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The Binding Energy of the Deuteron

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In a recent paper,[1] Chadwick, Feather and Bretscher have published their results on the photodisintegration of the deuteron. For the determination of the energy of the photo-protons from their range they use the range-energy relation of Blackett and Lees;[2] this gives 185 kv for the energy of the photo-protons produced by the 2.62 Mev γ-rays from Th C″, and therefore 2.25 Mev for the binding energy of the deuteron. However, recent experimental[3–5] and theoretical[3] investigations show that the energy of a slow proton is, for a given range, considerably higher than is indicated by the relation of Blackett and Lees. Because of the fundamental importance of the binding energy of the deuteron for nuclear theory, it seems worth while to analyze the data of Chadwick, Feather and Bretscher in terms of the most recent range-energy data.

A very careful study of the range-energy relation of slow (up to 2 Mev) protons in air was recently carried out by Parkinson, Herb, Bellamy and Hudson.[4] We have applied some obvious corrections to the data published by P. H. B. H. such as using the mean[6] instead of the extrapolated voltage, using the distance to the center of the ionization chamber as the "mean range" and reducing the data to standard conditions (15° and 760 mm). The range-energy relation thus obtained[7] was used in the following. Its accuracy is difficult to estimate; probably the greatest error arises from the definition of the end of the range which may be different in cloud chamber and ionization chamber work; we shall assume that this error amounts to 0.1 mm standard air (depth of ionization chamber in experiments was 0.4 mm, range of photo-protons about 2 mm).

Chadwick, Feather and Bretscher used in their cloud chamber a mixture of about 60 percent He and 40 percent CH_4. In this mixture, the mean range of the photo-protons was 0.613 cm, with a mean error (due to statistics) of 0.010 cm or 1.6 percent. The stopping power (S.P.) of the mixture was calibrated with α-particles of about 2 cm range, and it was found that 2.91 cm of the gas were equivalent to a mica foil of 0.902 cm air equivalent.[8] If the S.P. of the gas relative to air were the same for slow protons as for 2 cm alphas, the range of the photo-protons in air would be $0.613\times 0.902/2.91=0.190$ cm.

Actually, the S.P. of the gas is considerably higher for slow particles. The S.P. of hydrogen and helium relative to air were measured by Gurney[9] for α-particles of various energies and by Gerthsen[10] for very slow protons. Blackett and Lees[2] have given an interpolation table for the S.P. of hydrogen for protons and α-particles. From this table, the average S.P. for protons in the last 0.23 cm of their range[11] is 0.33, that for alphas of 2 cm range[12] 0.217.

For helium as a stopping gas, Gurney found a very small variation of the S.P. from about 0.175 at medium to 0.179 at small velocities. This result may seem somewhat surprising since helium is so much lighter than air. However, it is

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[1] Chadwick, Feather and Bretscher, Proc. Roy. Soc. **163**, 366 (1937).

[2] Blackett and Lees, Proc. Roy. Soc. **134**, 658 (1932).

[3] Livingston and Bethe, Rev. Mod. Phys. **9**, 246 (1937), especially §95, p. 261–68. (Theory, and discussion and references to experimental papers up till 1936.)

[4] Parkinson, Herb, Bellamy and Hudson, Phys. Rev. **52**, 75 (1937).

[5] Blewett and Blewett, private communication (Experiments on the range-nergy relation of slow α-particles, carried out in the Cavendish laboratory).

[6] I.e. the voltage at which, for a given position of the ionization chamber, the number of protons reaching the chamber has dropped to one-half the number at high voltage.

[7] A limited number of blue-prints of this relation has been sent to some of the chief laboratories in nuclear physics.

[8] It is not stated how the mica foil itself was calibrated. We assume in the following that it was also calibrated with 2 cm alphas. If, instead, Ra C′ alphas were used, its actual air equivalent would be about 1.1 percent less, owing to the variation of the stopping power of mica. This would increase the binding energy of the deuteron by 5 kv. This correction is negligible compared with the uncertainty in the range-energy relation.

[9] Gurney, Proc. Roy. Soc. **107**, 341 (1925).

[10] Gerthsen, Ann. d. Physik **5**, 657 (1930).

[11] Actual range in air found for the photo-protons.

[12] For this case, Fig. 33a of reference 3 gives 0.231. This difference may indicate the probable error. Since only the *relative* stopping powers for slow protons and 2 cm alphas matter, we prefer to use one set of data (those of Blackett and Lees) consistently.

immediately understood theoretically because the important quantity for the S.P. is the average excitation potential of the electrons of the stopping atoms. Now, the K electrons of air contribute only a very small amount to the S.P. for slow particles (about 5 percent for 2 cm alphas). The average excitation potential of the *L electrons* of air is 40.3 volts (from experimental data, reference 3, p. 267); that of helium (K electrons) is 42.7 volts,[13] i.e., almost the same.[14] Therefore the relative S.P. of He should remain almost constant up to about 2 or 3 cm range, and should decrease slowly for greater ranges when the K electrons of air become effective.

The S.P. of carbon changes very slightly, from about 0.935 at 2 cm range to 0.945 at zero range (cf. Fig. 33a, reference 3).

The stopping power of the mixture of 60 percent He and 40 percent CH_4 becomes then for 2 cm alphas:

$$0.6\cdot 0.175+0.2\cdot 0.935+0.8\cdot 0.217=0.466,$$

for slow protons (last 2.2 mm of range):

$$0.6\cdot 0.179+0.2\cdot 0.945+0.8\cdot 0.33=0.560.$$

Therefore the range of the photo-protons in standard air

$$0.190\times 0.560/0.466=0.228 \text{ cm}.$$

According to the range-energy relation of P.H.B.H. this corresponds to an energy of 228 kv. Since the energy of the Th C″ γ-rays is 2.62_3 Mev, the binding energy of the deuteron becomes

$$\epsilon=2.62_3-0.45_6=2.17 \text{ Mev}.$$

The probable errors may be estimated as follows:

Statistical error in determination of range of protons	1.6 percent
Error in range-energy relation of P.H.B.H.	5 percent
Error in relative stopping power of gas for slow protons and 2 cm alphas	5 percent
Error in calibration of stopping power for 2 cm alphas, including error due to variation of S.P. of mica	2 percent

Mean square total error about 8 percent (of 0.46 Mev), i.e., 0.04 Mev.

Therefore the binding energy of the deuteron is

$$\epsilon=2.17\pm 0.04 \text{ Mev}.$$

This value agrees very closely with the original result of Chadwick and Goldhaber[15] who used the ionization produced by the photo-protons (2.14 Mev±0.16). It also makes it understandable why Ra C γ-rays, of maximum energy 2.198, cause an observable photo-disintegration of the deuteron.[16] From the mass-spectroscopic difference between the masses of H_2 and the deuteron, *viz.* 1.53 ± 0.04 milli-mass-units $=1.43\pm 0.04$ Mev, we obtain for the difference between the masses of neutron and hydrogen atom

$$n-H=0.74\pm 0.06 \text{ Mev} =0.80\pm 0.06\times 10^{-3} \text{ mass units}$$

and for the mass of the neutron

$$n=1.008\,93\pm 0.000\,05$$

as against 1.008 97 in Table 73 of reference 3.

The masses of all nuclei which are obtained from disintegrations involving neutrons must be changed accordingly. The most important changes in Table 73 of reference 3 are

$$\begin{aligned} He^3 &= 3.017\,11 \text{ (against } 3.017\,07), \\ C^{14} &= 14.007\,63 \text{ (`` } 14.007\,67), \\ N^{13} &= 13.010\,08 \text{ ( `` } 13.010\,04). \end{aligned}$$

From the new mass of N^{13} and $C^{13}=13.00761$ the expected energy of the nitrogen positron comes out to be 1.28 Mev (previously 1.24 Mev), still in good agreement with the "inspection limit" of 1.25 Mev and in disagreement with the Konopinski-Uhlenbeck limit of 1.45 Mev.

The changes in the calculations of nuclear force constants are very small.

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[13] For K electrons, $I_k=1.103\, Z_{\text{eff}}{}^2 Ry$ (reference 3, p. 264), where $Z_{\text{eff}}=Z-0.31$, the screening constant 0.31 being obtained from the well-known variation calculation. For $Z=2$, this gives $I_k=1.103\cdot 1.69^2\cdot 13.54=42.7$ volts.

[14] The slight excess of I_{He} over I_{air} compensates partly the influence of the K electrons of air.

[15] Chadwick and Goldhaber, Proc. Roy. Soc. **151**, 479 (1935).

[16] Banks, Chalmers and Hopwood, Nature **135**, 99 (1935); Mitchell, Rasetti, Fink and Pegram, Phys. Rev. **50**, 189 (1936).