rates of formation of the two activities is about 1 : 3. Correcting this ratio for the difference in abundance of the two parent isotopes, Sr^{87} and Sr^{88}, it would appear that the ratio of proton captures by Sr^{87} to deuteron captures by Sr^{88} is about 7 : 2.

Other possibilities for the production of active isotopes of yttrium would be expected from the bombardment of zirconium with deuterons or fast neutrons, followed by the emission of an alpha-particle or a proton, respectively. Preliminary experiments showed that isotopes of yttrium are actually produced in these bombardments and there is indication of the two yttrium periods just reported, but as yet the work of complete identification has not been attempted. Similarly, bombardment of rubidium with alpha-particles of 12–13 Mev has yielded yttrium activities too weak for exact analysis.

Beta-Ray Spectra

It has been possible to obtain tentative values for the upper limits of the energy of the beta-ray spectra of the two isomeric forms of Sr^{89}, for Y^{88} and for Y^{90} by photographing the tracks in a cloud chamber. More than 600 tracks of each kind were measured in a magnetic field of 330 gauss. In Figs. 3 to 6 are shown curves obtained from these data. In each case is given the distribution histogram actually obtained and the Konopinski-Uhlenbeck plot. In Fig. 4, where the tracks photographed were from both the 3-hour and the 55-day periods, the K-U plot is fitted best by two straight lines. The line yielding a higher energy limit is in agreement with the results for the 55-day period, shown alone in Fig. 3. The extrapolated K-U upper energy limits may be summarized as follows:

Sr^{89} (3-hour period) 0.61 Mev;
Sr^{89} (55-day period) 1.9 Mev;
Y^{88} (120-min. period) 1.2 Mev;
Y^{90} (60.5-hour period) 2.6 Mev.

It will be noted that the energy limit for the strontium isomer with the longer half-life is considerably greater than that found for the 3-hour activity. Present concepts of isomeric nuclei place the two excited states very close together. In view of the beta-ray energies found above, this would require the gamma-ray activity associated with the three-hour period to have an energy of about 1.3 Mev, i.e., the difference in energy of the beta-activity from the two isomers. Measurements of the observed gamma-ray energy are yet to be made.

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Field Theory of Nuclear Interaction

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The problem of explaining the recent results on nuclear interaction by means of a field theory is studied. Fermi's theory of the electron-neutrino field is used as a model which is sufficient to account for the symmetries of the problem, although it fails to explain the order of magnitude of the forces. The equality of forces between like and unlike particles is exactly accounted for by introducing interaction terms involving the emission of electron pairs or neutrino pairs. The interaction law may be stated very simply with the aid of an isotopic spin variable for light as well as for heavy particles. The ratio of force constants obtainable from the theory of mass defects may be accounted for in detail by a suitable choice of the light particle field. However, it is difficult to explain any law involving more than one potential function $J(r)$.

§1. General Considerations

THE problem of the forces governing the constitution of nuclei has recently been studied with a considerable amount of success. From the increasing experimental data and their comparison with continually improving theoretical computations it has been possible to gain a great deal of knowledge about the nuclear interaction potentials. Among other facts it has lately been apparently well established[1] that, apart from the Coulomb repulsion of the protons, the interaction is not only the same between two protons as between two neutrons, but also between a proton and a neutron, if these are in an antisymmetrical state. Thus the electric charge of the particles seems to be irrelevant as far as the specifically nuclear forces are concerned. For brevity we will call this the charge-independence hypothesis and refer to it as CIH.

For these purposes it has been sufficient to use nonrelativistic wave mechanics, the interaction then being described by a static potential function $J(r)$. However, a characteristic additional assumption that has been generally adopted is that the forces are predominantly of the "exchange" type. This is expressed mathematically by multiplying $J(r)$ into a linear combination of certain exchange operators, the interaction taking the form[2]

$$U=(MP_M+HP_H+W-BP_HP_M)J(r)\,; \quad (1)$$

P_H denotes the operation of exchanging the spatial and spin coordinates of the two interacting particles, P_M the exchange of spatial coordinates only; M, H, W and B are constants.

In the formalism of the "isotopic spin" shown by Cassen and Condon[3] to be extremely practical if the CIH be accepted, the interaction assumes the form

$$U=[W+\tfrac{1}{2}B\{1+(\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2)\}-\tfrac{1}{2}H\{1+(\boldsymbol{\tau}_1\boldsymbol{\tau}_2)\} -\tfrac{1}{4}M\{1+(\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2)\}\{1+(\boldsymbol{\tau}_1\boldsymbol{\tau}_2)\}]J(r), \quad (2)$$

$\boldsymbol{\sigma}_i$ and $\boldsymbol{\tau}_i$ being the spin and isotopic spin operators of particle i.

The reason for the use of exchange forces is that only these seem capable of explaining the practically linear increase of binding energy with atomic weight in the region of medium and heavy nuclei. It should be remembered, however, that when originally proposing a nuclear exchange force, Heisenberg[4] did not choose this unusual type of interaction simply to account for this one experimental fact. The proposal was founded on a definite physical idea of the mechanism of interaction, Heisenberg's conception being that the binding is effected by an actual exchange of electric charge between the two particles.

If, as is in any case necessary in a relativistic treatment of the problem, the force is considered to be transmitted from one particle to the other by means of an intermediate field, then Heisenberg's picture requires that, in the intermediate state, the charge pertains to the field.

Such a charge-bearing field had been independently introduced into nuclear physics in Fermi's theory of β-decay,[5] and it was therefore natural to seek to connect the two phenomena and to attempt to explain nuclear binding in terms of Fermi's electron-neutrino field. As is well known these attempts have met with little success. If one assumes the interaction term to have the magnitude deducible from the probability of β-decay, the resulting nuclear forces are far smaller than experimental knowledge requires. No satisfactory suggestions for the interaction essentially different from Fermi's have yet been studied.

However, the application of Fermi's theory to these questions may still serve a useful purpose. In the study of the dependence of nuclear forces on the spins of the particles it may be expected to yield correct results. The exhaustive discussion of this problem by Fierz[6] indeed shows that the dependence required by experiment can be completely accounted for.

If one considers the dependence of the nuclear forces on the charge, it seems even more likely that essentially only the symmetry properties of the intermediate field can be of importance, and therefore relevant results should be obtainable from a theory of the Fermi type. For instance

[1] Breit, Condon and Present, Phys. Rev. **50**, 825 (1936).
[2] The more recent suggestion of using a sum of different linear combinations of the exchange operators multiplied into different functions $J_i(r)$ will be briefly considered in § 3.
[3] Cassen and Condon, Phys. Rev. **50**, 846 (1936).
[4] Heisenberg, Zeits. f. Physik **77**, 1 (1932).
[5] Fermi, Zeits. f. Physik **88**, 161 (1934).
[6] Fierz, Zeits. f. Physik **104**, 553 (1937).

the CIH can obviously not be well explained with Heisenberg's charge-bearing field alone, since this field gives interaction forces between particles of like charge in higher approximation only. Therefore the natural assumption is that further interaction terms must exist which involve a field with zero total charge.[7, 8] In the language of Fermi's theory this would mean that the emission of two electrons of opposite sign or of two neutrinos by a proton or a neutron should be possible in addition to the originally assumed emission of one electron together with one neutrino.[9] We shall call the interaction in Fermi's original theory β-interaction of the first kind, the additional interaction here considered being denoted as of the second kind.

It is not obvious that this assumption is sufficient to account for the CIH, but the following considerations will show that this is actually the case.[10] A number of similar considerations can profitably be added and the general result deduced that the ratio of the coefficients W, B, H and M in (2) that is in best agreement with experimental data can well be accounted for by a simple form of β-theory. Any modification of Fermi's theory that leads to forces of the correct magnitude can however be shown to be incompatible with the saturation conditions required in the theory of heavy nuclei. In deriving these results the Fermi interaction should therefore, as has already been stated, be regarded merely as a "model" suitable for our purpose, without being necessarily generally correct.

§2. Derivation of the Interaction Law

We shall apply the method of the isotopic spin in the case of the protons and neutrons, and it is therefore natural to accept a corresponding description for the light particles forming the interaction field.[11] This is only practical if one may assume that the mass difference between electrons and neutrinos can be neglected, as otherwise the undisturbed energy of the light particles would already be a function of $\boldsymbol{\tau}$ and the simplifications gained by introducing this variable would be largely lost. It is well known that the mass difference in question is in fact essential in describing the shape of β-spectra, but as the electrons and neutrinos of high energies give by far the greatest contribution to nuclear forces, the neglect is certainly permissible for our purposes. The same simplification has actually been made in all previous investigations. The minor corrections to the forces due to the mass difference of course necessarily violate the CIH, and this is equally true of the effects due to the Coulomb field of the electrons, but deviations of that order of magnitude are certainly not essential to our considerations.

It is most practical to define the isotopic spin operator of the light particles in such a way that τ_z is $+1$ for the *positron* -1 for the neutrino. This definition implies that we describe the positron as a "particle" and the negative electron as its "antiparticle." The "neutrino" is then considered to be emitted in β^--decay, the "antineutrino" in β^+-decay. We shall designate the quantized wave functions of the heavy particles by Ψ, those of the light particles by ψ. For brevity we will omit the operators acting on the spins of light and heavy particles. Their effect is known from the work of Fierz[6] and does not differ for β-interaction of the second kind. It is easy to remedy the omission in our final formulae; this will be done in §3.

When determining all possibilities for the dependence of the interaction on $\boldsymbol{\tau}$ we must note that the total charge of heavy + light particles must be conserved in all transitions. Then the most general expression for the interaction energy W is seen to have only six possible linearly independent terms:

$$\begin{aligned}4W=&g_1\Psi^*(\tau_x+i\tau_y)\Psi\cdot\psi^*(\tau_x-i\tau_y)\psi\\&+g_1^*\Psi^*(\tau_x-i\tau_y)\Psi\cdot\psi^*(\tau_x+i\tau_y)\psi\\&+g_2\Psi^*(1+\tau_z)\Psi\cdot\psi^*(1+\tau_z)\psi\\&+g_3\Psi^*(1-\tau_z)\Psi\cdot\psi^*(1+\tau_z)\psi\\&+g_4\Psi^*(1+\tau_z)\Psi\cdot\psi^*(1-\tau_z)\psi\\&+g_5\Psi^*(1-\tau_z)\Psi\cdot\psi^*(1-\tau_z)\psi.\end{aligned}\quad(3)$$

[7] Wentzel, Helv. Phys. Acta **10**, 108 (1937).
[8] Gamow and Teller, Phys. Rev. **51**, 289 (1937).
[9] The compatibility of these further interaction terms with experiment is clearly stated by Wentzel (reference 7).
[10] This result has been evolved in a discussion with Professor G. Wentzel who has kindly permitted the writer to point out that the statement to the contrary contained in his paper (reference 7) p. 110, footnote 2, needs correction.
[11] Feenberg, Phys. Rev. **51**, 777 (1937).

g_1 to g_5 are constants, all but g_1 being necessarily real. The first two terms in (3) describe the usual interaction of the first kind, the four others give transitions of the second kind and correspond to the four possibilities of either electron or neutrino pairs being emitted by either a proton or a neutron.

To derive the forces between heavy particles we have to determine the second order perturbation energy arising from (3). This calculation differs in no essentials from the similar ones performed previously for the neutron-proton force alone, e.g. by v. Weizsäcker.[12] We can therefore confine ourselves to stating the result. The perturbation energy is found to be

$$H_W = +\frac{1}{64\pi^3\hbar c}\cdot\int d\mathbf{x}\int d\mathbf{x}' F(r)$$
$$\times \sum_{\alpha\alpha'\beta\beta'} \Psi_\alpha^*(\mathbf{x})\Psi_{\alpha'}^*(\mathbf{x}')\omega\Omega_{\alpha\alpha',\beta\beta'}\Psi_\beta(\mathbf{x})\Psi_{\beta'}(\mathbf{x}') \quad (4)$$

with

$$\Omega_{\alpha\alpha',\beta\beta'} = 2|g_1|^2(\boldsymbol{\tau}_{\alpha\beta}\boldsymbol{\tau}_{\alpha'\beta'})$$
$$+[(g_2+g_3)^2+(g_4+g_5)^2]\delta_{\alpha\beta}\delta_{\alpha'\beta'}$$
$$+[(g_2-g_3)^2+(g_4-g_5)^2-2|g_1|^2]\tau_{z,\alpha\beta}\tau_{z,\alpha'\beta'}$$
$$+[g_2^2+g_4^2-g_3^2-g_5^2][\tau_{z,\alpha\beta}\delta_{\alpha'\beta'}+\delta_{\alpha\beta}\tau_{z,\alpha'\beta'}]. \quad (5)$$

The suffixes $\alpha\alpha'$ and $\beta\beta'$ refer to the columns and rows of the isotopic spin matrices and the corresponding components of the wave functions; ω symbolizes the spin dependent operators which we have refrained from putting down in detail. In Fermi's theory $F(r)$ can only be reasonably calculated for distances $r=|\mathbf{x}-\mathbf{x}'|\gg\hbar/Mc$, then it is

$$F(r)=r^{-5}. \quad (6)$$

Our main concern, however, is the study of Ω. It is seen immediately that the last term in (5) results in forces of different sign for two protons as compared with two neutrons. This must be excluded, so that we have to postulate:

$$g_2^2+g_4^2=g_3^2+g_5^2. \quad (7)$$

Similarly the third term of (5) would give an attractive force for unlike particles and a repulsive force of equal magnitude for like particles. The CIH therefore requires its coefficient to be zero:

$$(g_2-g_3)^2+(g_4-g_5)^2=2|g_1|^2. \quad (8)$$

The remaining two terms are in agreement with the CIH and exactly of the form implied by (2).

Let us now put

$$(g_2+g_3)^2+(g_4+g_5)^2=2f^2, \quad (9)$$
$$|g_1|^2=g^2. \quad (10)$$

Omitting the suffixes we then have

$$\tfrac{1}{2}\Omega=f^2+g^2(\boldsymbol{\tau}\boldsymbol{\tau}'). \quad (11)$$

It is easily seen that in spite of the restrictions (7) and (8) f and g can assume any real values independently, but it should be noted that it appears impossible to derive negative coefficients for either of the terms on the right side of (11). Let us further note that the constant g is essentially Fermi's original constant and is therefore determined by the probability of β-decay; f on the other hand is free to be determined to fit the facts of nuclear interaction alone.

The general form of Ω are given by (11) can now easily be shown to be derivable from a specially simple case of (3). To demonstrate this, let us put

$$g_2=g_5=\tfrac{1}{2}(f+g);\quad g_3=g_4=\tfrac{1}{2}(f-g). \quad (12)$$

The Eqs. (7), (8), (9) together with (10) are then satisfied. Inserting (12) in (3) we obtain

$$2W=f\Psi^*\Psi\cdot\psi^*\psi+g(\Psi^*\boldsymbol{\tau}\Psi\cdot\psi^*\boldsymbol{\tau}\psi). \quad (13)$$

It thus appears quite unnecessary to use any more complicated form of the β-decay law than (13) in order to obtain the most general result (11) for the heavy particle interaction.

§3. Discussion of Results

In order to compare our results with the usual interaction expressions of the type (2), it is necessary to include the term involving the spin dependence that has hitherto been omitted. According to Fierz the most general type of spin dependence is

$$\omega=\alpha+\beta(\boldsymbol{\sigma}\boldsymbol{\sigma}')+\gamma(\boldsymbol{\sigma}\mathbf{r})(\boldsymbol{\sigma}'\mathbf{r})r^{-2}. \quad (14)$$

The peculiar third term is of a type not usually considered in the calculations on nuclear binding,

[12] v. Weizsäcker, Zeits. f. Physik **102**, 572 (1936).

and it is at present best to restrict oneself to those special interactions in which its coefficient is zero. We therefore assume

$$\gamma=0 \tag{15}$$

and note that by Fierz' formula β must then be positive, so that we can put

$$2\beta=b^2. \tag{16}$$

A similar restriction for α is not deducible from Fierz' formula, but we can easily see that it is necessary for physical reasons. Breit and Feenberg[13] showed that the linear increase of nuclear binding energies is explicable only if a certain inequality is satisfied by the force constants. In our notation this inequality reads

$$\alpha f^2 \geqslant 0. \tag{17}$$

Therefore we can restrict ourselves to the consideration of interactions with positive α, and put

$$2\alpha=a^2. \tag{18}$$

The interaction potential is then proportional to

$$\omega\Omega=[a^2+b^2(\boldsymbol{\sigma\sigma}')][f^2+g^2(\boldsymbol{\tau\tau}')]. \tag{19}$$

From (19) we find that the potential in the ground state of the deuteron is proportional to

$$(a^2+b^2)(f^2-3g^2). \tag{20}$$

In order that this may result in an attractive force we must then have

$$f^2<3g^2. \tag{21}$$

A choice of an f appreciably larger than g, such as has been suggested in order to account for the true magnitude of nuclear forces is thus shown to be incompatible with the generally accepted saturation conditions of the type (17).

It should be observed that the expression (19) is only valid if one makes the natural assumption that the spin dependence of all five terms in (3) is identical, as was implied by the notation $\omega\Omega$. It is possible to abandon this assumption and then one evidently can obtain an interaction proportional to the more general expression

$$\omega\Omega=A+B(\boldsymbol{\sigma\sigma}')+C(\boldsymbol{\tau\tau}')+D(\boldsymbol{\sigma\sigma}')(\boldsymbol{\tau\tau}'). \tag{22}$$

[13] Breit and Feenberg, Phys. Rev. **50**, 850 (1936).

Any ratios of the coefficients A to D that are in agreement with the condition used in (17) and the other "saturation conditions" given by Breit and Feenberg[13] or Volz[14] and by the present writer,[15] can be seen to be derivable from suitable assumptions for the β-interaction, though it is still impossible to obtain the correct order of magnitude of the forces. It is, however, much more satisfactory to assume equal spin dependence of all terms and in consequence to restrict oneself to the more special form (19) for the interaction. It is interesting to note that in this special case all saturation conditions are always satisfied. In the form given by the author[15] they require:

$$\begin{aligned} &\text{(i)} && a^2f^2\geqslant 0,\\ &\text{(ii)} && a^2(f^2+g^2)\geqslant 0,\\ &\text{(iii)} && (a^2+b^2)f^2\geqslant 0, \end{aligned} \tag{23}$$

and are evidently true. More specially the considerations of Volz if modified to satisfy condition (iii) show that the best agreement with experiment is attained if one puts

$$f=0,\quad a^2:b^2=3:5, \tag{24}$$

and the use of the restricted expression (19) is indeed possible.

Thus, β-theory extended so as to involve the isotopic spin of the light particles succeeds in accounting completely for the assumptions as to the nature and ratios of nuclear forces used in current theory. Apart from the question of the absolute magnitude of the forces a further difficulty may however present itself if, as has been recently suggested,[16] it is found necessary to assume that interactions of different exchange character must be combined with different radial functions $J_i(r)$. There appears no other way of accounting for this but to suppose that a number of fields of different nature are responsible, each involving a different dependence on $\boldsymbol{\sigma}$ and $\boldsymbol{\tau}$. From the point of view of field theory it would clearly be very unsatisfactory to accept this.

[14] Volz, Zeits. f. Physik **105**, 537 (1937).
[15] Kemmer, Nature **140**, 192 (1937).
[16] Rarita and Present, Phys. Rev. **51**, 788 (1937).