The Kinetic Theory of the Motion of Ions in Gases

A paper with this title,[1] written in 1915, has received so little notice that at the request of the editor I send a brief note describing it and giving references to two papers in which results contained in it have been rediscovered. The paper applies Boltzmann's equation to the motion of ions and electrons in gases. Hilbert reduced the equation to an integral equation when the collisions are those of elastic spheres. The paper begins with a reduction when the law of force is any function of the distance between ion and molecule, the corresponding extension for a single gas having been announced by Lunn in the preliminary draft of a paper which has apparently never been published. Lunn also pointed out that the equation is reduced to one of one degree of freedom by the use of Legendre's coefficients. I am not here giving any references which can be found in my paper.

As a first application the coefficient of diffusion of a simple gas into itself is found for elastic collisions, putting the mass of the ion equal to that of the molecule. The distribution function is found numerically. When the mass of the ion is small, it is shown that Lorentz' formula applies at most to the initial stages of the motion, since there can be no final steady velocity of a stream of ions in an electric field when the molecules are immovable. If the mass of the molecule is finite, it is possible to have a final state in which the average kinetic energy of an ion exceeds that of a molecule, which was first put in evidence by Townsend. The rise of energy in weak fields is regulated by the quantity $b=2e^2h^2E^2M/3N^2\pi^2o^4m$, where E is the electric force, M the mass of a molecule and m that of an ion, and the rest of the notation is usual in the kinetic theory of gases. The distribution function is approximately proportional to $(p^2+b)^b \exp(-p^2)$, where p is proportional to the velocity. This formula has been rediscovered by Davydov.[2] The ratio k of the mean kinetic energy of ion and molecule is calculated for small values of b, and as b tends to zero the limiting ratio of $k-1$ to v^2/Ω^2 is $3\pi/8$, where v is the velocity of drift and Ω the root-mean-square velocity of a gas molecule. When the mass of an ion is much larger than that of a molecule, Boltzmann's equation reduces to a partial differential equation.

The kinetic theory of the motion of ions in a magnetic field was first treated by Gans. My paper deals partly with an inverse fifth power law of repulsion and partly, like Gans, with ions of small mass colliding elastically. The equations of diffusion in a magnetic field are shown to contain cross terms, and a geometrical argument by which Townsend sought to justify his equations without cross terms is shown to be capable of another interpretation. These results have been rediscovered by Tonks.[3] Lastly, the work on elastic collisions is extended to collisions of constant restitution less than unity, and some previous results of the author are corrected.

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December 14, 1937.

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[1] Pidduck, Proc. London Math. Soc. **15**, 89 (1915).
[2] P. Davydov, Physik. Zeits. Sowjetunion **8**, 59 (1935).
[3] L. Tonks, Phys. Rev. **51**, 744 (1937).

Heavy Beta-Rays from RaE

Equation (4) of the previous letter by the writer[1] reads

$$m_0c^2/(1-\beta_0^2)^{\frac{1}{2}}=pm_0c^2/(1-\beta^2)^{\frac{1}{2}}, \tag{1}$$

where m_0 is the rest mass of the ordinary electron, β_0c is the maximum speed with which the beta-rays of a radioactive transformation are ejected, pm_0 is the rest mass of the beta-rays which are ejected with a velocity βc. For RaE the maximum kinetic energy is 1.05 Mev. Since $m_0c^2=0.51$ Mev, (1) may be written

$$p=3.06(1-\beta^2)^{\frac{1}{2}}. \tag{2}$$

Applying the writer's own theory to the published curves for the magnetic spectrum of beta-rays from RaE, the writer estimates that the beta-rays are most prevalent for a value of p about 2.75. From (2) the corresponding β is 0.44. By means of a velocity selector in a mass spectrograph I obtained beta-rays of speed $0.44c$. The condenser plates of the selector were 1.05 mm apart. A sample of RaE was placed at one end of the plates. The beta-rays traveled a length of 5 cm in the crossed electric and magnetic fields into a space in which the magnetic field alone was present. The deflection of the beta-rays was measured on a photographic film placed perpendicular to the line of travel of the beta-rays between the plates and at a distance of 3.5 cm from the end of the condenser plates opposite to that at which the RaE was placed. The voltage V across the plates and the deflection s from the undeviated beam are shown in Table I.

Table I

Film	H	V	β	s	p
1	247	3440	0.441	0.80	2.4
2	199	2740	0.437	0.58	2.6
3	374	3950	0.335	1.09	3.8

The experimental values of p for the first two films agree reasonably well with the theoretical value 2.75. The experimental value of p for the third film differs from the theoretical value 2.88 by an amount greater than is comfortable. When correction is made for the fact that the range of βc allowed through the selector is an estimated $\pm 0.08c$ and that for values of p between 2.75 and 3.06 lighter electrons are more prevalent than the heavier, the average βc let through is not $0.335c$ but more like $0.375c$. The experimental value of p is then 3.34 which is within experimental error of the theoretical value 2.88. The estimated error is perhaps 0.5 in p. The writer believes that at present the best chance of showing the effect above described is to select those beta-rays which are the most prevalent. These have a velocity of $0.44c$.

The subject of this and the previous letter was presented at an informal meeting of the Physical Society at Indianapolis on December 30, 1937.

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[1] G. E. M. Jauncey, Phys. Rev. **53**, 106 (1938).