

Ultrasonic Interferometry for Liquid Media*

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A resumé of the theory of the ultrasonic interferometer is given and adapted for use with liquid media where the term ρv is very large (approximately 10^8) in comparison with that occurring in gases (approximately 40). The differences of the observed phenomena due to this factor are discussed together with a method of obtaining the coefficient of absorption of ultrasonic waves in liquids, and the "effective" coefficient of reflection of the waves at a liquid-metal boundary. Measurements at 2.79 and 8.37 megacycles yield for the "frequency-free" coefficient of absorption in water approximately 19×10^{-17} while the coefficient of reflection varies from 0.7 to 0.9 at boundary surfaces of monel metal and brass.

INTRODUCTION

EARLY investigators using the acoustic interferometer developed by Pierce¹ assumed that the observations used in computing the absorption coefficients could be interpreted as though produced by a single reflection. Although multiple reflections take place before the energy is entirely dissipated, few such reflections occur; and in gases, where the absorption is large, the effect of successive reflections is small even when the path length is short, and becomes negligible as the path length is increased. This assumption was confirmed by comparing the results obtained by interferometry with those secured by measurements with a torsion vane.²

Hubbard³ developed the theory of the acoustic interferometer to include measurements over the entire range of path length, emphasizing especially the importance of the variations occurring when the path length is short (several half wave-lengths of the sound in a gaseous medium) and the part played by the coefficient of reflection between the reflector and the gas. His measurements of the absorption of ultrasonic waves of frequencies in the neighborhood of 500 kilocycles brought out the interesting fact that the coefficients of reflection were much further removed from unity than would be expected.

The investigations of Quirk⁴ indicated that the absorption of ultrasonic waves in liquids can be

detected by the interferometric method. In his analysis, however, no attempt was made to include the effects of the many reflections that must occur in most liquids over an observable interferometric path length, nor to discuss the role of the coefficient of reflection in the phenomena observed.

Following the theoretical method of Hubbard, the present paper seeks to develop a method of determining the coefficient of absorption in liquids, and includes the results of measurements made with distilled water by means of which the coefficient of absorption together with the coefficient of reflection between the water and the reflector were obtained at frequencies of 2.79 and 8.37 megacycles.

THEORY OF THE LIQUID INTERFEROMETER

Hubbard³ has extended the theory of an electromechanical resonator, consisting of a piezoelectric resonator driven by an independent source, to include the effect of a fluid column mechanically coupled to the piezoelectric plate in such a manner as to set up longitudinal vibrations in the fluid column. In his treatment it is shown that the coupled fluid column can be represented by modifying the equivalent resistance and piezoelectric capacity of the equiva-

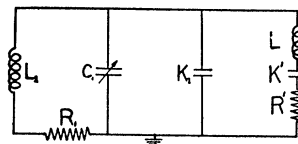


FIG. 1. The electrical and equivalent electrical network of the pick-up circuit and interferometer.

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¹ Pierce, Proc. Am. Acad. 60, 27 (1925).

² See Pielmeier, Phys. Rev. 34, 1184 (1929).

³ Hubbard, Phys. Rev. 38, 1011 (1931); 41, 523 (1932).

⁴ Quirk, Doct. Diss., Cath. Univ. Am. (1934).

lent electrical circuit of the quartz resonator. He then considers such a circuit in conjunction with a driving electrical circuit, solving for the current flowing in the associated electrical resonant circuit as a function of the electrical

and equivalent-electrical constants, including constants and variables dependent on the acoustical behavior of the coupled fluid column. For the equivalent circuit of Fig. 1 he obtains the expression:

$$\sigma^2 = \frac{\varphi_1^2 \{ \varphi_2^2 + [q + K' / (C_1 + K_1)]^2 \}}{\{ p q - \varphi_1 \varphi_2 - (1 - p) K' / (C_1 + K_1) \}^2 + \{ \varphi_2 p + \varphi_1 [q + K' / (C_1 + K_1)] \}^2}, \quad (1)$$

where L_1 , R_1 , C_1 are the inductance, resistance, and capacity of the pick-up electrical resonant circuit; K_1 , K' , R' and L are the corresponding equivalent electrical constants of the quartz resonator with a coupled fluid column, K_1 being the dielectric capacity of the resonator, while K' is its modified piezoelectric capacity, and

$$\begin{aligned} p &= 1 - (C_1 + K_1) L_1 \omega^2, \\ q &= 1 - L K' \omega^2, \\ \varphi_1 &= R_1 (C_1 + K_1) \omega, \\ \varphi_2 &= R' K' \omega, \\ \sigma &= i / I_0, \end{aligned}$$

i = current in the $L_1 R_1$ branch. $I_0 = E_0 / R_1$, the maximum current in the resonant circuit when the interferometer branch is disconnected, E_0

being the maximum induced voltage in the inductance L_1 .

The terms R' and K' include the modifications introduced by the coupled fluid column, thus:

$$R' = R + A B \rho v P, \quad (2)$$

$$1 / K' = 1 / K + A B \rho v \omega Q, \quad (3)$$

where R is the equivalent resistance of the quartz resonator near its response frequency; K is its equivalent piezoelectric capacity; A the effective area of the resonator face exposed to the fluid column; B a piezoelectric constant of quartz; ρ the density of the fluid; v the velocity of sound in the fluid; and ω is 2π times the frequency of the voltage induced in L_1 .

P and Q are further defined:

$$P_x = \frac{[e^{-\alpha x} - \gamma^2 e^{-\alpha(4r-x)}] \cos(\omega x/v) + \gamma [e^{-\alpha(2r-x)} - e^{-\alpha(2r+x)}] \cos[\omega(2r-x)/v]}{1 - 2\gamma e^{-2r\alpha} \cos(2r\omega/v) + \gamma^2 e^{-4r\alpha}}, \quad (4)$$

$$Q_x = \frac{[e^{-\alpha x} + \gamma^2 e^{-\alpha(4r-x)}] \sin(\omega x/v) + \gamma [e^{-\alpha(2r-x)} + e^{-\alpha(2r+x)}] \sin[\omega(2r-x)/v]}{1 - 2\gamma e^{-2r\alpha} \cos(2r\omega/v) + \gamma^2 e^{-4r\alpha}}. \quad (5)$$

In these expressions α is the attenuation factor of particle velocity in the fluid (one-half the intensity coefficient of absorption); γ is the coefficient of reflection at the distant boundary of the fluid column, and r is the length of the fluid column. The P and Q terms are in general valid for any position x in the fluid column, but interest us only at the position $x=0$, i.e., at the quartz face exposed to the fluid. Here they take the form:

$$P_{x=0} = \frac{1 - \gamma^2 e^{-4r\alpha}}{1 - 2\gamma e^{-2r\alpha} \cos(2r\omega/v) + \gamma^2 e^{-4r\alpha}}, \quad (6)$$

$$Q_{x=0} = \frac{2\gamma e^{-2r\alpha} \sin(2r\omega/v)}{1 - 2\gamma e^{-2r\alpha} \cos(2r\omega/v) + \gamma^2 e^{-4r\alpha}}, \quad (7)$$

Since Eq. (1) has been discussed by Hubbard (and a somewhat similar equation by Dye⁵) this need not be repeated here, but by imposing certain experimental conditions a much simplified expression is obtained which is used by Hubbard for the case where the fluid is a gas, and by means of which his determination of the coefficients of absorption and reflection in gases was carried out. We repeat in substance the discussion of the expression for the case where the fluid is a gas in order to contrast the behavior in liquids with that in gases.

As the frequency is varied σ^2 rises to the familiar resonance maximum where p is zero at

⁵ Dye, Proc. Lond. Phys. Soc. 38, 399 (1935).

the resonance electrical frequency of the L_1C_1 circuit. If the constants of the electrical resonance circuit are adjusted so that p is zero in the region where $[q+K'/(C_1+K_1)]$ is zero, which occurs at the response frequency of the quartz resonator, there is a very sharp dip or "crevasse" in the resonance curve. By adjusting C_1 and ω it is possible to find the value of the frequency for which p and $[q+K'/(C_1+K_1)]$ are zero at the same time, that is, the crevasse divides the peak of the electrical resonance curve, since the response frequency is the same as the resonant frequency of the L_1C_1 circuit. These adjustments are ideally made with the quartz vibrating in vacuum where there would be no modification because of the fluid column. Practically they can readily be made in air in the manner described by Hubbard. A crevasse is obtained while the reflector has any random setting, and the $\lambda/2$ spacing is determined by observing the change of the crevasse minimum as a function of the reflector displacement, a sharp rise occurring where the reflector is set at $n\lambda/2$ positions. The reflector is then set midway between two such positions, and under these conditions C_1 and the frequency may be adjusted as outlined above since Q is zero and P is zero to a very close approximation. Under these restrictions the simplified form of σ^2 is obtained as a function of the reflector displacement.

$$\sigma^2 = \frac{[(1+SP)^2 + (SQ)^2]}{[(1+SP+C)^2 + (SQ)^2]}, \quad (8)$$

where the current ratio σ is given in terms of electrical and equivalent electrical constants, and constants depending on the fluid and the reflector displacement, since in Eq. (8)

$$C = 1/R\omega\phi_1(C_1+K_1) \quad \text{and} \quad S = AB\rho v/R, \quad (9)$$

while P and Q vary periodically with r .

The adjustments outlined above, in which $p=q=Q=0$ and P is approximately zero, determine the constant C . Calling the value of σ under these conditions σ_0 ,

$$C = (1 - \sigma_0)/\sigma_0.$$

We now discuss Eq. (8) as a function of the reflector setting in order to show the striking differences that occur when a liquid medium is used instead of a gas. With a gas the P and Q

terms are small for the greater portion of the $\lambda/2$ spacing of the reflector, becoming large only near the $\lambda/2$ setting of the reflector. The value of ρv that characterizes gaseous media is of the order of 40, R is of the order of a thousand ohms, while C may have values from 1 to 10 depending upon the electrical circuit and the quartz resonator. The terms SP and SQ are thus very small compared to C for all reflector settings except in the immediate neighborhood of an $n\lambda/2$ position, so that the value σ_0 is nearly constant as the reflector is moved away from the resonator, but rises sharply to peak values at the $n\lambda/2$ positions.

When a liquid such as water is used the term ρv is 1.48×10^9 , so that it is evident that for reflector positions where neither P nor Q is zero, although small, SP and SQ will have large values that keep σ very close to unity, and only in the immediate neighborhood where $Q=0$ and P is a minimum will there be any large departure of σ from unity. This is made evident from the P and Q curves for liquid media. This dip occurs at the $(2n+1)\lambda/4$ setting of the reflector where both C and SP determine the minimum value of σ , and as the reflector is moved from such a setting the growth of the Q term quickly causes σ to rise. Thus in gases we find sharp peaks in the σ curve at $\lambda/2$ reflector setting, while in liquids the characteristic phenomenon is a rather narrow dip at $(2n+1)\lambda/4$ positions, the depth and width of the dip depending upon the experimental constants used.⁶

⁶ The effect of liquid loading on the "crevasse" curves of a quartz resonator may also be seen from a consideration of the σ^2 curve as a function of frequency. We may write Eq. (8) in the form:

$$\sigma^2 = \frac{1 + [(\epsilon' + SQ)/(1 + SP)]^2}{\{[1 + C/(1 + SP)]^2 + [(\epsilon' + SQ)/(1 + SP)]^2\}},$$

where

$$\begin{aligned} \epsilon' &= [1 - LK\omega^2 + K/(C_1 + K_1)]/RK\omega; \\ S &= AB\rho v/R; \\ C &= 1/[R_1R(C_1 + K_1)^2\omega^2] \end{aligned}$$

by assuming that the frequency variation is not far from the response frequency of the resonator in a vacuum, and that during such variation p does not change greatly from zero. For an unmounted resonator in a vacuum, C may be of the order of 10, while for one mounted for use in liquid interferometry C may be 2.

In gases for any reflector setting $(\epsilon' + SQ)$ may be made zero, but if Q is not zero the 1 in the denominator is increased by the fraction $C/(1 + SP)$, say $10/(1 + 16)$ or 0.6 in a typical case, and for the frequency where $(\epsilon' + SQ) = 0$ there is still a decided departure of σ^2 from unity to form the "crevasse" at the response frequency of the combination of resonator and column, regardless of

There is another factor that contributes to this behavior in the values of P and Q that occur when the reflecting boundary is water-metal, or water-glass instead of air-metal. With the liquid case the values of the coefficients of reflection are correspondingly smaller than those occurring in the gas case, and this prevents the minimum values of P from ever becoming negligibly small. Under certain conditions when C is small, as often happens in liquid interferometry, the minimum value of P may still be so large that only a slight dip is obtained in the measured current as the reflector is moved through a critical setting, while under the optimum conditions the dip will hardly take σ under 0.5. The behavior of the curves is shown in Fig. 2 for typical cases over a wave-length variation of r , together with the corresponding P and Q curves. The values of the constants, etc., for gases are taken from Hubbard's papers, while those for liquids are computed from values measured during this investigation.

DETERMINATION OF γ AND α FOR LIQUIDS

To deduce the values of γ and α from a series of current readings i^2 as a function of the path length r , Eq. (8) is used. Knowing the resonant frequency of the quartz plate, values of L_1 and C_1 are used that will give a resonance peak in the pick-up circuit at about the same frequency. The driving oscillator is then coupled loosely to the system and its frequency adjusted till a crevasse is observed. The reflector is then set at

the reflector setting. Of course the deepest crevasse is obtained when Q and ϵ' are zero simultaneously. Since the frequency making ϵ' zero is the response frequency of the resonator and its electrodes, and the response frequency of the combination including the fluid column is that making $(\epsilon' + SQ)$ go to zero, the well-known fact that a vibrating quartz crystal may have its response frequency changed by the presence of a coupled gas column is here brought out. At the same time it is seen that each such shift of the response frequency (from that at which $\epsilon' = 0$) by tuning the vibrating column results in increased damping.

When liquid columns are used S is much larger and C is usually smaller so that when the reflector is set at random, $(\epsilon' + SQ)$ may still be brought to zero by changing the frequency but SP may be so large (and except for reflector setting where P and Q are very small, this is always so) that the denominator term $2/(1+SP)$ is roughly $2/(300+1)$ and σ^2 does not depart appreciably from unity. Thus a "crevasse" in the σ^2 curve as a function of frequency variation can only occur for a limited range of reflector settings near a $(2n+1)\lambda/4$ setting of the reflector.

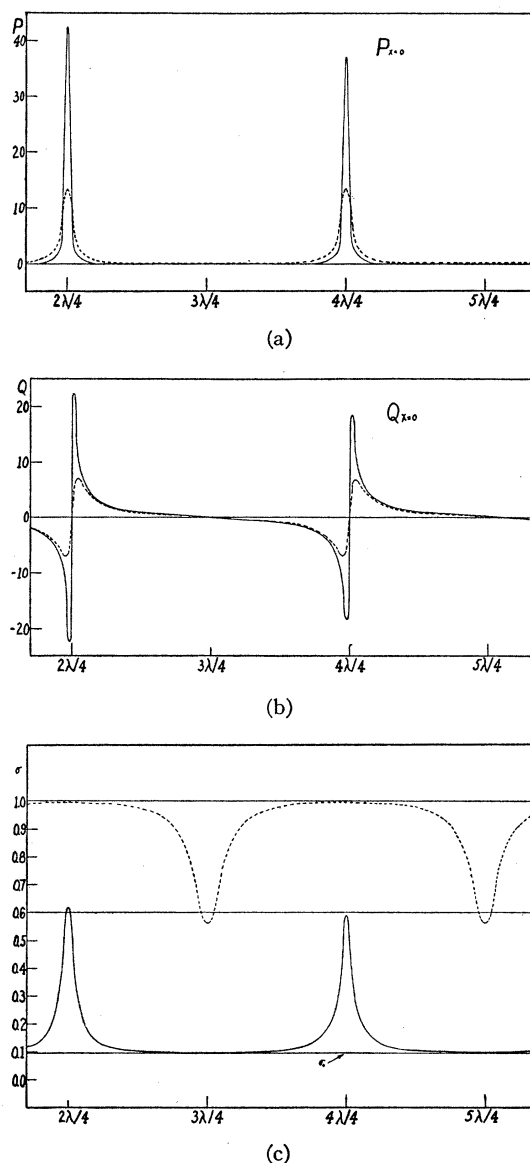


FIG. 2. (a), (b), (c). Typical P , Q and σ curves for gases (full line) and liquids (broken line). Constants for the curves for air: $\lambda=0.05858$ cm, $\gamma=0.96$, $\alpha=0.11$, $C=9.2$, $S=0.33$. Constants for water: $\lambda=0.0525$ cm, $\gamma=0.862$, $\alpha=0.0015$, $C=1.89$, $S=19.0$.

a $(2n+1)\lambda/4$ position, determined as outlined above, the fluid column being air. The value of C_1 is then adjusted till the crevasse as a function of frequency appears at the center of the resonance peak of the pick-up circuit, and is symmetrical about the minimum i_0^2 . This frequency which causes i^2 to fall to the bottom of the crevasse is also the response frequency where

$[q+K'/(C_1+K_1)]$ is zero, and is then held constant throughout the rest of the readings. The interferometer is disconnected and C_1 adjusted to determine the resonance maximum I_0^2 , after which the interferometer is reconnected and C_1 restored to the original value for the symmetrical crevasse. From the reading of i_0^2 at the bottom of the crevasse in air with the reflector at a $(2n+1)\lambda/4$ position σ_0 , and therefore C , is obtained. The liquid is placed in the interferometer, the reflector adjusted parallel to the resonating surface, and a series of readings of i^2 as a function of reflector settings is taken. Enough points are recorded in the neighborhood of the first dip to graph it accurately and then the reflector is moved from the face of the resonator until a change is noticeable in the reading of i_m^2 at the bottom of the dip ($i_{m_i}^2$). From the width of the first dip the coefficient of reflection is determined and this together with the observed σ_{m_i} enable one to compute the coefficient of absorption.

γ —THE COEFFICIENT OF REFLECTION

In order to determine γ , we confine our attention to a dip occurring when the reflector displacement r is very small, for example, at the first dip. At the minimum $\sin(2r\omega/v)=0$, $\cos(2r\omega/v)=-1$, giving $Q=0$, and

$$P = P_m = (1 - \gamma e^{-2r\alpha}) / (1 + \gamma e^{-2r\alpha}), \quad (10)$$

$$\sigma_m = (1 + SP_m) / (1 + SP_m + C), \quad (11)$$

$$SP_m = [C(\sigma_m) - (1 - \sigma_m)] / (1 - \sigma_m); \quad (12)$$

if now r is small any term $\gamma e^{-2r\alpha}$ departs from unity almost entirely by the factor γ , since α is very small in liquids. Writing $e^{-2r\alpha}=1$ we obtain

$$P = (1 - \gamma^2) / [1 - 2\gamma \cos(2r\omega/v) + \gamma^2], \quad (13a)$$

$$P_m = (1 - \gamma) / (1 + \gamma), \quad (13b)$$

$$Q = [2\gamma \sin(2r\omega/v)] / [1 - 2\gamma \cos(2r\omega/v) + \gamma^2]. \quad (14)$$

From the i^2 readings the values of σ are obtained, and plotted for the dip against the reflector displacement, and the width of the dip is thus measured for a value of σ_i about halfway between σ_m and 1. Calling this width $2\Delta r$, the argument of the circular functions occurring in

P and Q are computed from $2r\omega/v = 4\pi\Delta r/\lambda$ where r is the displacement of the reflector at σ_i from the setting at σ_m . A value of γ is chosen arbitrarily and P_m computed which in turn determines a value of S from the knowledge of C and σ_m . This value of γ is then used to compute P_i and Q_i for the displacement Δr , which together with S yield a computed σ_i . Of course this is in general different from the observed σ_i . If the computed value of σ_i is greater than that observed a smaller value of γ is chosen to yield a lower value of the computed σ_i until one of the computed values falls below that observed. The computed σ_i are then plotted as ordinates against assumed γ as abscissa and where the curve intersects the value of " σ_i observed" the corresponding γ is taken as an approximation. By this method the value of γ may be obtained by successive approximations to any degree of accuracy justified by the accuracy of the experimental data.

α —THE COEFFICIENT OF ABSORPTION

Knowing S we can obtain α from a series of $\sigma_{m_i}^2$ readings at various reflector displacements. The reflector is moved from the resonator till a measurable difference is observed in the i_m^2 values, readings being made over as long a path length as is compatible with the assumption of a plane wave front. For all such values of σ_m we have $Q=0$, and placing $\gamma=e^{-\beta}$

$$P_{m_i} = [1 - e^{-(2r_i\alpha+\beta)}] / [1 + e^{-(2r_i\alpha+\beta)}] = \tanh [(2r_i\alpha+\beta)/2]. \quad (15)$$

As $(2r_i\alpha+\beta)/2$ is small compared with unity we expand $\tanh [(2r_i\alpha+\beta)/2]$ and obtain

$$P_{m_i} = (2r_i\alpha+\beta)/2 - (2r_i\alpha+\beta)^3/24, \quad (16)$$

of which we need only take the first term. From Eq. (12)

$$r_i\alpha+\beta/2 = C\sigma_{m_i}/S(1-\sigma_{m_i}) - 1/S$$

$$\text{or} \quad r_i\alpha = C\sigma_{m_i}/S(1-\sigma_{m_i}) - (2+\beta S)/2S. \quad (17)$$

From this we can most conveniently determine α by setting $(\sigma_{m_i})/(1-\sigma_{m_i})=y_i$ and plotting y_i as a function of r , which should be a straight line of slope M such that $\alpha = CM/S$.

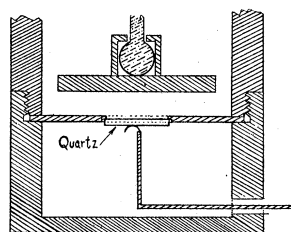


FIG. 3. Interferometer with quartz resonator exposed directly to the liquid column.

EXPERIMENTAL PROCEDURE

During the early part of these investigations the interferometer was used as described by Quirk and Rock,⁷ the ultrasonic wave being transmitted into the liquid by means of a drop of oil placed on the vibrating quartz plate, the oil being in contact with a thin diaphragm, and the liquid resting on the diaphragm. Although it was possible to measure changes in σ_m^2 under these conditions, the dips themselves were very small as indicated by the thermogalvanometer and the term $\sigma_m/(1-\sigma_m)$ is therefore ill-conditioned for the determination of α . But a greater difficulty lies in the failure of such an arrangement to approximate the assumption made in the analysis that the quartz plate gives to the first liquid layer its own periodic motion by direct coupling, since there are in reality two intervening media each having boundary conditions which were not considered. The interferometer was accordingly modified as shown in Fig. 3 to approximate the assumptions made in the analysis. The quartz plate was sputtered with platinum, the lower face before cementing to the metal diaphragm, and the upper face after the quartz was fixed in position, this face being maintained at ground potential, together with the rest of the interferometer. This method of mounting the resonator changes of course its equivalent electrical constants from the values obtaining when the resonator is vibrating freely in air or vacuum, but these are redetermined experimentally before each run as indicated above in the measurement of the constant C . With this method very efficient transmission into the liquid is obtained, and not only is the assumption embodied in the analysis fulfilled, but the current dips are increased to such an extent that

variations in σ_m^2 are measurable by the thermogalvanometer (a General Radio vacuum thermocouple and a critically damped GM 2551A galvanometer provided with a number of shunts to change the sensitivity). In order to make this more direct procedure practicable it was desirable to work at high frequencies, since the absorption will be expected to increase with the square of the frequency. A resonator with a fundamental frequency of 2.79 megacycles was finally chosen from among the plates available in this frequency range, it being the only one having an isolated response frequency far enough removed from any small neighboring response frequency to be used confidently in this investigation. The importance of using a crystal having but a single response frequency over a fairly large range of frequency excitation is obvious from a consideration of the fact that the presence of the liquid greatly widens the crevasse curve of the crystal, so that neighboring crevasses that would normally be completely separated if the crystal were vibrating in air, merge into a single widened crevasse when the crystal has a face exposed to a liquid column. This is strikingly illustrated in the dips of the σ^2 curve as a function of the reflector setting, the maximum dip occurring for the main response frequency when the reflector is set at a $(2n+1)\lambda/4$ position corresponding to the λ of this frequency, but as the reflector is displaced the σ curve rises, until the setting of the reflector corresponds to a $(2n+1)\lambda/4$ setting for the neighboring response frequency, where a small dip occurs on the side of the main dip. If the frequencies are sufficiently close, as often happens, the result is a single dip, but wider than would be the case if a single frequency response were contributing to its formation. The values of γ and α are accordingly modified, such unsuitable resonators exhibiting a periodic variation in σ_m^2 values depending upon

TABLE I. Observed values of the current (i^2) in the pick-up circuit, expressed in arbitrary units with the reflector set for minimum of the first dip, as a function of "added R " in the pick-up circuit.

R (ohms)	I_0^2	i_m^2	σ_m	γ
0	44.70	25.50	0.756	3.105
10	24.75	15.70	0.808	4.219
20	16.05	11.40	0.843	5.376

⁷ Quirk and Rock, Rev. Sci. Inst. 6, 6 (1935).

the $(2n+1)\lambda/4$ spacings for the response frequencies involved. The need of a careful investigation of the resonator's response spectrum is obvious and it was the result of such investigations that governed the choice of the crystal used. The same crystal was found to vibrate vigorously when excited by the third harmonic of its fundamental response frequency, giving a 20 percent dip in the σ curve when in the liquid, while the variation in σ_m with increasing path length is easily measured on the galvanometer, so that measurements of the absorption made at this frequency are considered to be the most satisfactory obtained thus far in the investigation.

EXPERIMENTAL RESULTS

The first experimental objective was to check the constancy of the term S which depends upon the characteristics of the coupled liquid column. At the minimum of a dip, $P=P_m$ and $Q=0$, and $\sigma_m = (1+SP)/\{1+SP+1/[R\omega\phi_1(C_1+K_1)]\}$ (18) or $\sigma_m/(1-\sigma_m) = [(1+SP)R\omega^2(C_1+K_1)^2]R_1$, (19) where R_1 is the total effective resistance of the

TABLE II. Observed values of i^2 in arbitrary units as a function of reflector settings " r " (in divisions—arbitrary zero) in the neighborhood of the first dip, and at the minima of the dips denoted under " d ."

Reflector Frequency 10^{-6} Wave-length (divs) i_0^2 i_0^2	I		II		III	
	Monel 2.79 104.8 50.00 15.4		Brass 2.79 105.0 50.00 6.00		Monel 8.37 34.92 50.00 15.30	
	r	i^2	r	i^2	r	i^2
	85.0	39.45	42.0	40.50	36.0	43.90
	86.0	35.05	43.0	38.70	36.5	41.60
	87.0	29.40	44.0	35.20	37.0	38.30
	88.0	25.50	45.0	30.40	37.5	35.60
	88.1	25.40	46.0	23.70	37.8	35.05
	89.0	28.75	47.0	18.00	38.0	35.30
	90.0	37.40	48.0	15.65	38.5	37.65
			49.0	16.40	39.0	40.95
			50.0	20.50	39.5	43.25
			51.0	28.00		
			52.0	33.50		
			53.0	37.00		
			54.0	40.50		
	d	i_m^2	d	i_m^2	d	i_m^2
	1	25.40	1	15.65	1	35.00
	51	25.60	101	16.10	101	35.60
	101	26.00	201	16.60	201	36.05
	151	26.10	301	16.95	301	36.60
	201	26.35	401	17.30	401	37.05
	251	26.65			501	37.50
	301	26.90			601	38.00

NOTE.—1 division reflector setting = 0.0005 cm. All current values are expressed in arbitrary units (gal. def. in cm).

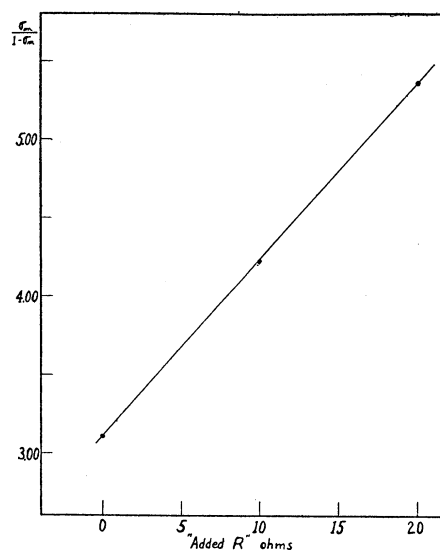


FIG. 4. Variation of y for the same dip as a function of resistance added to the pick-up circuit.

pick-up circuit. As R_1 is easily varied by adding resistance to this circuit, values of σ_m^2 were observed for various values of "added R " (ΔR). If we now place $\sigma_m/(1-\sigma_m) = y$ and plot y against ΔR we should obtain a straight line provided S remains constant during the measurements. Table I gives the results of a series of such observations, and Fig. 4 shows the straight line obtained, the intercept on the ΔR_1 axis giving the initial resistance of the pick-up circuit, and indicating that the factors in the expression $(1+SP)R\omega^2(C_1+K_1)^2$ are essentially constant at reflector settings giving the minimum of the dip.

In order to test the theory as the reflector setting varies, many curves of σ plotted against

TABLE III. Results: computed from data of Table II, giving the values of the constants C and S , the computed and theoretical values of the coefficient of reflection, the slope of the line determined by the change in σ_m with increasing path length, the coefficient of absorption for the frequency used and the frequency-free (α/ν^2) coefficient of absorption.

	I	II	III
C	0.800	1.890	0.805
S	19.80	19.0	18.50
γ (computed)	0.906	0.862	0.710
γ (theoretical)	0.928	0.910	0.928
M	0.0344	0.0154	0.309
α	0.00139	0.001534	0.0134
$\alpha/\nu^2 \times 10^{17}$	17.9	19.7	19.2

NOTE.—Case I fitted for γ at $2\Delta r/\lambda = 0.04$ where $\sigma_i = 0.857$. Case II fitted for γ at $2\Delta r/\lambda = 0.06$ where $\sigma_i = 0.774$. Case III fitted for γ at $2\Delta r/\lambda = 0.07$ where $\sigma_i = 0.906$.

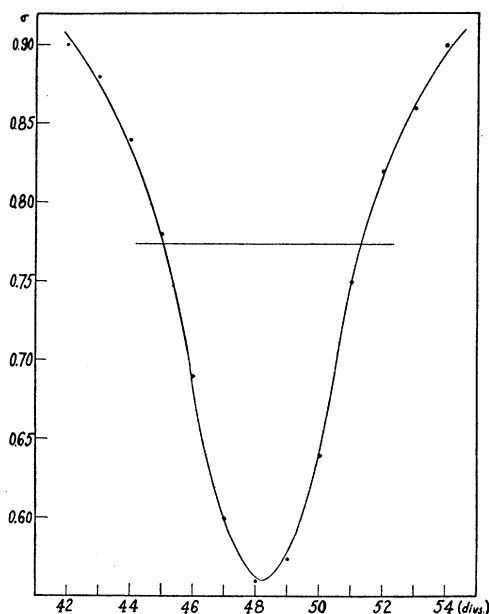


FIG. 5 (a). Case II. Change in σ as a function of the reflector setting in the neighborhood of the first dip.

reflector setting were made in the neighborhood of a dip. The constants S and γ were then determined by fitting these constants to a point corresponding to about $(1 + \sigma_m)/2$, or a σ_i value about half way down the side of a dip as described before, and then computing the rest of the dip from Eq. (8), this curve being compared with the one experimentally obtained. Fig. 5 (a) gives an example taken from the data for case II where the curve is fitted for the σ_i value indicated by the horizontal line, the curve being theoretical, the points experimental. Again the close agreement shows the essential constancy of S .

Figure 5 (b) shows y_{mi} plotted against $r_i\alpha$, i.e., $\sigma_{mi}/(1 - \sigma_{mi})$ as a function of reflector displacement, the r_i value of the first point being somewhat in doubt due to the fact that the reflector cannot be brought into direct contact with the resonator itself, but is of no importance since only the slope of the line need be known to obtain α . The experimental points lie, as well as could be expected from the accuracy of the measurements, on a straight line.

Measurements made using different resistances in the L_1C_1 circuit, different types of crystal mounting and different reflectors indicate the

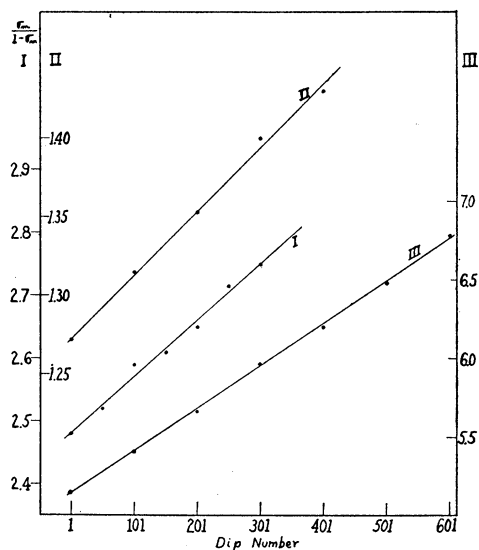


FIG. 5 (b). Variation of y at the dip minima as a function of the path length. The abscissa is given as the dip number, the distance between dips being the half wave-length spacing for the frequency and liquid used.

essential validity of the assumptions made in the analysis. A table of the measurements made for two different reflectors (the liquid being distilled water in all cases) at 2.79×10^8 cycles and one at 8.37×10^8 cycles.

DISCUSSION OF RESULTS

These values of α and α/ν^2 must be doubled to give the intensity attenuation factor. Although they are several times the value to be theoretically expected from a consideration of viscosity (8.0×10^{-17}) as ordinarily measured, they are smaller than those observed by other investigators. Quirk's⁸ value of 8.5×10^{-8} for intensity absorption factor at 1459.5 kilocycles gives $\alpha/\nu^2 = 200 \times 10^{-17}$. Biquard's⁹ observations give 26.8×10^{-17} for α/ν^2 at 7.96 kilocycles and at 20°C as obtained by an optical method, and from 31.5×10^{-17} to 27.2×10^{-17} in the same region using a torsion pendulum to measure the change in pressure exerted by the ultrasonic radiation. Sörensön¹⁰ at 950 kilocycles and at 18.5°C gives 0.011 as intensity absorption coefficient or $\alpha/\nu^2 = 610 \times 10^{-17}$. It is to be noted that in the last mentioned case the driving voltage was of

⁸ Quirk, *Doct. Diss.*, Cath. Univ. Am. (1934).

⁹ Biquard, *Theses Univ. de Paris* (1935).

¹⁰ Sörensön, *Ann. d. Physik* **26**, 120 (1936).

the order of several thousand volts, and both Sörenson and Biquard mention an increase of measured absorption with the amplitude of the ultrasonic waves used. In the present investigation the power used was very low, the voltage applied to the crystal being of the order of one volt as measured by a vacuum tube voltmeter across the crystal faces. It is possible that some factor such as this may be responsible for the wide range of values obtained by different observers.

It is difficult to arrive at an estimate of the accuracy of the measurements obtained. From a consideration of the measuring devices used one would expect the variation to be within 10 percent and a consideration of the agreement of α/ν^2 as determined using different frequencies seems to justify this. It is to be noted that the measurements at the higher frequency are by far the most accurate set obtained in the investigation. No effort was made to measure the frequency with accuracy greater than 1.0 percent, the only precaution being that during an experimental determination the frequency did not change. This was insured by heterodyning some harmonic of a Standard Signal Generator with the driving oscillator so that a 1000 cycle beat note was produced and compared with a 1000 cycle electrically driven tuning fork. Any change in the beat note is detected immediately as the frequency of either oscillator varies. I_0^2 was measured before and after each set of readings and if either this or the frequency changed during the run, the readings were discarded.

With the coefficient of reflection it is even more difficult to account for the variation of the observed values from those to be theoretically

expected (at a brass-water boundary about 0.910, and for monel-water about 0.928). There is some indication that the mounting of a crystal may change the "effective coefficient of reflection" measured. It might be more correct to say that the measured coefficients are due to energy losses not only at the reflector boundary but also to surface losses at the resonator itself, e.g., where it is attached to the support. Then, too, the fact that the reflector itself is not infinitely thick probably cannot be neglected, and there is a possible explanation of the departure from expected values, suggested by such an assumption, because of the chance of the reflector thickness being near to some integral number of quarter wave-lengths of the sound in the reflector itself. This departure from expected values was also noticed by Hubbard in gaseous media, where the departure from expected values was relatively much larger. A further analysis of the reflector system might help to clear up the difficulty, but for the present one simply classifies the results as "effective coefficients" of reflection.

No attempt was made to maintain the temperature of the water constant since the effect to be expected from small temperature changes is certainly masked by the experimental errors. In all the determinations ordinary distilled water was used, the only additional precaution being that of heating to drive out absorbed air.

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