Shot Effect of Secondary Electrons from Nickel and Beryllium*

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The secondary emission ratios and shot effects from nickel and beryllium were measured over a range of primary energies up to 1600 volts. The results are consistent with the following conclusions. At high primary energies the true secondary electrons, as distinguished from the reflected primaries, are emitted in groups whose distribution in size about the average is closely Bernoullian. There are many primaries which are buried in the body of the metal and produce no secondaries; their number has been determined. Disregarding reflected and buried primaries, the secondary emission ratio is nearly proportional to the square root of the primary energy. At low primary energies the effects of reflection, both internal and external, can be followed in detail.

INTRODUCTION

RECENTLY one of us[1] has published measurements of the shot effect of secondary electron currents, and has shown that such measurements are capable of giving information on the details of the mechanism of secondary emission. The results of that paper, which will be referred to as (*I*), have in general been confirmed by several other papers[2–5] dealing in part with the same subject.

The measurements of (*I*) were made on surfaces coated with the oxides of barium and strontium. It was found that complications were introduced by the relatively thick oxide coatings and their instability. We now present the results of measurements, over a wider range of primary energies, on two other surfaces, nickel and beryllium.

THE EXPERIMENTAL TUBES

The tubes used were small triodes having cylindrical grid and plate, and straight axial filament. This form was used in preference to the more elaborate electron gun types because of the large currents (0.1 to 1 milliampere) needed for shot effect measurements; and because of the necessity of eliminating space charge effects in the primary beam.

All the electrodes were brought out through the usual type of squash. For some of these assemblies we are indebted to Mr. B. J. Thompson of the RCA Radiotron Corporation. The plates were of nickel sheet 0.12 mm thick and were 13 mm in diameter. The grids were also of nickel in all cases, and were of the helical type, made of 5 mil (0.13 mm) diameter wire, with a pitch of 25 mils (0.64 mm) and diameter 7.5 mm. The filaments were of thoriated tungsten and were considerably shorter than the plates and grids, to avoid certain end effects.

The plates were outgassed by continuous electron bombardment and periodic induction heating; the grids were similarly treated. The plates were raised to temperatures of 1000 to 1100°C for intervals totaling half an hour to an hour. The process was carried on until reproducible secondary emissions were obtained. When getter was used the secondary emission after gettering was the same as before, nor did the sealing off affect the results. All shot effect measurements were made on tubes sealed off from the pumps.

The one beryllium surface studied was produced by evaporating beryllium in high vacuum onto a nickel plate, in a separate tube. The lumps of beryllium were held in a coiled tungsten

* Presented before the Physical Society, April 29, 1937. See Phys. Rev. **51**, 1007A (1937).

† This work was done at the University of Michigan during the summers of 1935 and 1936.

[1] L. J. Hayner, Physics **6**, 323 (1935); L. J. Hayner and A. W. Hull, Phys. Rev. **33**, 281A (1929).

[2] M. Ziegler, Physica **3**, 1 and 307 (1936).

[3] W. H. Aldous and N. R. Campbell, Proc. Roy. Soc. **A151**, 694 (1935).

[4] F. M. Penning and A. A. Kruithof, Physica **2**, 793 (1935).

[5] V. K. Zworykin, G. A. Morton, and L. Malter, Proc. I. R. E. **24**, 351 (1936).

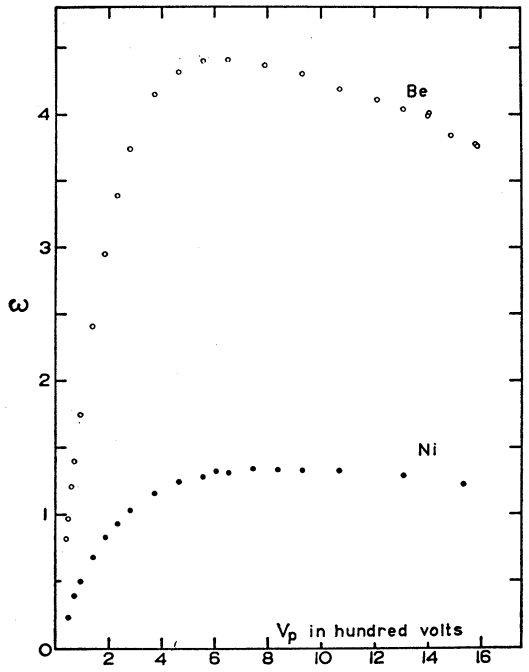

FIG. 1. Secondary emission ratio ω for nickel and beryllium plotted against primary energy.

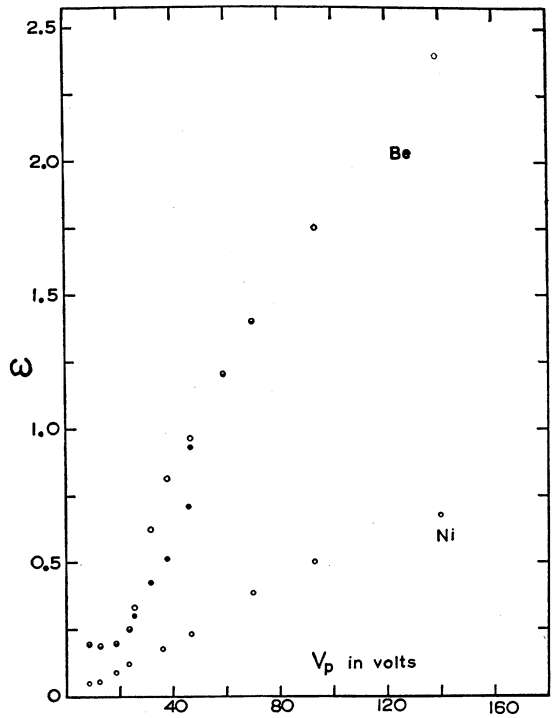

FIG. 2. Low voltage portion of Fig. 1 enlarged and extended. In the beryllium curve the solid circles are for increasing primary energy, the open circles for decreasing primary energy, the half-open circles for both.

filament. The thickness of the beryllium layer was about 0.01 mm, estimated from the weight of beryllium used. The plate was then incorporated into the tube in which measurements were to be made, and was there given the treatment described above.

SECONDARY EMISSION MEASUREMENTS

The secondary emission ratio ω was calculated from the plate current and filament emission, assuming the fraction of primary electrons φ which strike the grid to be given by the grid dimensions.[6] For the cylindrical tubes used this assumption has been found to be true over a wide range of the ratio of grid to plate voltage. The fraction φ was found to be 0.250, of which 0.200 was due to the grid wires and 0.050 to the two grid supports. The secondary emission ratio is given by

$$\omega = 1 - i_p/(1-\varphi)i_0.$$

The emission current i_0 and plate current i_p were measured with multirange meters good to 0.1 percent of full scale.

[6] J. M. Hyatt, Phys. Rev. **32**, 922 (1928).

There are two factors which may cause errors in ω. First, the grid emits secondary electrons, some of which have sufficient energy to reach the plate, and thus have the effect of decreasing φ. If the geometrical value of φ is then used in the calculations, ω will come out too small. This effect has been minimized by keeping the grid voltage equal to $\frac{3}{2}$ the plate voltage. Under these conditions not more than 5 percent of the grid secondaries will have sufficient energy to reach the plate, and of these about half may return to grid wires, so that with $\varphi = 0.250$ the percent error in $(1-\varphi)$ or $(1-\omega)$ will be less than $\frac{1}{2}$ percent times the value of ω for the grid. In all cases the error in ω due to this source should be less than 1 percent. This has been verified by experiments in which the plate voltage was held constant and the grid voltage increased.

This source of error was responsible for the decrease in ω with increasing plate voltage, found in (*I*), where at the highest plate voltages the grid voltage was not high enough.

The second source of error in ω results from

the fact that some secondary electrons leaving the plate obliquely and with large velocities may strike the plate again, rather than go to the grid. A simple calculation shows that for the dimensions and voltage ratio of our tubes only secondaries having more than half the primary energy can return to the plate even if emitted at grazing angles. Estimated errors due to this factor are also less than one percent.

Shot Effect Measurements

The shot effects were measured with a five-stage amplifier tuned to a frequency of 110 kilocycles. A low impedance tuned input circuit was used ($L=230$ microhenrys, $C=9000\mu\mu$f., $R=4.6$ ohms, from which $Z\sim5600$ ohms). About half the resistance was added deliberately to reduce the effect of the parallel resistance of the shot tube.

The secondary emission shot effects were measured by direct comparison with temperature-limited shot effects from a diode connected in parallel with the experimental tube. The diode shot effects were superimposed on those from the experimental tube; this procedure made it unnecessary to correct for the resistance of the latter. A correction was made for the resistance of the diode. This method is rapid and gives values good to 1 percent or better.

Results

The values found for ω are shown as a function of primary energy in Fig. 1. The values shown for nickel are in general within 10 percent of the values of Rao.[7] The values for beryllium are some 20 percent lower than those of Copeland.[8] Doubtless neither his beryllium surface nor ours was free of oxygen.[9]

In Fig. 2 the values of ω for energies below 150 volts are shown on a larger scale. At the lowest voltages both curves become horizontal. The beryllium curve has a loop, the values for increasing voltages lying below the values for decreasing voltages. We shall discuss this effect subsequently.

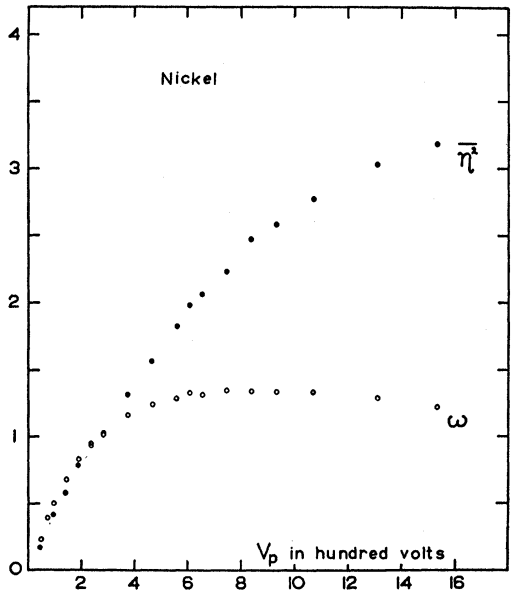

FIG. 3. $\langle\eta^2\rangle_{Av}$ and ω for nickel plotted against primary energy.

Any discontinuities such as those studied by Rao and others were not detectable in our curves, being entirely smoothed over by the potential drop of 3 to 4 volts along the filament.

The shot effects J_p were in every case measured at the plate of the experimental tube, and were expressed in units of equivalent temperature-limited current. From these measurements we have computed J_2, the shot effect of the secondary electrons alone; κ, the ratio of J_2 to i_2 the secondary current; and $\langle\eta^2\rangle_{Av}$, the mean square deviation of the individual secondary emission ratios from the average value ω. These quantities are related to each other and to the primary current i_1, by the equations:

$$J_p=i_1\{(1-\omega)^2+\langle\eta^2\rangle_{Av}\}, \quad (1)$$

$$J_2=i_1\{\omega^2+\langle\eta^2\rangle_{Av}\}, \quad (2)$$

$$\kappa=(\omega^2+\langle\eta^2\rangle_{Av})/\omega. \quad (3)$$

The principal feature of the values of J_2 is that they continue to increase far beyond the voltage at which ω is maximum. The maximum values of J_2/i_1 obtained are about 4.7 for nickel and about 36 for beryllium.

Figures 3 and 4 show $\langle\eta^2\rangle_{Av}$ for each surface,

[7] S. R. Rao, Proc. Roy. Soc. **A128**, 41 and 57 (1930).
[8] P. L. Copeland, Phys. Rev. **46**, 167 (1934).
[9] Dr. Malter of the RCA Manufacturing Co. has informed us that in unpublished measurements during deposition of the beryllium surface he has found values considerably lower than ours. See also H. Bruining and J. H. de Boer, Physica **4**, 473 (1937).

the values of ω being included for comparison. Here again the most striking feature is the continued increase in $\langle\eta^2\rangle_{Av}$ as the maximum of ω is passed.

Discussion of the Results

A complete theory of secondary emission would enable us to calculate the values of $\langle\eta^2\rangle_{Av}$ as well as those of ω for any given metal. Lacking this, it is necessary to proceed empirically to interpret the results. This was done in (*I*) by dividing the primary electrons into three classes: (1) a fraction x reflected without appreciable loss of energy; (2) a fraction y which is buried in the metal without producing any secondaries; (3) the remainder z which is responsible for the true secondary emission. It was shown that the following relations hold:[10]

$$\omega = x + z\omega_z, \tag{4}$$

$$J_p/i_1 = (1-\omega)^2 + \langle\eta^2\rangle_{Av} = y + z\{(1-\omega_z)^2 + \langle\eta_z^2\rangle_{Av}\}. \tag{5}$$

In order to carry this analysis further it became necessary to make auxiliary assumptions,

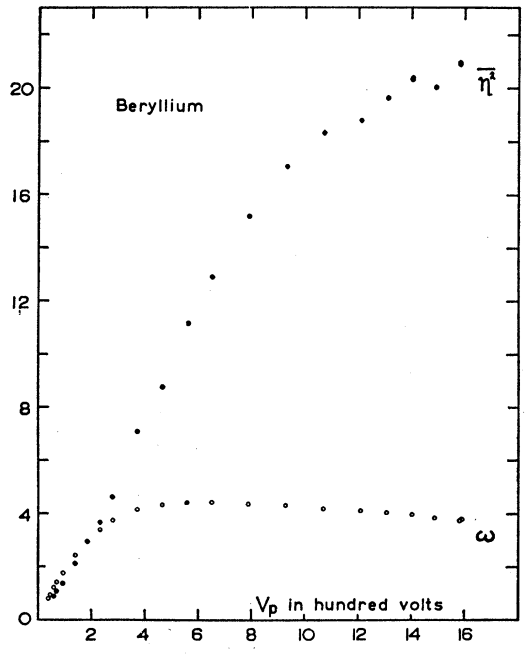

FIG. 4. $\langle\eta^2\rangle_{Av}$ and ω for beryllium plotted against primary energy.

[10] (*I*) p. 332, Eqs. (16) and (17); except that in the latter the first bracket was misplaced by a typographical error. The subsequent equations there given are correct.

since each of the four unknowns in these two equations is a function of the primary energy.

In (*I*) it was shown that while $\langle\eta_z^2\rangle_{Av}$ may have any value from zero to ω_z^2, neither of these extreme values would fit the data then available. The assumption was made that "because of the large number of random factors involved in the secondary emission process" the actual number of secondaries ejected by one primary "varies in purely random fashion. In this case $\langle\eta_z^2\rangle_{Av}$ would be approximately equal to ω_z itself, at least for high primary voltages."

This assumption has been criticized by Ziegler[11] in both papers. His criticisms are unjustified for the following reasons. First of all he has incorrectly translated our assumptions into his notation. For instance ω_z is not $\delta-\beta_1$, as he states (p. 11) but $(\delta-\beta_1)/(1-\beta_0-\beta_1)$; and his expression for $\langle\eta_z^2\rangle_{Av}$ is similarly in error. Furthermore he has confused randomness due to time independence with randomness due to the many space factors involved. Although the secondaries from a given primary electron are all produced within a negligible time interval, they are produced at different positions, have different velocities both in direction and in magnitude, and experience different energy losses and scattering on their way out.

We therefore believe it is rational to assume that $\langle\eta_z^2\rangle_{Av}=\omega_z$ whenever the maximum possible number of secondaries per primary is large, i.e. at high primary energies.

Next we shall assign certain values to the fraction x. At the lowest voltages, where the curves of Fig. 2 are nearly horizontal, the fraction z and the associated ω_z are certainly negligible, so that $x=\omega$; i.e. there is only reflection, no true secondary emission. This has been found also by investigators who have studied the energy distribution of the secondary electrons. At higher energies there are fewer and fewer reflected primaries; but even at beta-ray energies there are still one percent or more. A smooth but slow decrease of x with increasing voltage is therefore required by the experimental evidence.

The exact type of law which this decrease follows is unknown. For a square barrier the theory of transmission of electrons through

[11] Reference 2, especially the notes on pages 11 and 311.

potential barriers gives $x \sim W/4V_p$, when $V_p \gg W$, where W is the height of the barrier.

For a barrier due to an image force slightly smaller values are found. Such reflection is actually found at our lowest voltages. At high voltages the decrease in x is certainly less rapid than $1/V_p$, probably because of Bragg reflections or other large angle scattering from the interior of the metal.

As a reasonable compromise we shall assume that in the case of nickel x decreases linearly from 5 percent at 20 volts to 2.5 percent at 1500 volts. Even complete neglect of x, however, will not seriously alter our results at the higher voltages. The case of beryllium at low voltages will be discussed separately below; for high voltages we shall use the same values as for nickel.

TABLE I. *Values of ω_z, z and $\omega_z/(V_p-V_0)^{\frac{1}{2}}$ or A, calculated on the assumption that $\langle \eta^2{}_z \rangle_{Av} = \omega_z$. Below 300 volts the values of A rapidly decrease. For the first three points of both nickel and beryllium it is preferable to use the corresponding points of Table II.*

V_p	ω	κ	x	z	ω_z	A
			Nickel			
300	1.065	2.12	0.045	0.87	1.16	0.069
350	1.130	2.24		.84	1.29	.071
400	1.185	2.37		.80	1.42	.072
450	1.230	2.48		.76	1.54	.074
500	1.265	2.59		.74	1.64	.075
600	1.307	2.78	.04	.69	1.83	.076
700	1.330	2.96		.64	2.02	.077
800	1.340	3.10		.61	2.16	.077
900	1.344	3.23	.035	.57	2.29	.077
1000	1.338	3.34		.54	2.40	.076
1100	1.325	3.45		.51	2.50	.076
1200	1.310	3.54	.03	.49	2.60	.076
1300	1.285	3.63		.47	2.68	.075
1400	1.260	3.71		.45	2.76	.074
1500	1.230	3.80	.025	.42	2.86	.074
1600	1.185	3.91		.39	2.97	.075
			Beryllium			
150	2.55	3.42	0.047	>1	2.46	0.212
200	3.08	4.10		0.97	3.15	.232
250	3.52	4.66		.94	3.70	.242
300	3.83	5.18	.045	.89	4.23	.250
350	4.08	5.66		.86	4.71	.257
400	4.21	6.00		.83	5.05	.258
450	4.30	6.34		.79	5.39	.258
500	4.35	6.60		.76	5.66	.257
600	4.40	7.10	.04	.71	6.15	.254
700	4.40	7.51		.66	6.57	.253
800	4.37	7.87		.63	6.93	.247
900	4.31	8.17	.035	.59	7.23	.243
1000	4.25	8.42		.56	7.48	.238
1200	4.12	8.78	.03	.52	7.84	.228
1400	3.97	9.06		.49	8.09	.218
1600	3.74	9.30	.025	.44	8.34	.210

Our last assumption is that

$$\omega_z = A(V_p - V_0)^{\frac{1}{2}}, \tag{6}$$

where A is a constant and V_0 is the limiting voltage below which no true secondaries are emitted. This is essentially the type of law predicted by Fröhlich's theory[12] of secondary emission for a moderate energy range above V_0.

These assumptions are sufficient not only to yield values of z and ω_z over a long voltage range, but also to provide a check of the assumptions. This is shown as follows. Using Eqs. (3), (4), (5), together with $\langle \eta_z{}^2 \rangle_{Av} = \omega_z$ we get

$$z = (\omega - 2x)/(\kappa - 1), \tag{7}$$

neglecting a term x^2/ω in the numerator. With these values of z, ω_z follows from Eq. (4) and A from Eq. (6). From inspection of Fig. 2 we have assigned V_0 the value of 15 volts, which is close to the theoretical values, and also to the values obtained experimentally by others. The uncertainty of V_0 is no greater than the uncertainty in V_p itself due to contact potential and filament potential drop.

The results are shown in Table I. For nickel the values of A are remarkably constant, varying only a few percent down to $\omega_z = 1.5$, near 400 volts. For beryllium the constancy is satisfactory. In both nickel and beryllium, though more pronounced in the latter, there is a decrease in A at the highest voltages. The reason for this is not known.

The question arises how sensitive these results are to the assumption $\langle \eta_z{}^2 \rangle_{Av} = \omega_z$. To answer this, we set $\langle \eta_z{}^2 \rangle_{Av} = B\omega_z$; then Eq. (7) becomes $z = (\omega - 2x)/(\kappa - B)$ approximately, and the values of A will be altered in the ratio $(\kappa - B)/(\kappa - 1)$. Thus a one percent deviation of A from the average value, from $\kappa = 3.9$ to $\kappa = 2.5$ (in nickel) requires $|B - 1| = 0.08$; for beryllium about four times as much deviation is possible. $\langle \eta_z{}^2 \rangle_{Av}$ is therefore much closer to ω_z than to $\omega_z{}^2$ or to $2\omega_z$.

Similar calculations can be made with Ziegler's data for barium oxide coated surfaces. These data do not extend below 50 volts, so that experimental information about x is not avail-

[12] H. Fröhlich, Ann. d. Physik **13**, 229 (1932).

able. Neglecting x, the values of A range from 0.33 at 800 volts to 0.28 at 400 volts. With normal values of x the constancy would be improved and extended. Similarly the data of Penning and Kruithof[13] for a caesium-oxygen-silver surface give values of A from 0.30 at 600 volts to 0.35 at 100 volts. The trend of these values seems to be opposite to the normal trend; however, their data are in some cases given to 10 percent only.

Without further knowledge of the mechanism of secondary emission, the similarity of the A values for these surfaces must be regarded as accidental.

In the low voltage range we must not expect $\langle\eta_z^2\rangle_{Av}=\omega_z$ to hold. Retaining the other assumptions, we have calculated $\langle\eta_z^2\rangle_{Av}$ from Eqs. (1)–(5), in the form

$$\langle\eta_z^2\rangle_{Av}=\{(\kappa\omega-x)/(\omega-x)-\omega_z\}\omega_z, \qquad (8)$$

using values of ω_z extrapolated from the high voltage range[14] by means of Eq. (6). For this purpose we have taken A as 0.075 for nickel and as 0.255 for beryllium. This procedure yields the values shown in Table II.

These values of $\langle\eta_z^2\rangle_{Av}$ lie below the values of ω_z. This is to be expected, for in the limiting case where there are only single secondaries we must have $\langle\eta_z^2\rangle_{Av}=\omega_z-\omega_z^2$, i.e., $\kappa=1$. This limit is almost reached at the lowest voltages in nickel. (Below 25 volts the beginning of space charge effects made shot effect measurements uncertain, although the ω values were not affected.)

The situation in the beryllium tube at the lower voltages was very complex. The loop already mentioned, in the region of Fig. 2 between 20 and 50 volts, evidently represents some type of change in the surface. This change may involve sudden removal of a layer of atoms from the surface when the primary voltage reaches the point where the lower curve jumps to the upper one. We shall assume that the electron reflection coefficient of the surface is altered, and with it x and z, the latter because of internal reflection of outward bound secondaries. We assume also that ω_z and $\langle\eta_z^2\rangle_{Av}$ are functions of the conditions in the interior, and are not altered by surface change.

TABLE II. *Values of $\langle\eta_z^2\rangle_{Av}$ and z, calculated on the assumption that $\omega_z=A(V_p-V_0)^{\frac{1}{2}}$; $A=0.075$ for nickel, 0.255 for beryllium.*

V_p	ω	κ	x	ω_z	$\langle\eta_z^2\rangle$	z
			Nickel			
25	0.13	1.04	0.05	0.24	0.20	0.33
30	.16	1.06		.29	.23	.38
35	.185	1.08		.34	.25	.40
40	.215	1.11		.38	.29	.43
50	.268	1.16		.45	.38	.49
75	.408	1.28		.58	.43	.62
100	.540	1.38		.69	.50	.71
150	.715	1.58		.87	.65	.77
200	.855	1.80		1.02	.84	.79
250	.975	1.97		1.15	1.01	.81
300	1.065	2.12	.045	1.27	1.14	.80
350	1.130	2.24		1.38	1.26	.79
400	1.185	2.37		1.48	1.51	.77
			Beryllium			
25	0.336	1.07	0.17	0.81	0.27	0.21
	.296	1.03	.19		.23	.13
30	.542	1.25	.08	.99	.30	.46
	.380	1.12	.19		.25	.20
35	.740	1.43	.05	1.14	.37	.60
	.473	1.25	.185		.32	.25
40	.855	1.55	.05	1.28	.39	.63
	.570	1.38	.18		.36	.31
50	1.04	1.76	.05	1.51	.43	.66
75	1.50	2.23	.05	1.98	.57	.73
100	1.86	2.64	.05	2.35	.78	.77
150	2.55	3.42	.05	2.96	1.51	.86
200	3.08	4.10	.05	3.47	2.32	.89
250	3.52	4.66	.05	3.90	3.12	.90
300	3.83	5.18	.045	4.32	3.93	.89

To test these assumptions we have assigned different values of x to several points on the two branches of the loop, using for the lower branch the values found at the lowest voltages (about 20 percent), and for the upper branch values similar to those for nickel (about 5 percent). We then calculated z and $\langle\eta_z^2\rangle_{Av}$, as in the case of nickel using the same values of ω_z for each branch. The results show two interesting features: (1) the values of $\langle\eta_z^2\rangle_{Av}$ for the two branches are equal within their precisions; (2) the ratio of the z values for the two branches is nearly constant, being 0.43, 0.42 and 0.49 for the three points not lying in the transition regions, at 30, 35, and 40 volts. This last result leads to an estimate of the internal reflection

[13] F. M. Penning and A. A. Kruithof, Physica 2, 800 (1935).

[14] It may seem surprising that ω_z can have values less than 1; but the fraction z must include primaries which produce secondaries of which only one or none escape from the surface.

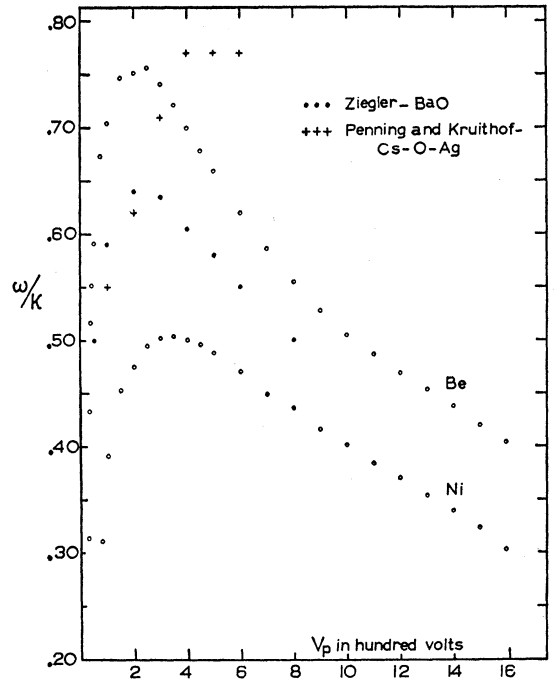

FIG. 5. ω/κ for various surfaces plotted against primary energy.

coefficient for the lower branch. If the upper branch (large z) represents a condition with no internal reflection, the lower branch will involve about 55 percent internal reflection. This is not an abnormally high value, considering that the average secondary electron energy is 2–4 volts, and that the primaries at 15–20 volts are 20 percent reflected.

These results are consistent with the assumptions made. We have tried other assumptions which led to contradictions. It seems therefore that shot effect measurements can be used to distinguish surface effects from volume effects in the mechanism of secondary emission.

The values of z for nickel and those for beryllium are very similar, the latter being slightly larger. It is difficult to say what relative values one would expect, let alone to estimate absolute magnitudes. The atomic number, the atomic density (which is 30 percent higher for beryllium than for any other element), and the value of ω_z are among the factors which must be considered.

It appears from these results that the decrease of ω at high voltages is due to the increase of the number of buried primaries, rather than to any decrease in the number of secondaries for those primaries which are effective. This suggests that secondary emission is in large part due to primaries which have been scattered through angles of nearly 180° without loss of energy. There is other evidence pointing in the same direction: the existence of reflected primaries, together with the fact, established with thin foils, that an outward bound fast electron produces more secondaries than an inward bound one of the same energy. If this is so the theory of Fröhlich, which in its present form applies only to inward primaries and neglects scattering, must be considerably extended.

PRACTICAL BEARING OF THE RESULTS

As a matter of practical interest we have given in Fig. 5 the values of ω/κ, the ratio of signal power amplification to shot effect power amplification by a single stage of secondary emission, assuming that the primary current is 100 percent modulated by the signal. This ratio is also important in multistage electron multipliers. The shot effect in multistage multipliers has been discussed by Shockley and Pierce,[15] who have given the general relations involved, and have shown that the principal factor determining the multistage shot effect is a quantity b equal to $(\kappa/\omega)-1$. It will be seen that at low voltages this particular beryllium surface is definitely superior to the other types in this respect, having the highest values of ω/κ, or lowest b.

In conclusion we wish to express our gratitude to Professor H. M. Randall, who placed the facilities of the physics laboratories of the University of Michigan at our disposal; and especially to Professor N. H. Williams, who furnished the amplifier and other apparatus, and was most generous and helpful in many other ways.

[15] In process of publication. We are indebted to Dr. Shockley for information on this subject, and for his kindness in sending us a copy of the manuscript.