a flip coil inserted in the gap. The fine measurement was made with the beam itself in a manner to be described.

The collimating slits were mounted on a plate which set in a groove cut transversely through the magnet. The slit mount could be displaced any desired distance by means of a screw. One of the slit jaws had a vertical slot cut through it wider than the gap in the magnet. The detector was the usual oxide-coated tungsten filament, 0.025 mm in diameter, eccentrically mounted on a ground joint and provided with a microscope for very accurate setting of position. In addition, the apparatus was provided with vertical beam stops not shown in the diagram, which limited the height of the beam and were adjustable during the course of a run not only in vertical position but also in height.

Adjustment

The slits were made parallel by setting each slit and the detecting wire separately, parallel to a plumb line. A rough alignment of the slits was effected optically; all finer adjustments were made with the beam itself. These adjustments consist in setting all the slits and the detector filament in the same plane, and in setting this plane parallel to the pole face of the magnet and at a known distance from it. There is a certain optimum position where the magnitude of the field is constant over the greatest height.

The adjustments were usually made with a beam of KCl molecules which is not as difficult to handle as In. The oven is heated to an appropriate temperature and the collimating slit is pulled out of position to allow a broad beam to come down the apparatus through a slot in the slit jaw. The filament is moved until it detects this broad beam. The separating slits are made wide enough to permit this without any very accurate aim. The collimating slit is then moved in. When the slit is in line with the filament and oven slit, a beam comes through to the detector. The beam is then moved by moving the oven, separating slits and filament in step until it is cut off by an edge placed at a known distance from the convex edge of the pole piece nearest the oven. The position of the detecting filament is then read on the telemicroscope focused on the filament. The oven is then moved back, pivoting the beam about the collimating slit as a center, until it is cut off by another edge on the end of the pole face nearest the filament. From the interval between these two filament positions, the length of the beam and the length and location of the magnet, the position of the collimating slit can be calculated. The beam is then made parallel to the pole face by moving the filament back the proper distance from the cut-off point and setting the oven for the maximum intensity. The collimating slit can be moved a known interval by setting the filament at the proper point and pivoting the beam about the oven. After the collimating slit is set, the parallelism to pole face is restored by displacing the filament the proper amount and correspondingly resetting the oven. After this setting, the isolating slits are centered with respect to the beam by cutting the beam off first with one jaw and then the other, and setting the slit at the midpoint between the two positions.

It is important for the experiment that the detecting filament is accurately centered with respect to the beam; an error of 0.0004 mm represents a 1 percent error in the final value of the moment. The centering is effected by placing the filament midway between two points of equal intensity on the side of the beam. This method is very sensitive and corresponds to the center better than the point of maximum intensity which cannot be read as precisely. Although it may seem that the long apparatus would not be rigid enough to the extent desired, it was found that if it is shielded from drafts and from direct radiation and convection from the pump heaters, the average shift of the filament with respect to the beam center was less

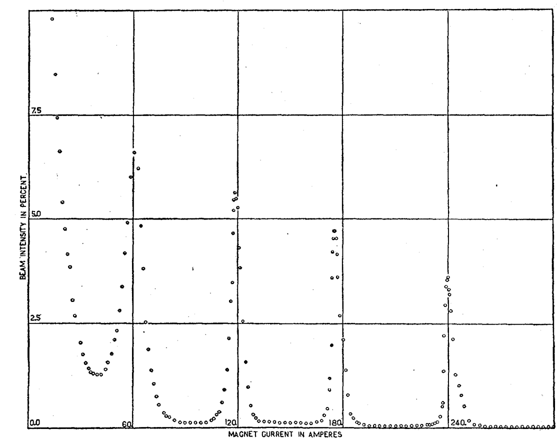

FIG. 3. Plot of the beam intensity against magnet current with detector fixed at the center of the beam.

than 0.001 mm in a period of about fifteen minutes. This is about the time consumed in the measurement of the fine structure of a peak.

Another serious difficulty was the possibility of a shift in the position of the beam when the field is turned on, because of warping of the magnet which is attached to the main tube. Such warping would not be detected with the indium beam itself. However, the effect could be investigated with a KCl beam, which is not magnetic except for nuclear and rotational moment. It was found that there was indeed a beam shift, but not much different from the expected shift of 0.0025 mm at 7500 gauss calculated from the known diamagnetic susceptibility of KCl. The apparatus itself stayed practically fixed. Had there been a shift, its effect could have been neutralized by displacing the filament beforehand by the same amount.

RESULTS

(a) The spin

In Fig. 3 is shown a plot of the indium beam intensity against magnet current. It was verified that the four approximately equidistant peaks were the only peaks from atoms in the $^2P_{1/2}$ state. The spin is therefore an odd half-integer and equal to 9/2.

(b) The h.f.s. separation

From Eq. (2) it is apparent that the h.f.s. $\Delta\nu$ can be evaluated when the field at any of the peaks is known because there x is $-2m/(2i+1)$. Since the ordinary experimental methods of measuring an inhomogeneous field in a small confined region lack precision, recourse was had to atomic beam methods. The last peak of Cs′, of which the $\Delta\nu$ is known[8, 9] to 0.2 percent, lies very close (within 1 percent) in magnet current to the first peak of In. Since the intervals in current between the first and second, and second and third peaks of In are equal (Fig. 3), and since these intervals must be equal in field, we know that the magnetization curve of field against current is linear in this region. The extrapolation from the last Cs peak to the first In peak is therefore very accurate. The experimental field ratio H(3rd peak of Cs)/H(1st peak of In) is 1.007. This yields 0.381 cm^{-1} for the $\Delta\nu$ of the $P_{1/2}$ state of In 115, as compared with the spectroscopic result of 0.380. This result is important for the absolute moment measurements because the value of g_J enters the calculation for $\Delta\nu$. The agreement between the two values shows that g_J is accurately $\frac{2}{3}$, as is to be expected from a pure $^2P_{1/2}$ term.

(c) The absolute moment

Figure 4 shows a detailed plot of the fourth peak. It differs from Fig. 3 in that much lower beam stops were used (0.7 mm instead of 2.5 mm), and smaller and more accurately measured steps in current were taken. Fig. 5 is a similar curve for the third peak, and Fig. 6 for the second. The fine structure is very easily measurable. The position of the beam was always rechecked after a complete curve of the type of Figs. 4, 5, 6 was taken to see if any accidental shift had occurred. When small shifts occurred, corrections were made by introducing a known

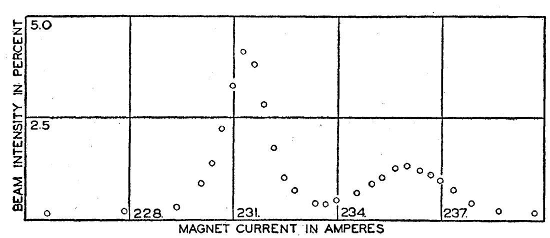

FIG. 4. Detail of fourth peak.

[8] Millman and Fox, Phys. Rev. 50, 220 (1936).
[9] Granath and Stranathan, Phys. Rev. 48, 725 (1935).

shift and measuring the effect on ΔI. A large number of curves were taken to diminish the effect of accidental errors.

The difference in intensity between the low and high field components of the double peak noticeable on all these curves arises from the fact that the field is necessarily inhomogeneous, and therefore varies across the width of the beam. The low field component represents atoms, the moments of which decrease with increasing field, while the high field peak comes from atoms for which the reverse is true. In the inhomogeneous field, this circumstance results in a slight focusing of the beam in the former case and a spreading in the latter.

Only the second and third peaks are suitable for the calculation of final results. The fourth peak occurs at a value of the field (9830 gauss) where the slope of the magnetization curve is changing and it is difficult to evaluate ΔH, the separation in field, from ΔI, the separation in current. At the third peak, the linearity is very good, and ΔH can be obtained accurately from ΔI by comparing it with the known field and current separation between the second and third peaks. The second peak is not as clearly resolved as the third, but the results from the two are in agreement and afford a check on the procedure.

The value of $\Delta H/H$ for the third peak, obtained by averaging 35 curves, is 0.0193; the average deviation from this mean is 0.0004. Substituted in Eq. (3) with $m=-3$ and $\mu_J=\frac{1}{3}\mu_0$ we obtain the value $\mu_i=6.65$ nuclear magnetons ($\mu_0/1838$). This value has to be corrected for the diamagnetic moment induced in the indium atom by the field. The magnitude of correction is easily evaluated by adding this moment to Eq. (1) which becomes, after setting $\mu_i=-g_i i$ and adding the diamagnetic moment $\mu_\alpha=-\alpha H$,

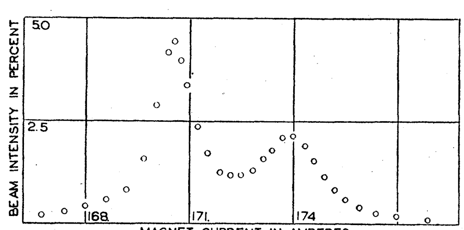

FIG. 5. Detail of third peak.

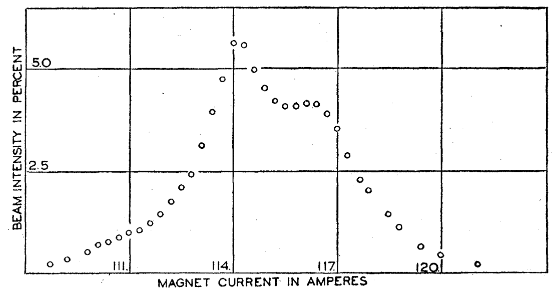

FIG. 6. Detail of second peak.

$$\mu_m=\frac{m}{i}\left(\mu_i-\alpha H\frac{i}{m}\right)\mp\frac{g_J-g_i}{2}\,\frac{x+(2m/2i+1)}{\{1+(4mx/2i+1)+x^2\}^{\frac{1}{2}}}.$$

Since the value of m for these zero moment peaks is negative, the effect is that the value for μ_i which we obtain is too large by the amount $\alpha Hi/m$, which for the third peak is $1.5\alpha H$. If we assume a reasonable value for the molar diamagnetic susceptibility of indium 66×10^{-6}, we have a correction to the moment of about 0.25, leaving it at 6.40. This value should be correct to about 3 percent, including the error in estimating the diamagnetic contribution.

The evidence of the second peak strongly supports these values. The value of $\Delta H/H$ from 14 curves is 0.0210. The value of m is (-2). This leads to a moment of 6.31. The diamagnetic correction is the same as for the third peak, since the ratio of H/m is the same. The final value is 6.06 nuclear magnetons. This result should not be considered as a disagreement with the value obtained from the third peak, but rather as a support, since any systematic error in reading the interval between two peaks when they overlap is in the direction of making the interval smaller. The precision of the determination is naturally much less than that of the third peak.

We therefore consider the value of the moment of In^{115} to be 6.40 nuclear magnetons.

THE METASTABLE $^2P_{3/2}$ STATE

The ratio of the number of atoms in the $^2P_{3/2}$ state to those in the normal state is given by the Boltzmann factor multiplied by the ratio of the statistical weights, and is equal to 2 exp

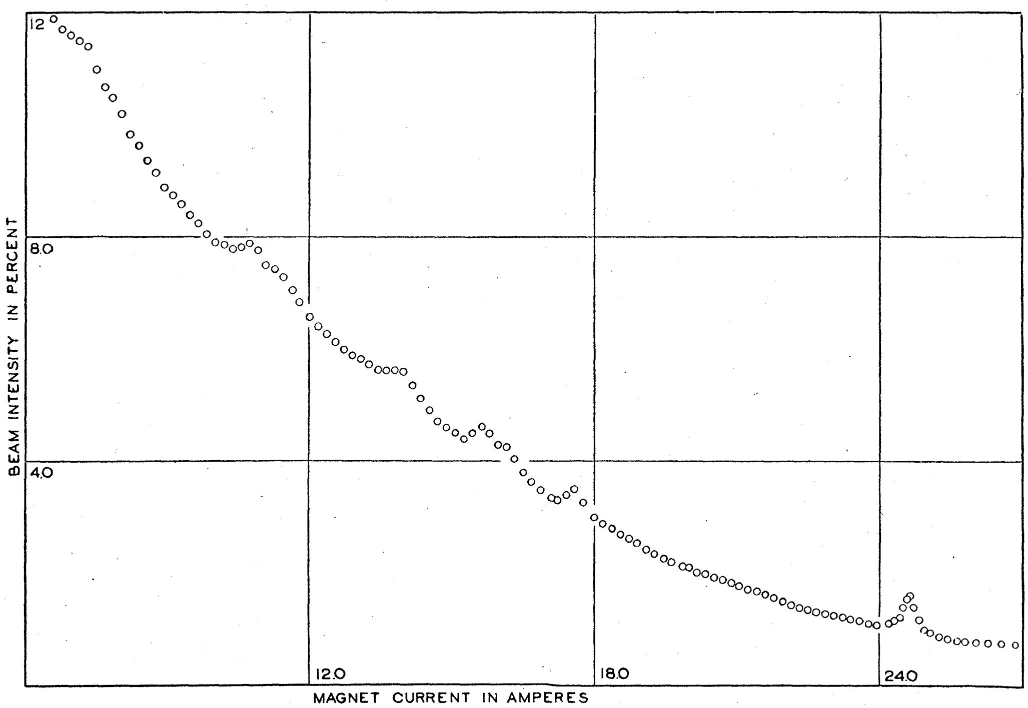

FIG. 7. Detail of the low current part of Fig. 3 showing the peaks arising from the metastable $^2P_{3/2}$ atoms against the background of undeviated $^2P_{1/2}$ atoms.

$-hc\Delta\nu/kT$. The absolute temperature was about 1500°K. At this temperature the excitation of other states may be neglected. The abundance of atoms in the higher state should be about 19 percent.

The expression for the magnetic moment as a function of the field can no longer be given as a simple quadratic function, but comes from a solution of a set of quartic equations, because the secular equation is of the fourth order. Each value of m results in a different quartic, the roots of which have been obtained by numerical methods. There are 40 such roots, but only the roots of the equations which come from negative values of m give "zero moment crossing." There are fourteen of these and in general the values of the field at which these zero moments occur are different for the different states. The fraction of atoms in a zero moment state should therefore be 0.19/40 of the total number, or about 0.5 percent.

In Fig. 7 we have a plot of the intensity against magnet current for the very early part of the curve in Fig. 3. The small peaks arise from the metastable atoms, and after subtracting the background of atoms in the normal state, we find the fraction of atoms to be 0.5 percent.

The reason they occur at such low field values is firstly the fact that the value of g_J is $\frac{4}{3}$ rather than $\frac{2}{3}$, and secondly the fact that the value of the h.f.s. separation for this state is about $\frac{1}{5}$ that of the $^2P_{1/2}$ state. This results in a very much larger value of x for a given field H.

The location of each of these peaks is a function of g_J, g_i and the value of the energy of each of the four h.f.s. levels of the h.f.s. multiplet. Since there are fourteen peaks to determine these quantities, it will be possible to ascertain any departures from the interval rule with great accuracy, and hence to study the quadrupole or higher moments of this nucleus. These experiments are being continued in this laboratory with an apparatus more suitable for low fields and high resolution.

DISCUSSION

A comparison of our result with the moment calculated from the h.f.s. of various states is given in Table I.

Our value is in fair agreement with the value calculated from the h.f.s. of the normal $^2P_{1/2}$ state. It is, however, impossible to decide from present knowledge whether the marked deviations from the true value of the moments, obtained from the s states, arise from an error in the theory of Fermi and Segrè or whether there is a specific electronic interaction with the nucleus not contemplated in the theory. It would be of great value in this connection to have a more precise theoretical calculation of the moments from these s states. The low value obtained from the $^2P_{3/2}$ state may perhaps be explained by some perturbation as in the case of thallium.

In these experiments, a careful search was made for small peaks arising from the isotope In 113, which has an abundance of 4.5 percent. Although our resolution was high enough to detect such peaks even if the spins of the two isotopes were the same and the moments differed by as little as 1 percent, we observe no such peaks. If we conclude from the work of Bacher and Tomboulian,[10] who failed to detect any lines from In 113, that the h.f.s. separations of the two isotopes are of approximately equal magnitude, we can be certain that the spin of In 113 is certainly 9/2.

In conclusion, we wish to thank Mr. Sam Cooey, chief mechanician of the physics department, whose remarkable skill in the construction of the magnet and other vital portions of this apparatus made the whole experiment feasible. The research has been aided by a grant from the Research Corporation.

Appendix

The derivation of the moment formula for $J=\frac{1}{2}$, i arbitrary, is best obtained from the F representation in which we take the Z axis as the direction of the field H.

TABLE I

In I	$6s$	5.02
	$5p\,^2P_{\frac{3}{2}}$	3.8
	$5p\,^2P_{\frac{1}{2}}$	6.1
In II	$5s$	5.46
Atomic beam		6.40

[10] Bacher and Tomboulian, Phys. Rev. **52**, 836 (1937).

The Hamiltonian of the interaction is

$$H=g(j)\mu_0 J_z H+g(i)\mu_0 i_z H. \quad (1)$$

The well-known matrix elements of J_z and i_z are given by Condon and Shortley.[11] They are

$$\left.\begin{aligned} J_z(F\,m;\,Fm) &= (1/2F(F+1)) \\ &\quad\times[F(F+1)-i(i+1)+J(J+1)]m \\ i_z(F\,m;\,Fm) &= (1/2F(F+1)) \\ &\quad\times[F(F+1)-J(J+1)+i(i+1)]m \end{aligned}\right\} \quad (2)$$

$$J_z(F\,m;\,F-1\,m)=J_z(F-1\,m;\,Fm)=f$$

$$i_z(F\,m;\,F-1\,m)=i_z(F-1\,m;\,Fm)=-f \quad (3)$$

$$f=\left\{\frac{(F+J+i+1)(i+J+1-F)(F+i-J)\times(F+J-i)(F^2-m^2)}{4F^2(2F-1)(2F+1)}\right\}^{\frac{1}{2}}.$$

The secular equation for almost degenerate states is

$$|W^0{}_{F'}\delta_{F',\,F''}+H'_{F'',\,F'}-\delta_{F'F''}W|=0. \quad (4)$$

In this equation $W^0{}_{F'}-W^0{}_{F''}$ is the observed separation between the two h.f.s. levels F and $F-1$, W is the eigenvalue of the energy in the field and H' is as in Eq. (1). When these quantities are inserted we obtain

$$W_m=\frac{W^0{}_{i+\frac{1}{2}}+W^0{}_{i-\frac{1}{2}}}{2}+g(i)\mu_0 H\pm\frac{\Delta W}{2}\left(1+\frac{4m}{2i+1}x+x^2\right)^{\frac{1}{2}}, \quad (5)$$

where x is $\dfrac{(g_j-g_i)\mu_0 H}{\Delta W}$ and $\Delta W=W^0{}_{i+\frac{1}{2}}-W^0{}_{i-\frac{1}{2}}$.

The moment of the atom in a state m is given by $\mu_m=-\partial W_m/\partial H$. This differentiation leads to Eq. (1) of the text, since the first term on the right-hand side of Eq. (5) is a constant. For values of J larger than 1/2 the procedure is identical, but the secular equation is of higher degree and the solution is best obtained with numerical methods.

Equation (3) of the text is obtained by setting for negative values of m

$$g(i)m+\frac{x_1-((2|m|)/(2i+1))}{(1-((4|m|)/(2i+1))x+x^2)^{\frac{1}{2}}}\frac{g_j-g_i}{2}=0, \quad (6)$$

$$g(i)m-\frac{x_2-((2|m|)/(2i+1))}{(1-((4|m|)/(2i+1))x+x^2)^{\frac{1}{2}}}\frac{g_j-g_i}{2}=0.$$

From the definition of x we have

$$\frac{x_1-x_2}{\frac{1}{2}(x_1+x_2)}=\frac{2(H_1-H_2)}{H_1+H_2}=\frac{\Delta H}{H}. \quad (7)$$

The value of x_1-x_2 and x_1+x_2 from Eq. (6) yields the desired formula if one neglects quantities of the order of $(g_i/g_j)^2$ and also drops g_i in the second term on the left-hand side of Eq. (6). This introduces an error of only about 0.1 percent. It is apparent that all reference to the nature of the interaction between the nuclear spin and the extranuclear J is eliminated from the final formula 7.

[11] *Theory of Spectra.*