An Improved Method for the Determination of the Specific Charge of Beta-Particles

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In the following article is described a method for the determination of the ratio e/m of beta-particles. The method represents a modification of the experiments of Bucherer and of Neumann in which the positions of the source and the detector are effectively interchanged and the photographic plate is replaced by a Geiger counter in a fixed position. A general discussion is given for the behavior of electrons in crossed fields inside a shallow condenser, and application is made to the special case here described. A semi-graphical procedure is then outlined for the determination of the shape and the intensity of the peaks to be observed by the Geiger counter as the voltage on the condenser is varied. The effect of scattering on such experiments and the importance of resolution calculations are then pointed out. In this latter regard the modified experiment appears to have considerable advantage over the original method of Bucherer. In fact, as a result of these considerations, doubts are raised as to the validity of the usually accepted interpretation of Neumann's experimental data at the higher velocities. A more complete discussion of the interpretations of the classical experiments of Neumann is promised for a later date. The possibilities of the method for measurements on electrons of very high velocities are discussed. Experimental results obtained by application of the present method to the determination of the specific charge of pure disintegration electrons will be given in the paper immediately following.

THE method here described was developed for the purpose of determining the ratio e/m for pure disintegration electrons as distinguished from the secondary, or discrete beta-particles.[1] As is well known, Bucherer's classical experiment consisted in placing a source of beta-rays at the center between the plane plates of a circular condenser in vacuum and then superimposing a uniform magnetic field parallel to the condenser plates. The crossed fields and the condenser plates form a velocity filter with a resolving power depending on the plate separation and on the length of path in the condenser. The filtered velocity depends on the ratio of the electric and the magnetic field intensities, E/H, and on the angle θ between the magnetic field and the radius of projection into the condenser ($v=cE/H\sin\theta$). After leaving the condenser the electrons are deflected in the magnetic field alone and finally produce a trace on a photographic plate. Each point on the trace corresponds to a different angle θ and hence to a different filtered velocity v. The deflection depends on the momentum of the electron, and the ratio e/m can be calculated very simply from the measured values of E, $H\sin\theta$, and the deflection. Neumann's modification consisted in working with only one velocity at a time, with the magnetic field and the electron path mutually perpendicular ($\theta=\pi/2$). Hence he placed a source at the end of a rectangular condenser.

For the above-mentioned purpose it was thought that the use of a Geiger counter as detector might eliminate the inconveniences and the inaccuracies of the photographic method as well as permit the use of weak sources. A Geiger counter, used simply in Neumann's arrangement to replace the photographic plate, would have to be moved about to explore the region covered by the plate. This would cause considerable inconvenience, particularly if it were necessary to use the counter in vacuum, as in the case of beta-rays of low energy. It will, however, be seen in the sequel that these difficulties, as well as other serious limitations of the Bucherer-Neumann method, can be successfully eliminated by the use of the following modification. The method consists essentially in interchanging the positions of the source and the detector. The source S, Fig. 1, is placed some distance from the first end of the condenser plates and somewhat below the central plane; and the rays transmitted are detected by means of a Geiger counter G placed at the other end, or at any distance along the appropriate circular arc drawn from the condenser. The electrons will be transmitted only when, on their arrival at the entrance to the condenser, they are directed approximately

[1] For preliminary results see C. T. Zahn and A. H. Spees, Phys. Rev. **52**, 524 (1937).

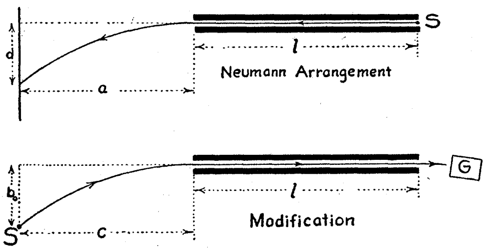

FIG. 1. Arrangement of source and detector in the Neumann experiment and in the present modification.

along the central plane and when at the same time the electric field is so adjusted that their velocity $v=cE/H$. The relative geometry of the source and the condenser specifies within a certain small tolerance a momentum $H\rho$, and the observed value of E permitting transmission serves to determine the velocity associated with this momentum. Here the momentum is selected first, and then the velocity is measured; whereas in the Bucherer experiment the velocity is selected first and then the momentum is measured.

With this modification the geometry is kept fixed, and all the geometrical calibrations can be made once for all, after which all subsequent measurements can be made exclusively by the use of electric meters and the Geiger counter. With this fixed geometrical arrangement one can then obtain extremely accurate *relative* measurements independent of all geometrical factors; whereas in the photographic method all individual results are subject to errors in the microphotometric measurements on rather poorly resolved lines. Electrons of various values of the momentum $H\rho$ can be studied simply by varying H. A further simplification is provided by the fact that an experiment may be repeated at will without the necessity of opening the vacuum chamber as in the photographic method.

Other advantages are to be seen in connection with certain objections which have been raised from time to time against the interpretations of the Bucherer-Neumann data. As will be remembered, a long discussion between Bestelmeyer and Bucherer centered around the question as to whether the limited resolving power of the condenser could cause a dissymmetry in the line density on the photographic plate, sufficient to displace the observed maximum appreciably from the theoretical position corresponding to perfect resolution. Bucherer made some calculations for the case $\beta=0.3$ and found no serious dissymmetry, but he does not seem to have made similar calculations for the higher velocities—although it is just at the higher velocities that one would expect such difficulties, and although in fact he (as did Neumann later) found it impossible to obtain sharp traces for $\beta>0.8$, because of a spreading and fogging of the photographic line! It is clear that the resolution should become poorer as the velocity increases and that one should expect just such a limitation as was observed. Therefore the question raised by Bestelmeyer still seems pertinent, at least for the higher velocities; that is, for velocities near the region where the resolution becomes poor. It is of further interest to note that Neumann, after repeating Bucherer's measurements with slight alterations, stated that: for $\beta<0.7$ the agreement with the relativity theory was satisfactory, but that for $\beta>0.7$ the data showed a definite trend from the theory, it not being quite clear whether or not the discrepancy was within the limits of experimental error. Later Schaeffer, after remeasuring Neumann's plates with an improved microphotometer, seemed satisfied that the small systematic discrepancies could be explained by imperfections in the original photometric measurements. Nevertheless, not until a thorough theoretical investigation of the system of rays in such an experiment is made, will it be possible to state definitely within what limits of error and over what region of velocities the theory of relativity has been verified experimentally by Neumann's results. It is desired here simply to point out the difficulties in interpretation associated with Neumann's data and the improvement achieved in this regard by the use of the present modification. A complete analysis of the electron "optics" of the *modified* experiment will be given in the following. A similar treatment of the Bucherer-Neumann apparatus will, however, be deferred to a later article, in which it is planned to give a critical discussion and interpretation of Neumann's results.

In the interpretation of Neumann's data an idealized theory was used, which takes into account neither the imperfect resolution of the apparatus nor the possibility of scattering from the condenser plates. The possibility of scattering has been mentioned, but it was assumed, with questionable justification, that the scattering is of negligible amount, or at least that it is so spread out after reaching the photographic plate as to be of negligible, or of uniform density in the region of the line. Now it is well known that a considerable number of electrons is scattered diffusely from material surfaces. The coefficient of diffuse back-scattering may, in fact, vary from 10→80 percent, depending on the incident energy and roughly on the density of the scatterer. For silver it is of the order of 40→60 percent, and for aluminum or glass, 10→40 percent. With such high reflection coefficients it seems possible, in an arrangement like that of Bucherer, that a very considerable amount of multiply scattered radiation could emerge from the condenser in addition to that expected on the idealized theory in which it is assumed that all electrons are absorbed after impinging on the condenser plates. In fact, it does not seem impossible that for heavy scatterers the multiple reflections could produce a scattered component much greater than the direct component. It might at first seem that scattering would not cause serious difficulty, since, even after scattering and the accompanying energy reduction due to range depth in the scatterer, the particles would still not escape from the condenser unless they had the proper velocity for compensation. But it should be remembered that the scattering will occur at all points along the condenser plates, and that for points near the exit the resolution becomes very poor, since the effective path length is small. Hence, while the intensity of the beam may be considerably enhanced by the presence of scattering, the effective resolving power may be greatly reduced. Scattering would then tend to aggravate the difficulties associated with limited resolving power, and to a degree that is difficult to estimate.

This resolution difficulty will exist for any actual experiment; and, if one should wish to extend the region of experimental verification of the theory of relativity to higher velocities, one would finally be limited accordingly. It is therefore of importance first to increase the ideal resolving power to the greatest possible value consistent with intensity requirements, and then to eliminate scattering insofar as is possible. In view of these considerations it will be seen that the present modification offers considerable advantage over the original method. First, by the use of the Geiger counter it should be possible to work with lower intensities (neglecting the undesirable increase due to scattering) and therefore with higher resolving power. Further, it may be noted that in Neumann's experiment, for example, about one-half the total radiation from the source enters the condenser space, while only about *one ten-thousandth* of this can possibly be transmitted directly to the photographic plate, the rest being absorbed or scattered. On the other hand, with the present modification, since the source is some distance from the condenser opening, the amount of radiation entering the condenser may be of the same order of magnitude as that which can be transmitted when the electric field is adjusted for a peak; and in fact, by the use of auxiliary slits any desired momentum resolution can be obtained. Also by placing the counter at some distance along the circular arc from the condenser and using further defining slits at the proper points, one can obtain a double momentum resolution and at the same time reduce the scattered component of radiation entering the counter. Scattering will, of course, not be entirely eliminated even here; for when the electric field is not adjusted for a peak, the electrons will impinge on the condenser plates; but the effect of scattering will obviously be greatly reduced when compared to that in the Bucherer experiment. Further, it is possible by the use of the auxiliary slits to restrict the momentum interval of the electrons so that when the condenser voltage is adjusted for a peak, there will be no scattering at all, since all the beam will pass freely through the condenser. Under these circumstances the chief effect of scattering will occur at voltages some distance on either side of the peak. On the other hand, in the Bucherer experiment, the scattered radiation is distributed all over the peak; and, in fact, it is possible that the peak intensity itself may be largely due to scattered radiation of various degrees of resolution.

Electrons in Crossed Fields

In studying electron paths in crossed fields it is a simple matter to obtain certain general rela-

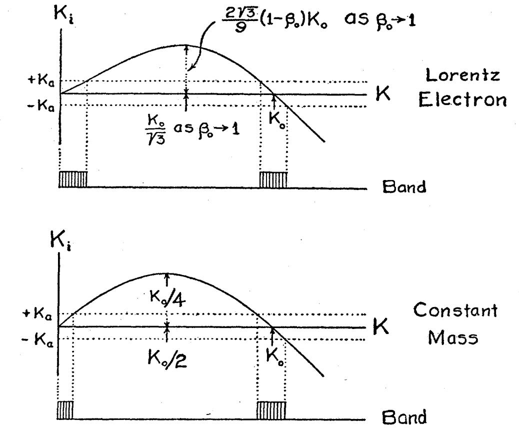

FIG. 2. The curvature of the electron path inside the condenser for constant electric and magnetic field but for various values of the curvature of the electron path in the magnetic field alone.

tions which apply to cases where the condenser separation is small compared with its length. The force is then always very nearly perpendicular to the direction of motion, *for those electrons* which eventually pass all the way through the condenser; and therefore the velocity and the mass are invariant under the motion. The equation of motion may then be written:

$$mv^2k_i = Hev - ceE, \tag{1}$$

where m is the mass of the electron; v, its velocity; and k_i, the curvature of the trajectory inside the condenser. To a close approximation the electrons in question will describe circular paths of curvature k_i. One may further write:

$$mv^2k = Hev, \tag{2}$$

where k is the curvature the electron path would have in the magnetic field alone, and therefore a measure of the reciprocal of the momentum of the electron. If one further writes $cE/H \equiv v_0$, which is the velocity for which (1) vanishes, or the velocity for which the forces are compensated, it then follows from (1) and (2) that:

$$k_i/k = 1 - v_0/v \tag{3}$$

If the relation between the mass and the velocity is known, it will be possible in general to express k_i as a function of k and k_0 alone. For Lorentz electrons, for which $m = m_0/(1-\beta^2)^{\frac{1}{2}}$, it follows that:

$$k_i = k[1 - ((A^2+k^2)/(A^2+k_0^2))^{\frac{1}{2}}], \tag{4}$$

where $A = eH/m_0c$. For electrons of constant mass it would follow that:

$$k_i = k[1 - k/k_0]. \tag{5}$$

(The latter case was of special interest in connection with certain speculations mentioned in a previous Letter to the Editor, and to be discussed in the paper immediately following.)

It will therefore be seen that the resolution characteristics will depend on the particular type of variation of mass with velocity, and that a relativity distribution of electrons, with constant rest mass, may have a different resolution width from that of a distribution of constant actual mass.

Qualitative considerations: The general behavior of a shallow condenser in crossed fields may be understood by considering the relation (4), or (5). Two special cases are of interest: (*a*) k_0 fixed and k variable, as in the Bucherer experiment, and (*b*) k fixed and k_0 variable, as is approximately the case for the modified experiment.

Case (*a*) k_0 *fixed.*—With k_0 fixed; that is, with E and H fixed, one can determine the behavior of the velocity filter for a distribution of varying k. The relations (4) and (5) will have the general characteristics shown in Fig. 2. As the velocity increases beyond v_0 (i.e., $k < k_0$) a maximum value of k_i is reached, after which k_i falls to zero, for $v = c$ in the case of Lorentz electrons, and for $v = \infty$ in the case of constant mass. A simple consideration of the geometry of the condenser will show that there is a maximum value of $|k_i| \equiv k_a$, for which electrons could be transmitted at all. This maximum value will correspond to the special initial conditions indicated in Fig. 3. Whenever $|k_i| > k_a$ the electron can definitely *not* be transmitted; but if $|k_i| \leqq k_a$ the electron has a possibility of being transmitted, provided that the initial conditions at entrance

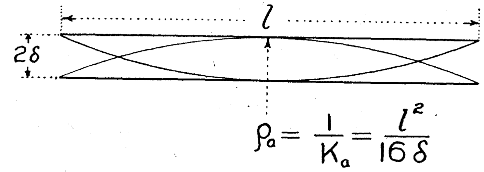

FIG. 3. Geometrical conditions which show that there is a maximum value of $|k_i| = k_a$ for which electrons can be transmitted.

to the condenser be suitable. If $k_a \ll k_0$ one sees from Fig. 2 that the condenser may transmit electrons only in two small bands, one around $k=k_0$, and another around $k=0$ (i.e., high velocity). The latter band around $k=0$ will usually be of no importance, unless it becomes very wide. As k_a is made smaller, by proper choice of the condenser geometry (see Fig. 3), the band narrows down and the resolution improves. But if k_a becomes larger one may reach the condition shown in Fig. 4, where the condenser behaves as a "by-pass" filter for all velocities greater than a velocity slightly less than v_0. Under these conditions the condenser cannot be regarded as a filter for a small band around $k=k_0$. It may also be noted that as k_a is decreased it requires smaller k_0 for a given required resolution; that is, for higher velocities the resolution becomes poorer with a given k_a.

From these considerations of the case (a) one sees clearly the imitations of the Bucherer experiment. In fact, when either k_a becomes too large, or v_0 becomes too large, the band around $k=k_0$ spreads out to meet the band around $k=0$, and the resolution becomes infinitely bad on the side of the higher velocities, while remaining only fairly good on the lower side.

Case (b) *k fixed.*—In the case of the modified experiment one may regard k as approximately fixed within the small limits defined by the narrow momentum interval. Then one is interested in the variation of k_i with k_0 where k_0 is varied by varying the electric field E. This variation is shown in Fig. 5 for two neighboring values of k, corresponding to the limits of the momentum selection before reaching the condenser. The shaded rectangle represents the band in k_0 for which transmission is potentially possible (*but further* contingent upon the proper choice

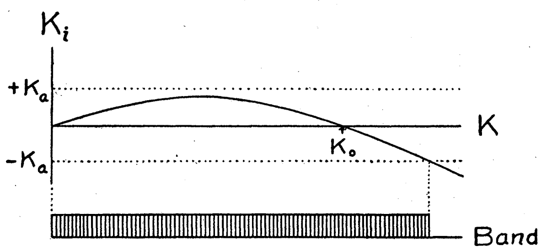

FIG. 4. Conditions under which the condenser system behaves as a "by-pass" filter for velocities greater than a velocity slightly less than v_0.

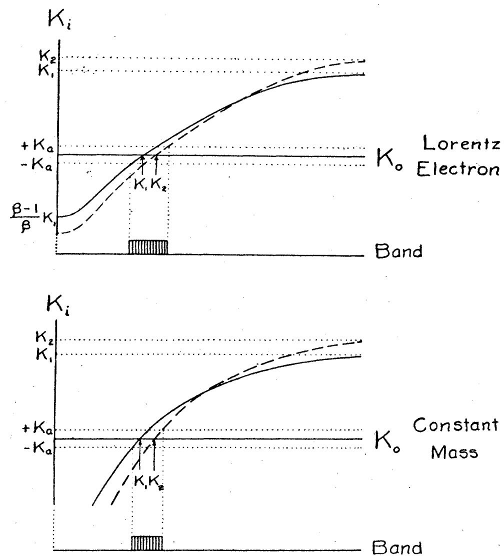

FIG. 5. Variation of k_i with k_0.

of initial conditions at entrance into the condenser). A comparison of these curves with those of Fig. 2 shows the decided advantages of the modified arrangement as regards resolution. Whereas in the Bucherer experiment k_i has a definite maximum in the interval $k=0 \rightarrow k_0$; in the present case k_i increases steadily with k_0 and approaches the value k assymptotically. Nothing appears here corresponding to the high velocity band around $k=0$ in the former case. Besides, the over-all spread in k_i, for example, in the simple case of constant mass is $-\infty \rightarrow k_0/4$ in the former case; and $-\infty \rightarrow k$ in the latter, which indicates that k_a may be much larger in the latter case and still give good resolution. The same is true of the case with Lorentz electrons. In addition, here the behavior around $k_0=k$ will be much more nearly symmetrical than that around $k=k_0$ in the Bucherer experiment.

Conditions for Cut-Off

Up to the present one has considered simply the overall limits of k, or of k_0 permitted by the geometry of the condenser alone; but transmission will be further contingent on the actual initial conditions at entrance into the condenser. The position of the source will in general determine these initial conditions, but it will be of more general use to determine the cut-off conditions in terms of the initial conditions them-

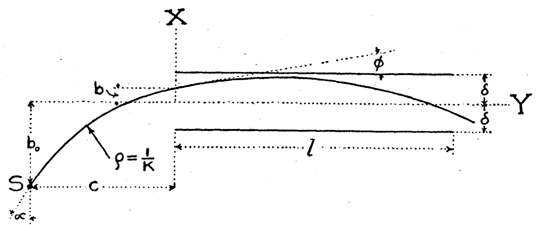

FIG. 6.

selves, regardless of how the source may give rise to them. One can specify the initial conditions in terms of b and ϕ (of Fig. 6), where b represents the distance of the point of entrance above the central plane of the condenser; and ϕ, the angle between the initial direction of motion and the same plane. The equation of the trajectory of the electron may be written, for small deflections and in the interval inside the condenser space, as:

$$x=b+\phi y-k_i y^2/2 \quad \text{for} \quad 0 \leqq y \leqq l.$$

It is then easy to show that the conditions for cut-off may be written:

(I) $+\delta \leqq b \leqq -\delta$ (cut-off at entrance),
(II) $+\delta \leqq b+\phi l-l^2 k_i/2 \leqq -\delta$ (end cut-off),
(III) simultaneously: $0 \leqq \phi/lk_i \leqq l$ and
$+\delta \leqq b+\phi^2/2k_i \leqq -\delta$,

which latter pair of simultaneous conditions expresses the fact that there is an extremal value of x between the two ends of the condenser and that at the same time this extremal value itself is outside the interval of the condenser space, or outside the interval $+\delta \leqq x \leqq -\delta$.

These conditions involve the three variables b, ϕ, and k_i; and, if one uses the previous relation between k_i and k, they may be expressed in terms of b, ϕ, and k. They are generally applicable, and if one specifies the position of the source at $y=0$ and $x=b$, they refer to the conditions of the Bucherer-Neumann experiment. If, on the other hand, one places a source in front of the condenser, as in the modified arrangement of Fig. 1, the variables b, ϕ, and k are no longer independent. In fact, for small variations around the central ray it is easily seen that;

$$\phi=-k'c-\alpha'(1-b_0k_{00}), \tag{6}$$

$$b=-k'b_0/k_{00}-\alpha'c, \tag{7}$$

where $\alpha' \equiv \alpha-\alpha_{00}$ and $k' \equiv k-k_{00}$; whence also:

$$\phi=b(1-b_0k_{00})/c-k'b_0/k_{00}c. \tag{8}$$

Any two of the four variables ϕ, b, α', and k' will suffice to specify the trajectory completely; but since b and k' enter rather simply into the cut-off conditions, it is convenient to choose these as the independent variables. Now, for any given value of k', or for any given momentum, there will be a definite interval in b for which rays are transmitted; and because of the linear relationship (7) this interval in b will be proportional to the corresponding interval in α'; that is:

$$\Delta b=-c\Delta\alpha', \text{ for constant } k'. \tag{9}$$

But the interval $\Delta\alpha'$ is a measure of the relative intensity of rays transmitted with momentum specified by k', if one assumes that the source is isotropic and its distribution function is practically constant throughout the small interval in k' permitted by the apparatus. According to (9) then, so also is Δb a measure of the relative intensity of the same rays. The problem of determining the actual intensity-momentum band transmitted for given values of E and H resolves itself into determining Δb as a function of k'.

In order to apply the cut-off conditions it is necessary to specify k_i in terms of k. For Lorentz electrons the relation (4) holds; and for electrons of constant mass, relation (5). (It may be recalled that k_{00} refers to the central ray defined by the position of the source relative to the condenser; and k_0, to the compensated value of k specified by the ratio E/H.) In either of the above two cases, since the interval in k' is assumed small, one may write:

$$k_i=B(k-k_0), \quad \text{where } B=(\partial k_i/\partial k)_{k=k_0}.$$

Then, if $k_0-k_{00}\equiv\Delta$, it follows from the definition of k' that $k_i=B(k'-\Delta)$. Δ is then a measure of the amount that the condenser voltage is off adjustment for the peak of transmission. For Lorentz electrons it can be shown that $B=-(1-\beta_0^2)$; while for the case of constant mass, $B=-1$.

These relations, taken together with the cut-off conditions and the source conditions, should suffice to determine the momentum distribution of electrons for given values of Δ, and hence one can determine the value of Δ at which absolute cut-off occurs. For this purpose the cut-off conditions may be rewritten as follows:

(I) $+\delta \lesssim b \lesssim -\delta$,

(II) $+\delta \lesssim b[1+(1-b_0k_{00})l/c] - k'[lb_0/k_{00}c+l^2B/2]+l^2B\Delta/2 \lesssim -\delta$,

(III) (a) $0 \lesssim \{b(1-b_0k_{00})/c - k'(b_0/k_{00}c)\}/B(k'-\Delta) \lesssim l$,

(b) $+\delta \lesssim b+[b(1-b_0k_{00})/c - k'(b_0/k_{00}c]^2/2B(k'-\Delta) \lesssim -\delta$.

Conditions (I) and (II) restrict transmission to a parallelogram in the b, k' plane, and (III) further restricts it to a space lying inside two hyperbolae unless at the same time outside two wedgeshaped spaces defined by the two straight lines implied by (III) (a). An example of this restricted area is shown in Fig. 7 for $\Delta=0$. The allowed values of b, k' are those contained inside the heavy closed curve, and the spread Δb for any given k' is given by the section of the line, $k'=\text{constant}$, lying within the restricted area. The total intensity of radiation transmitted for the given value of Δ is proportional to the area inside the same curve. As Δ varies the restricted area decreases until finally for a certain value $\pm\Delta_m$ the whole region is excluded and no transmission is further possible. The values of this integrated area, plotted against the value of Δ show the shape of the peak to be observed by the Geiger counter as the voltage on the condenser is varied. (From the form of the cut-off conditions it is easily seen that in general this curve will be symmetrical around $\Delta=0$.) In the particular case shown in Fig. 7 the restricted area departs from the parallelogram only very slightly, and the extremal type of cut-off plays an unimportant role; but for other values of Δ the extremal cut-off may play the predominant role.

The particular type of cut-off which predominates for the value of Δ at which absolute cut-off occurs may depend on the particular problem, so that further generalizations become difficult to express in concise form; but *for particular cases* it is not too laborious a task to carry out graphically the method outlined above. (The integrated restricted areas can be conveniently obtained by the use of a planimeter.)

In this way one can determine whether the resolution width in k' is really sufficiently small to prevent shifts in the peak due either to the k_i, k relationship or to variations in the source intensity with k. Considerations of the latter type are of much greater importance in connection with the Bucherer experiment, where (as will be pointed out in a future discussion) it is possible to obtain focusing effects which may conceal a very poor resolving power, indeed, and permit large shifts in the density maximum of the photographic line. Such a determination of the expected shape of the observed peaks should also serve to distinguish between effects due to the direct and the scattered components of the radiation detected.

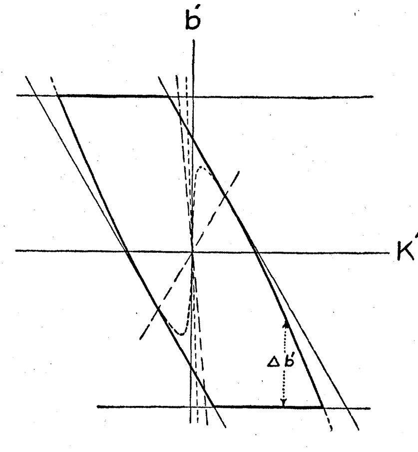

FIG. 7.

FOCUSING EFFECTS

The equation of the trajectory inside the condenser, expressed in terms of b and k', becomes for the modified experiment:

$$x=b[1+(1-b_0k_{00})y/c] - k'[b_0y/k_{00}c+By^2/2]+B\Delta y^2/2.$$

From this one sees that if

$$y=y_1\equiv -c/(1-b_0k_{00}),$$

then x is independent of b, which means that all rays of the same energy, if continued backward with the same curvature k_i, will be congruent at a point on the line $y=y_1$ and of ordinate varying linearly with k' and Δ.

On the other hand, if

$$y=y_2\equiv 2b_0/(-Bk_{00}c),$$

then x is independent of k' and varies linearly with b and Δ, which means that an image of the front end of the condenser is projected on the plane $y=y_2$, provided $l \leqq y_2$, and with a magnification given by the coefficient of b and a shift proportional to Δ. These relations suggest graphical methods, but in order to obtain reasonable accuracy it would require apparatus of unwieldy proportions.

Resolution Limits at High Velocities

The actual spreads in k' and Δ can be obtained by the above method; but one can obtain an approximate idea of the behavior of the apparatus with increasing velocity β by considering the absolute over-all limits $k_i = \pm k_a$, *defined by the geometry of the condenser alone.* In general the actual resolution width will be somewhat smaller than that estimated in this way, so that such considerations will give an upper limit to the resolution width (since the source conditions further restrict the spreads involved). If one assumes that all the increments around the central compensated ray are small, then in general:

$$dk_i = (\partial k_i/\partial k_0)_{k_0=k} dk_0 + (\partial k_i/\partial k)_{k=k} dk = \pm k_a,$$

where one can set $k_0 = k_{00} = k$. (See Fig. 5.) If one remembers that also dk is limited by the same values $k_i = \pm k_a$; that is, $dk = \pm k_a/(\partial k_i/\partial k)_{k=k}$ then one sees that

$$(\partial k_i/\partial k_0)_{k_0=k} dk_0 = \pm 2k_a$$

or $dk_0 = \pm 2k_a/(\partial k_i/\partial k_0)_{k_0=k} = \pm 2k_a/(1-\beta_{00}^2)$

or the over-all spread $\Delta k_0 = 4k_a/(1-\beta_{00}^2)$. This provides an upper limit to the spread Δk_0 in terms of k_a and β, and for the case of Lorentz electrons.

One can also show, for Lorentz electrons, that

$$\Delta E/E = (1-\beta_0^2)\Delta k_0/k_0 = (1-\beta^2)\Delta k_0/k$$

and from the preceding relation that

$$\Delta E/E = 4k_a/k,$$

which shows that this absolute upper limit of the resolution width is independent of β. But it must be remembered that $\partial k_i/\partial k_0$ and $\partial k_i/\partial k$ were assumed constant over the intervals in question. In order that this be true β cannot be too near unity, since with increasing velocity the negative intercept of the curve of Fig. 5 approaches zero. From the figure one sees that the positive limit of k_i is always equal to $+k$; and the negative limit, to $-k(1-\beta)/\beta$. Therefore, in order to insure fairly small resolution width, as well as approximate symmetry around k, it will suffice to have $k(1-\beta)/\beta$ considerably larger than k_a. As a practical expression of this fact one might choose the limiting ratio between the negative intercept and k_a to be some number p which may be regarded as a "factor of safety:"

$$k(1-\beta)/\beta k_a = p$$

or $$\beta = 1/(1+pk_a/k) \underset{\beta\to 1}{\longrightarrow} (1-pk_a/k).$$

In this way the limiting value of β is expressed simply in terms of k_a and k, both of which quantities are limited by practical geometrical considerations. k is limited by the fact that the source has a finite extension relative to which b_0 must be large; whereas $k_a = 16\delta/l^2$ and hence is simply related to the geometry of the condenser. In Table I are shown the limiting values of β for different values of k/k_a, and for the two cases $p=5$ and $p=2$. In addition, the corresponding values of $H\rho$ and the kinetic energy of the electron are given. (For $p=2$ one should expect a rather large resolution width, but even for such a case one could probably calculate the errors caused by the resulting dissymmetry and the momentum distribution of the source,—if one wished to extend such experiments to the extreme limit of high velocities.)

In a particular application of the present method (to be described in the following experimental paper) the value of k/k_a was approximately 25, and with this arrangement one millicurie of radium E was found more than sufficient to locate the peak for $\beta=0.75$. It therefore seems definitely possible that an extension of the method to considerably higher velocities could be achieved by increasing the ratio k/k_a and using sources of greater intensity. There will, however, be various practical limita-

Table I. *Limiting values of β for different values of k/k_a for $p=5$ and $p=2$.*

	$p=5$			$p=2$		
k/k_a	β	$H\rho$	(Mev)	β	$H\rho$	(Mev)
25	0.83	2550	0.4	0.93	4350	0.9
50	0.91	3750	0.7	0.96	5850	1.3
100	0.95	5100	1.1	0.98	8450	1.8
250	0.98	7450	1.8	0.992	13400	3.5

tions. For example, as $\beta_0 \rightarrow 1$

$$H\rho_0 \rightarrow 1705/(1-\beta_0^2)^{\frac{1}{2}}.$$

Hence, for a given value of $\rho_0 = 1/k_0$, H increases rather rapidly. On the other hand, since $\beta_0 = E/H$, one sees that, as $\beta_0 \rightarrow 1$, E becomes proportional to H, approximately. Therefore, for a given value of ρ_0, E and H will increase approximately in the same ratio. In addition to the latter conditions there are also intensity requirements, which will probably be the final practical factor in such experiments. While it is at present difficult to predict how far it would be feasible to extend these experiments, it is quite clear that the present method offers the possibility of carrying the study of the variation of electron mass to higher energies of the order of ten times those used in the Bucherer-Neumann experiments.

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The Specific Charge of Disintegration Electrons from Radium E

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Rough preliminary measurements of the ratio e/m for the pure primary beta-particles from radium E and of momentum about 2000 $H\rho$ were described in a recent Letter to the Editor. The present article includes a more detailed description of these results as well as of more accurate results obtained with an improved experimental arrangement. In order to ascertain the origin of the side-peaks observed by the former measurements, in the new apparatus the resolving power was increased by doubling the length of the electric condenser, and the scattering was reduced by the use of aluminum for the construction of the condenser. In addition several auxiliary defining slits were inserted at points along the electron path, and all calibrations were rechecked so that the central peak could be located with moderate accuracy. In the preliminary measurements no special attempt had been made toward great accuracy, since the apparatus was designed primarily to distinguish between ordinary Lorentz electrons and the widely differing special type of heavy electrons required by the *speculation* that the well-known beta-ray paradox might be explained by variations, with velocity, of the rest mass of the electrons created in the nucleus, rather than by the neutrino hypothesis. With the improved apparatus extremely sharp peaks were observed by means of the Geiger counter, showing that the method described in the preceding article offers the possibility of very accurate determinations of the ratio e/m. The side-peaks were greatly reduced in height and separation, as compared with those in the preliminary experiments; and with variations in the slits behaved in such a way as to indicate that they are due to scattering from the condenser plates, as previously suspected, or possibly to small nonuniformities in the magnetic field. In any case the latter variations in the side-peaks were found to have no appreciable influence on the position of the central peak, and the side-peaks are therefore of no serious consequence. Further, an analysis along the lines indicated in the preceding article was carried out for the detailed resolution characteristics in this particular case, and it was found that no side-peaks should be expected in the absence of scattering, although second degree equations do occur in the cut-off conditions for the source-condenser system. The central peak was found to be so sharp that it was possible to locate it without difficulty to within 1/10 percent of the voltage at the peak. In fact, the accuracy in determining the peak voltage was limited, in the present arrangement, rather by fluctuations in the magnetic field and in the battery voltage. Similarly, the absolute determination of e/m was limited rather by the accuracy of calibrations, both electromagnetic and geometrical. In the present arrangement no special aim was made toward accurate absolute results to better than one or two percent, but the method offers the possibility of considerably greater accuracy; and if one requires only relative values of e/m, the same accuracy could be obtained without special precautions as to the calibrations. In addition, one is here free from the grave uncertainties (to be discussed in a later article) associated with the possibility of scattering in the Bucherer-Neumann experiments. The final corrected value of e/m was found to be in agreement with the theory of relativity to within $1\frac{1}{2}$ percent, which is well within the limits of experimental error. *The above-mentioned speculations as regards the special type of heavy electron are therefore untenable.*

In a previous Letter to the Editor[1] a brief description was given of a method for the determination of the ratio e/m for beta-particles, and at the same time preliminary results obtained by the use of this method were reported

[1] C. T. Zahn and A. H. Spees, Phys. Rev. **52**, 524 (1937).