tions. For example, as $\beta_0 \rightarrow 1$

$$H\rho_0 \rightarrow 1705/(1-\beta_0^2)^{\frac{1}{2}}.$$

Hence, for a given value of $\rho_0 = 1/k_0$, H increases rather rapidly. On the other hand, since $\beta_0 = E/H$, one sees that, as $\beta_0 \rightarrow 1$, E becomes proportional to H, approximately. Therefore, for a given value of ρ_0, E and H will increase approximately in the same ratio. In addition to the latter conditions there are also intensity requirements, which will probably be the final practical factor in such experiments. While it is at present difficult to predict how far it would be feasible to extend these experiments, it is quite clear that the present method offers the possibility of carrying the study of the variation of electron mass to higher energies of the order of ten times those used in the Bucherer-Neumann experiments.

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The Specific Charge of Disintegration Electrons from Radium E

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Rough preliminary measurements of the ratio e/m for the pure primary beta-particles from radium E and of momentum about 2000 $H\rho$ were described in a recent Letter to the Editor. The present article includes a more detailed description of these results as well as of more accurate results obtained with an improved experimental arrangement. In order to ascertain the origin of the side-peaks observed by the former measurements, in the new apparatus the resolving power was increased by doubling the length of the electric condenser, and the scattering was reduced by the use of aluminum for the construction of the condenser. In addition several auxiliary defining slits were inserted at points along the electron path, and all calibrations were rechecked so that the central peak could be located with moderate accuracy. In the preliminary measurements no special attempt had been made toward great accuracy, since the apparatus was designed primarily to distinguish between ordinary Lorentz electrons and the widely differing special type of heavy electrons required by the *speculation* that the well-known beta-ray paradox might be explained by variations, with velocity, of the rest mass of the electrons created in the nucleus, rather than by the neutrino hypothesis. With the improved apparatus extremely sharp peaks were observed by means of the Geiger counter, showing that the method described in the preceding article offers the possibility of very accurate determinations of the ratio e/m. The side-peaks were greatly reduced in height and separation, as compared with those in the preliminary experiments; and with variations in the slits behaved in such a way as to indicate that they are due to scattering from the condenser plates, as previously suspected, or possibly to small nonuniformities in the magnetic field. In any case the latter variations in the side-peaks were found to have no appreciable influence on the position of the central peak, and the side-peaks are therefore of no serious consequence. Further, an analysis along the lines indicated in the preceding article was carried out for the detailed resolution characteristics in this particular case, and it was found that no side-peaks should be expected in the absence of scattering, although second degree equations do occur in the cut-off conditions for the source-condenser system. The central peak was found to be so sharp that it was possible to locate it without difficulty to within 1/10 percent of the voltage at the peak. In fact, the accuracy in determining the peak voltage was limited, in the present arrangement, rather by fluctuations in the magnetic field and in the battery voltage. Similarly, the absolute determination of e/m was limited rather by the accuracy of calibrations, both electromagnetic and geometrical. In the present arrangement no special aim was made toward accurate absolute results to better than one or two percent, but the method offers the possibility of considerably greater accuracy; and if one requires only relative values of e/m, the same accuracy could be obtained without special precautions as to the calibrations. In addition, one is here free from the grave uncertainties (to be discussed in a later article) associated with the possibility of scattering in the Bucherer-Neumann experiments. The final corrected value of e/m was found to be in agreement with the theory of relativity to within $1\frac{1}{2}$ percent, which is well within the limits of experimental error. *The above-mentioned speculations as regards the special type of heavy electron are therefore untenable.*

In a previous Letter to the Editor[1] a brief description was given of a method for the determination of the ratio e/m for beta-particles, and at the same time preliminary results obtained by the use of this method were reported

[1] C. T. Zahn and A. H. Spees, Phys. Rev. **52**, 524 (1937).

for the beta-rays of radium E of momentum about 2000 $H\rho$. It was there mentioned that cloud chamber experiments of H. R. Crane and J. J. Turin[2] had suggested the possibility that primary beta-particles may have greater energy than that conventionally indicated by the curvature of their paths in a magnetic field, or by their momentum. This would require that their masses be different from the conventional mass $m_0/(1-\beta^2)^{\frac{1}{2}}$. An examination of the literature showed that Bucherer's and Neumann's classical experiments[3] for the determination of the ratio e/m of beta-particles had been made at a time when the differences in origin of the primary and the secondary particles had not yet been understood; and that the rays studied by these experimenters consisted chiefly of the secondary, or discrete beta-particles from radium and its equilibrium products. It then appeared that no actual experiments on the specific charge of the actual disintegration electrons had ever been made, after the identification of the latter with the continuous beta-rays.

The question of a possibly different rest mass for the primary beta-rays is closely connected with the well-known beta-ray paradox which led to the hypothesis of the neutrino. Lacking any definite experimental evidence to the contrary, one is tempted to make the speculation, that after all the neutrino hypothesis may not be necessary if the rest mass of the beta-particle were different from the conventional m_0. If one gives up the neutrino and still insists on the conservation of energy and a definite disintegration energy W, then by Einstein's principle of equivalence the *actual* masses of the electrons would all have the same value $W/c^2 \equiv M = M_0/(1-\beta^2)^{\frac{1}{2}}$, no matter what the velocity of emission of the particle. This would require that the beta-particle be a very special kind of electron, whose rest mass depended on the velocity of emission in such a way that $M_0 = (W/c^2)(1-\beta^2)^{\frac{1}{2}}$; that is, the rest mass of the slower electrons would be greater than that of the faster ones. If it further be assumed that the upper limit of the beta-ray energy, as *conventionally* calculated from the upper limit of the experimental momentum distribution, really represents the disintegration energy, and hence at the limiting energy there is no mass anomaly, then $M_0 = m_0$ at the upper limit and $M_0 > m_0$ for all other particles of the spectrum. Hence the electrons required by these speculations would be "heavy."

It need not be pointed out that there would be no violation of the theory of relativity involved here. It would simply mean that in the particular process of creation of beta-particles these particles would be created with such values of rest mass as to give them always the same actual mass, independent of their velocities at creation. The effect on certain experiments, however, would be the same as if one had a velocity distribution of particles of constant mass M, and the theory of relativity were not applicable. For example, this would be the case in experiments where the velocity remains constant in magnitude, as for electrons in magnetic fields or in electric fields which are perpendicular to the direction of motion. These latter conditions hold approximately for the case of electrons inside a condenser of small plate separation compared to the length, provided that one consider only such electrons as can pass all the way through the condenser. These same conditions are precisely the conditions which obtain for the Bucherer experiment and for the modification described in the preceding article.

It was also thought not impossible that the extra rest mass here involved might in some way be associated with abnormal values of the spin coordinate; and that the heavy electron might be created in metastable states, so that they need not return to normal until after a long time or after infrequent processes of "nuclear proportions;" and that therefore the extra energy might not be observable in the classical calorimeter experiment.

Inasmuch as no observations on the specific charge of actual disintegration electrons seemed to have been made, it was thought of importance to make at least a few observations of this kind and at the same time to keep in mind the values of e/m to be expected both in the case of conventional Lorentz electrons and in the case of

[2] See reference to Crane's report on the possible difference between nuclear beta-particles and electrons, G. Breit, Third Washington Conference on Theoretical Physics, Rev. Sci. Inst. **8**, 141 (1937).

[3] A. H. Bucherer, Ann. d. Physik, **28**, 513 (1909), G. Neumann, Ann. d. Physik, **45**, 529 (1914).

the above-mentioned speculations. In the latter of the two cases one would expect the greater departures from the ordinary value of e/m to occur at the lowest velocities of the spectrum. Therefore it would be desirable to measure e/m at the lowest velocity consistent with limitations of the apparatus due to absorption in windows, etc.

Preliminary measurements were made, applying a method fully described in the preceding paper, on the beta-rays from radium E and of momentum approximately 2000 $H\rho$. The latter momentum is sufficiently far from the upper limit of the spectrum that the observed heavy mass should have been about twice the value for normal electrons, if the above speculations were valid. For this purpose a carefully machined brass condenser, of 6 cm length and 0.5 mm plate separation and with quartz separators was used. The source was in the form of a nickel wire 1 cm long and 0.5 mm thick and covered with a radium E deposit of approximately 1 millicurie strength. This source was placed about 5.5 cm from the front end of the condenser and 1 cm below the central plane, corresponding to the value $\rho_{00}=16$ cm. (See preceding article for definition of constants.) The condenser and the source were placed in a vacuum chamber inside a pair of 18-inch water-cooled Helmholtz coils providing a magnetic field of 125 gauss (that is $H\rho_{00}=2000$). These coils were calibrated and tested for uniformity of field in the usual manner by means of a carefully wound exploring coil, a standard mutual inductance, and a ballistic galvanometer. A supply of voltages up to 2000 was provided for the condenser by small B batteries, and the voltage was measured by means of a carefully calibrated Jewell voltmeter with extra resistances. After leaving the condenser the rays passed through an aluminum window (of thickness 2 mils) on leaving the vacuum chamber, and then into a bubble counter provided with a scale-of-eight thyratron recording circuit.

Measurements with this preliminary arrangement were briefly described in the above-mentioned letter.[1] An example of the latter measurements is shown in Fig. 1, where the number of electrons recorded per minute by the Geiger counter is plotted against the condenser voltage V. The two arrows on the voltage axis

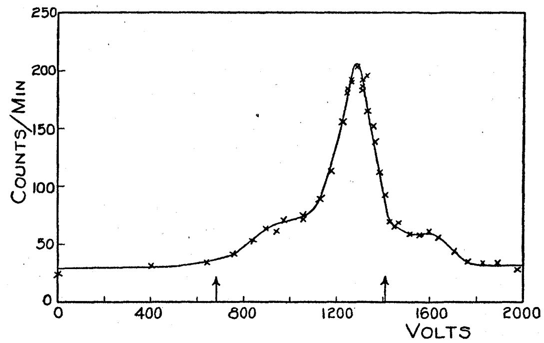

FIG. 1. Preliminary results showing number of electrons recorded per minute as a function of the condenser voltage.

indicate roughly the expected value of "peak" voltage for the two cases in question.

The latter values are calculated as follows. Assuming conventional mass at the upper limit of the spectrum, one obtains the disintegration energy W from the ordinary relativity relation between total energy and momentum:

$$W^2=c^2P_m{}^2+m_0{}^2c^4$$

or

$$W^2=c^2e^2(H\rho)_m{}^2+m_0{}^2c^4,$$

where P_m and $(H\rho)_m$ refer to the upper limit of momentum. For radium E the total energy at the upper limit is taken as 1.66 Mev; or the kinetic energy, as 1.15 Mev.[4]

Now the apparatus specifies a definite $H\rho_{00}$ independently of the values of mass and velocity. Hence the ratio of the masses in the two cases will be given by the inverse ratio of the observed velocities. For ordinary electrons one would have:

$$\beta_1=H\rho/[(H\rho)^2+(m_0c/e)^2]^{\frac{1}{2}}$$

and for the special type of heavy electron:

$$\beta_2=H\rho e/cM=H\rho ce/W$$

whence it is easily seen that

$$(\beta_1/\beta_2)^2=[(H\rho)_m{}^2+(m_0c/e)^2]/[(H\rho)^2+(m_0c/e)^2].$$

With $H\rho=2000$ and $(H\rho)_m=5280$ one obtains: $\beta_1=0.74$; $\beta_2=0.35$; and $\beta_1/\beta_2=2.1=M/m_1$. Since for a constant magnetic field H the velocity for the compensated electrons is proportional to the electric field E, the value of the condenser voltage at which one should expect a peak is inversely proportional to the mass of the electron of the given momentum $H\rho$.

[4] E. M. Lyman, Phys. Rev. 51, 1 (1937).

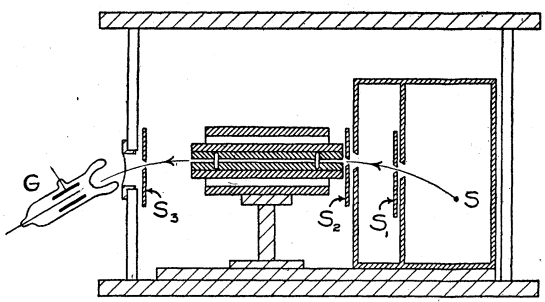

FIG. 2. Schematic diagram of the apparatus.

As was previously pointed out these preliminary data indicated a main peak near the value corresponding to the ordinary Lorentz electron and also two side peaks, the origin of which was not definitely understood. A rough estimate of the expected resolution of the condenser indicated that these side peaks probably must be due either to a bona fide mass effect or to scattering from the condenser plates. While these same results showed quite definitely that the assumption of the previously mentioned type of heavy electron is untenable, the main central peak seemed to be displaced from the expected value for the Lorentz electron by an amount possibly beyond the limits of experimental error. It was therefore decided to repeat the experiment with various improvements of such a nature as to enable one to distinguish between instrumental effects and bona fide mass effects. In this new arrangement it was also planned to aim at greater accuracy by recalibrating all the apparatus.

A schematic diagram of the improved arrangement is shown in Fig. 2. The distance from the source to the condenser was increased over that in the preliminary apparatus, so as to obtain both greater accuracy in establishing the geometrical ρ_{00} and at the same time greater effective momentum resolution. Because of limitations in machining the condenser plates it was not deemed advisable to reduce the plate separation, but the resolving power was further increased by doubling the length of the condenser. The condenser plates were made of aluminum 1 cm thick. Three 1-cm quartz interferometer separators were available to be used as separators for the condenser. The plates were carefully milled down on the sides by an amount sufficient to leave the plates separated by approximately $\frac{1}{2}$ mm. It was found possible in this way to obtain a uniform separation to considerably better than one percent of the separation. The plates were insulated by means of two pieces of plate glass, and the condenser was held rigid by three clamps (not shown) placed just over the quartz posts.

For the purpose of obtaining information concerning the nature of the side peaks the condenser was made of aluminum rather than brass, so as to reduce the scattering coefficient. In addition, adjustable slit systems S_1, S_2, and S_3 were inserted, chiefly to limit the radiation entering the condenser approximately to that momentum interval that could be transmitted by the condenser, and thus further to cut down the scattered component. The slit S_3 serves the same purpose by favoring the direct radiation and giving further momentum resolution. These slit systems could be raised or lowered by means of screws operated outside the vacuum chamber; and each system consisted of three interchangeable slits of widths approximately $\frac{1}{4}$, $\frac{1}{2}$ and 1 cm. Variations in the slit width enable one to ascertain whether scattering from the slits themselves plays an important rôle. The slit S_2 was placed about 1 mm from the front end of the condenser so as to confine the stray field of the condenser to small distances from the end of the plates, as well as to permit further definition of the momentum interval of the electrons reaching the condenser.

The source was enclosed in an aluminum box $\frac{1}{8}$ inch thick to absorb extraneous radiations, including the gamma-radiation; and, in order to avoid distortions in the magnetic field near the source, the latter was deposited on a nickel-plated, rather than a solid nickel wire. The wire itself was mounted on an aluminum frame. Further shielding with lead sheet between the counter and the condenser (with a small opening to allow the beam to pass through) considerably cut down the background of the counter.

TABLE I. *Constants of the apparatus.*

$2\delta=0.04663$ cm	$H\rho_{00}=1926.5$
$l=12$ cm	$\beta_1=0.7487$
$b_0=2.992$ cm	$W=1.66$ Mev
$(b_0^2+c^2)^{\frac{1}{2}}=9.77$ cm	$\beta_2=0.3481$
$\rho_{00}=15.94$ cm	$\beta_1/\beta_2=M/m_1=2.15$
$H=120.85$ gauss	

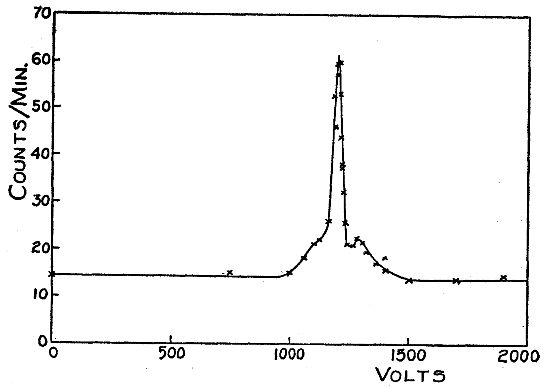

Fig. 3. Electron counts per minute as a function of condenser voltage when slits S_1, S_2, and S_3 of Fig. 2 are omitted.

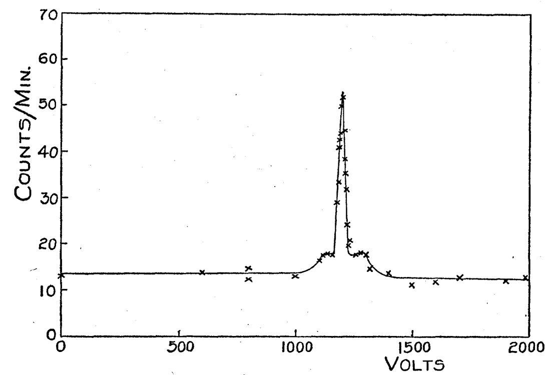

Fig. 4. Electron counts per minute as a function of condenser voltage when the slits S_1, S_2, and S_3 of Fig. 2 are inserted.

The vacuum chamber consisted of a Pyrex tube, of diameter 12 inches and length 8 inches, placed between two brass plates, 14 inches square and ½ inch thick. In the Pyrex tube was ground a 1 inch hole over which the aluminum window could be sealed.

The constants of the final apparatus are given in Table I, with the notation of the preceding article on the theory of the method.

With this arrangement a number of observations were made on the transmitted radiation for condenser voltages from zero up to 2000 volts. Counts were observed for time intervals of ten minutes at each voltage, and the total number of counts observed for voltages on the peak was about 600. It was found that variations of slit width produced negligible, if any shifts in the peak voltage. The curve in Fig. 3 corresponds to a case where the slits were all left out; and that in Fig. 4, to a case where all the slits were in position. All three slits were electrically grounded, and in order to prevent dissymmetries or shifts due to the grounding of the slit S_2 the condenser voltage was supplied in such a way as to permit grounding at the center of the battery, or at any other point. Variations in the position of this ground, however, were found to produce no appreciable effect.

These results, when compared with the preliminary results shown in Fig. 1 show clearly that the side peaks are of an instrumental origin, rather than due to a mass effect. Their separation and height are considerably reduced as compared to those in the previous case. Also the introduction of slits produces the same type of reduction. These facts suggest very strongly that the subsidiary peaks are really due to scattering from the condenser plates. One might, in fact, expect that scattering could produce the observed type of side peaks; for as one adjusts the voltage just off the peak value there will be scattering from the plates near the exit of the condenser, but under these conditions the scattered radiation will immediately escape from the condenser and be greatly dispersed because of the very poor resolution for rays originating near the exit from the condenser. Now as the voltage is adjusted farther off the peak the scattering occurs farther back in the condenser, and consequently there is a greater chance for multiple scattering and at the same time a greater resolution for the scattered radiation. On the other hand the probability for escape decreases as compared to that for absorption, because of the decreasing angular aperture. With these two opposing effects it seems likely that such side maxima might occur.

In order to be able to interpret intelligently the results obtained by this method a careful analysis was carried out for the resolution and intensity characteristics of the method used. Since part of the conditions for cut-off by the condenser plates involve second degree equations in the small variations assumed, it was thought possible that side peaks might be characteristic of the instrument itself, quite apart from scattering. For this reason it was considered advisable to complete the analysis outlined in the

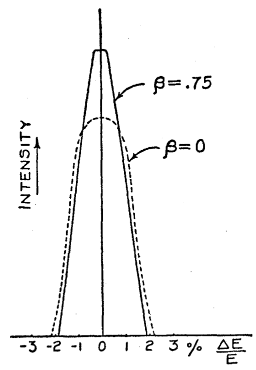

FIG. 5.

preceding article before attempting to interpret the data. Besides, it is always necessary to know the actual momentum spread of the transmitted electrons in order to insure that it is small enough that variations in the source intensity with momentum may be neglected, as well as variations of the slope of the k_i, k curve.

Resolution calculations were then carried out for the case $\rho_{00}=16$ cm, $b_0=3$ cm or $c=9.33$ cm, $l=12$ cm, and $\delta=0.025$ cm, which case approximates satisfactorily the conditions of the experimental arrangement. Since, as has been pointed out, the resolution depends on the particular type of variation of the mass with the velocity, the calculations were performed for the two cases: (1) Lorentz electrons around 2000 $H\rho$, or $\beta=0.75$ approximately; and (2) electrons of constant mass, and around the same momentum.

The results of these calculations are shown graphically in Fig. 5. In the notation of the previous article Δ really represents an increment in k_0, so that one can conveniently express the resolving power as $\Delta k_0/k_0=\Delta/k_{00}$. But one is interested more directly in the resolving power in terms of E, or in terms of the observed voltage V; that is, in $\Delta E/E=\Delta V/V$. Now it has also been shown that:

$$\Delta E/E=(1-\beta^2)\Delta k/k, \text{ for Lorentz electrons.}$$

For this reason the curves of Fig. 5 are plotted against $\Delta E/E$. It may be recalled from the previous article, that the over-all absolute half-width allowed by the condenser alone $\Delta E/E=2k_a/k$, which in the present case amounts to about 8.9 percent; and also that this limiting value was found to be independent of β. It is of interest to note, that, even though the position of the source improves the resolution by more than a factor of 4 : 1, nevertheless, in this case at least, also the *actual* voltage resolution for the two cases is about the same; namely, about 2 percent.

Figure 6(a) and Fig. 6(b) show the intensity-momentum distributions of the electron beams transmitted for different values of Δ, and for the two cases $\beta=0$ and $\beta=0.75$, respectively. The difference in the behavior shown in the two cases is related to the fact that the predominant type of cut-off, as Δ approaches its cut-off value, is different. One sees also from these figures that the expected half-widths, due to both the source and the condenser, have about the same value, 5 percent for *both cases;* whereas the over-all absolute limits of the half-widths defined by the condenser alone should be $\Delta k/k=k_a/(1-\beta^2)k$, or about 5 percent and 10 percent for the two cases, respectively. Hence the source does not further restrict the spread Δk appreciably for the case

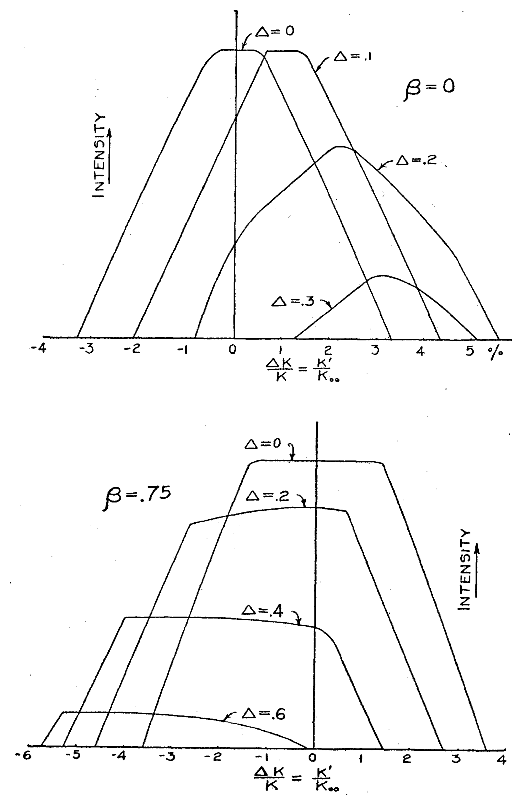

FIG. 6a AND 6b. Intensity-momentum distributions of electron beams transmitted for different values of Δ and for the two cases $\beta=0$ and $\beta=0.75$.

$\beta=0$; but for the case $\beta=0.75$, by a factor of about 1 : 2.

It is then clear that the voltage resolution obtained by the present method should be extremely good (2 percent half-width) as compared with that obtainable with the photographic method. As regards the effect of nonuniformities in the intensity-momentum distribution of the source, one must consider the above spreads in k itself. These spreads were found to be about 5 percent half-width, but because of the extreme sharpness of the voltage resolution effects of the source distribution will be very much minimized.

These results also show clearly that one should not expect side peaks for the direct, or unscattered radiation, and therefore further suggest that the latter are really due to scattering. It should, however, be noted that the effect of the finite extension of the source over the cylindrical wire has been tacitly neglected. The effect of this distribution would be to broaden the observed peak. To a close approximation one may regard this effect as due chiefly to a shift in the resolution curve without a change in width, which shift would be given by the change in k_{00} as the source position changes. In the present arrangement, with a $\frac{1}{2}$ mm source wire, one estimates the shift in k_{00} from points at the top to points at bottom of the wire to correspond to a half-width in k_{00} of about 0.7 percent or 0.4 percent in V. Now, if the source material is distributed uniformly over the cylindrical wire, the source intensity for small equal increments of k_{00} will be much greater for the values of k_{00} corresponding to the top and bottom of the wire than for the other points, since the electrons in question leave these points in directions tangent to the surface. Hence one might expect, under conditions where the "natural width" due to a line source is smaller than the extra width due to the source extension, that the integrated effect of the whole source could give rise to a double peak, between which one would expect a flat central portion, unless the peaks are not resolved, in which case one would expect simply a somewhat broadened central peak.

Since the natural resolution width was calculated as 2 percent half-width approximately; and the extra half-width due to the source, 0.4 percent, one would hardly expect the source distribution to produce resolved double peaks. At least, one would expect such peaks, if resolved, to appear only in the "fine structure" of the central peak, since the actual observed side peaks are separated by 13 percent in the case of no slits (see Fig. 3), or about ten times the separation estimated for the effect of the source distribution. This is further borne out by the preliminary data of Fig. 1, for which one would expect the source extension to be equivalent to a half-width of 1.3 percent in V, whereas the observed half-separation of the side peaks was about 20 percent.

Finally, it seems most probable that the side-peaks are due either to scattering from the condenser plates, as previously suggested, or to some other instrumental cause such as nonuniformity of the magnetic field (see later). In any case the presence of these peaks seems to be of no consequence as regards the position of the central peak, since variations in the separation and intensity of the side peaks, produced by slits and variations of the condenser resolution, caused no appreciable shifts in the central maximum. In fact, the observed half-widths of Fig. 3 and Fig. 4, 3.8 percent and 2.7 percent, respectively, when compared with the total theoretical half-width in the absence of scattering, $2.0+0.4=2.4$ percent, show that, at least *with* the auxiliary defining slits, the scattering does not even cause much broadening of the central peak.

Observed Value of e/m

In the preliminary experiments with the short brass condenser no special attempt was made toward great accuracy, since the apparatus was designed primarily to distinguish only semi-quantitatively between two cases of widely differing mass. Therefore no importance is to be attached to the previously mentioned possibility of a discrepancy of about 10 percent from the theory of relativity, particularly since it was later discovered that some of the rough calibrations were sufficiently at fault to explain away a large part of the discrepancy, and since in calculating the expected position of the central peak of Fig. 1 no correction was made for the stray field of the electric condenser and none for the effect of nonuniformity of the magnetic field of the Helmholtz coils.

End effect of the condenser: In order to take account of the attenuation of the electric field of the condenser, which in the idealized theory is assumed to occur abruptly at the end, Bucherer has pointed out that the effect is as if the ideal condenser were extended a distance p beyond the end of the plates; and he made some estimates based on Coffin's mathematical solutions of the end effect for certain types of condenser, which, however, do not approximate his own experimental conditions very well. For thin condenser plates an accurate solution is well known, but for thick plates like those of Bucherer the stray field may extend considerably farther than in the case of thin plates. Besides, in order to make the correction as Bucherer made it, it is necessary to be able to assume that the end effect extends only a short distance beyond the condenser plates, so that it may be treated as giving rise to a sudden impulse over a short distance. Since Bucherer's method of estimating p does not strictly apply to the actual conditions, Neumann attempted to treat the extra length p as a further constant to be determined from sets of data taken with two different values of the distance from the condenser to the photographic plate. But he found that his values of p, as calculated from his various sets of data, varied all the way from positive to negative values. Finally he had to satisfy himself that these fluctuations were due to the experimental difficulties associated with the determination of a small difference, and to take an average of all his values of p. (As will be seen in a later article on the interpretation of Neumann's data, there may have been other reasons for his difficulty in determining a constant value of p.)

In the present apparatus thick aluminum plates were used, for obvious mechanical reasons; and, as previously mentioned, in order to reduce the stray field suddenly to zero, a grounded slit was placed just in front of, and about 1 mm from the entrance to the condenser space. Hence a mathematical calculation for the stray field becomes impractical, but the effect may in no sense be negligible, if one aims at reasonable accuracy. Consequently measurements of the field attenuation were made by the well-known electric trough method on a large-scale model of the slit-condenser system. When the slit and the center tap of the battery are grounded, the central plane of the condenser becomes a plane of symmetry at zero potential. By the theory of images in a conductor this permits halving the model and placing a conducting surface of zero potential at the plane of symmetry. Further, in order to obtain the conditions of the approximate two-dimensional problem, it is necessary to construct only a shallow trough; since, as is well known, the effect of the insulating surfaces at the top and bottom of the electrolyte is the same as if the problem were a simple electrostatic problem with *insulating* dielectrics, but with the insulating surfaces of the air and the bottom of the trough replaced by surfaces of dielectric of specific inductive capacity zero (i.e. of negative polarizability). Under these conditions the latter surfaces of equivalent dielectric behave as perfect reflectors with *unchanged* sign of electric charge, in contradistinction to the case of reflections in conducting surfaces, where the sign of the charge is changed. The net effect of all the multiple reflections thus brought into play is such as to extend the model effectively to infinity, simulating a two-dimensional problem. (A more rigorous explanation in terms of boundary conditions is obvious.)

By this method the effective extra length p of the condenser was found to be $p=0.29$ mm, or about half the condenser separation. By a simple calculation it can be shown that the end effect is equivalent to a reduction in ρ_{00} of amount $(c/b_0)p$, which in the present case is $9.31\times.029/2.992=0.09$ cm, or $0.09/15.94=0.56$ percent of ρ_{00}.

Correction for nonuniformities in the magnetic field: The value $H=120.85$ gauss, given in Table I, refers to the value of the magnetic field intensity measured at the center of the Helmholtz coils. By exploring with a small coil it was found that the magnetic field dropped off by an amount of the order of a percent as one moved the coil from the center of the field to the extreme positions of the source and of the exit end of the condenser. Because of this nonuniformity it is necessary to make two small corrections when calculating the expected peak voltage for Lorentz electrons:

(I) In the space between the source and the condenser, since the average field will be less than the above-mentioned value of H, the momentum $H\rho$ of the electrons reaching the condenser with their velocity directed along the central plane will be smaller than the value given in Table I. One may take this effect into account by calculating the effective reduction in ρ_{00} corresponding to the value of H at the center of the field space. This correction may be made approximately by assuming that the field drops off linearly between the condenser and the source. (It would, of course, be more accurate to integrate over the region in question.) In this way one finds that the effect is equivalent to a reduction in ρ_{00} of 0.72 percent.

This reduction of 0.72 percent, taken together with the 0.56 percent due to the end effect of the condenser, gives a total reduction of 1.28 percent, whence the effective

$$\rho_{00} = 15.94(1-0.0128) = 15.72,$$

which corresponds to an effective momentum of $15.72 \times 120.8 = 1898$ $H\rho$, or to an effective velocity $\beta_1 = 0.745$, instead of 0.749 as given in Table I.

(II) Similarly one must correct for the small drop in magnetic field in the condenser space. For this correction one may again assume approximate linearity of the field variation with y. By making use of the original equations of the theory of the preceding paper, and replacing H by a field varying linearly with y, one may calculate the change in k necessary to compensate for the change in deflection at the exit of the condenser due to the small attenuation in the magnetic field. When this is done for the central ray ($b=0$ and $\phi=0$) at $k_0=k$, one finds that the effect is equivalent to a reduction in the expected peak voltage E_0 of amount 1.46 percent.

Final corrected value of expected peak voltage E_0: With the ideal values of Table I one calculates for the expected, but uncorrected value of the peak voltage E_0 the value 1266 volts; whereas the observed value 1222 volts differs from the uncorrected theoretical value by 3.5 percent. With the corrected value $\beta_1 = 0.745$ and the corrected value of k_0 one calculates the corrected value of the expected peak voltage as 1240 volts. This leaves a final discrepancy of about $\frac{3}{2}$ percent on the side of heavy electrons, but this corresponds only to about the estimated limit of experimental error, and on the basis of the present experiments cannot be considered as definite evidence for a bona fide anomalous mass effect. Besides, the corrections made for the nonuniformity in magnetic field are only rough approximations, since the field does not actually drop off linearly along the electron path; and no correction was made for the small effect of the earth's magnetic field. (It is also worth mentioning here that, while the shift in the central peak voltage due to the nonuniformity of the magnetic field, as given roughly by the latter calculations, is small; on the other hand the shape of the peak may possibly be considerably altered, and possibly in such a way as to give rise to small side peaks like those observed. In fact, a more detailed consideration of the maximum possible order of magnitude to be expected for the scattered component of the transmitted radiation makes it difficult to believe that the observed side peaks could be caused by scattering. In any case, since the side peaks have been shown to have no appreciable effect on the position of the central peak, they are of no further consequence in this connection.)

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