LETTERS TO THE EDITOR

Prompt publication of brief reports of important discoveries in physics may be secured by addressing them to this department. Closing dates for this department are, for the first issue of the month, the eighteenth of the preceding month, for the second issue, the third of the month. Because of the late closing dates for the section no proof can be shown to authors. The Board of Editors does not hold itself responsible for the opinions expressed by the correspondents.

Communications should not in general exceed 600 words in length.

A Theorem on the Effect of Vertical Divergence

Several corrections have been published[1–6] for the shift in the position of an x-ray line due to vertical divergence, but only those of Spencer[3] and Williams[4] check.

All writers agree that a monochromatic x-ray incident at an angle ψ with the horizontal plane containing the normal to the crystal, will be reflected at a horizontal angle $\theta+\delta\theta$ where θ is the Bragg angle and

$$\delta\theta=\tfrac{1}{2}D\lambda\psi^2=K\psi^2. \tag{1}$$

D is Allison's notation for the dispersion $d\theta/d\lambda$. For single crystal reflection $D\lambda=\frac{1}{2}\tan\theta$, while for the double crystal spectrometer $D\lambda=\frac{1}{2}(\tan\theta_A\pm\tan\theta_B)$.

The distribution of intensity in the vertical plane, which we shall call $f(\psi)$ is transformed into a horizontal distribution $F(x)$ where x stands for $\delta\theta$. We wish to study the nth moment of $F(x)$ defined by

$$\mu_n'=\int_{-\infty}^{\infty}x^nF(x)dx. \tag{2}$$

By substituting (1) and the transformation equation

$$F(x)dx=f(\psi)d\psi, \tag{3}$$

$$\mu_n'=K^n\int_{-\infty}\psi^2f(\psi)d\psi=K^n\mu'_{\psi 2n}. \tag{4}$$

Eq. (4) may be stated as a theorem: *The nth moment of the horizontal distribution is K^n times the 2nth moment of the vertical distribution.* This theorem obviates the necessity for changing variable.

Let us apply the theorem to the case in which radiation from a uniformly bright target is limited by two slits of total heights a and b with centers at the same height but distant L apart. The intensity distribution in the vertical plane is then a trapezoid, which is equivalent to the difference of two triangles

$$f(\psi)=L(\psi_2-|\psi|)-L(\psi_1-|\psi|), \tag{5}$$

where $\psi_2=\frac{1}{2}(b+a)/L$ and $\psi_1=\frac{1}{2}(b-a)/L$. (6)

Eq. (4) then becomes

$$\mu_n'=\frac{LK^n(\psi_2^{2+2n}-\psi_1^{2n+2})}{(n+1)(2n+1)}. \tag{7}$$

The center of gravity of $F(x)$, and hence, the shift in the center of gravity of the x-ray line, is

$$\mu_1'/\mu_0'=K(\psi_2^2+\psi_1^2)/6=(\delta_2+\delta_1)/6. \tag{8}$$

This agrees with the author's expressions[3] $\delta_m/6$ and $\delta_m/3$ for the special cases of $a=0$ and $a=b$. The maximum deviation is $\delta_m=\delta_2$. Eq. (8) also reduces to Williams' formula for the effective shift

$$\delta\theta_{\text{eff}}=[(a^2+b^2)/24L^2]\tan\theta, \tag{9}$$

which is to be applied to each crystal.

If (1) is divided by $D\lambda$, the fractional shift in the measured wave-length corresponding to $\delta\theta$ is

$$\delta\lambda/\lambda=\tfrac{1}{2}\psi^2. \tag{10}$$

If now we let $x=\delta\lambda/\lambda$ and $K=\frac{1}{2}$, (8) and (9) become

$$\delta\lambda/\lambda=(\psi_2^2+\psi_1^2)/6=(a^2+b^2)/24L^2, \tag{11}$$

which is the fractional shift in wave-length of the center of gravity of the x-ray line. When an error of a factor of 2 is rectified in Eqs. (16) and (17) developed by the author,[3] they are found to agree with (11).

The shift in the center of gravity approximates the shift in the peak, which is of greater importance. The peak may be located by means of the graphical solution developed by the author,[7] which makes use of the higher moments of $F(x)$.

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Brace Laboratory,
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[1] S. K. Allison and J. H. Williams, Phys. Rev. **35**, 1476 (1930), p. 1480.
[2] A. H. Compton, R. S. I. **2**, 365 (1931), Eq. (3).
[3] R. C. Spencer, Phys. Rev. **38**, 618 (1931), Eqs. (11)–(17).
[4] J. H. Williams, Phys. Rev. **40**, 636(L) (1932).
[5] L. G. Parratt, Phys. Rev. **47**, 882(L) (1935).
[6] Compton and Allison, *X-Rays in Theory and Practice* (Van Nostrand), p. 737.
[7] See reference 3, pp. 625–628.

Unidentified Interstellar Lines

The following absorption lines appear in stellar spectra but are believed to be of interstellar origin:

λ	Intensity
5780.55±0.03	3
5797.13±0.04	1
6202.99±0.06	1−
6269.99±0.04	1
6283.91±0.03	6
6613.9 ±0.1	2

These lines differ from other interstellar lines in being slightly diffuse. Their characteristics will be described in detail in an article to be submitted to the *Astrophysical Journal*. In addition to the lines in the above list there is a vague feature near λ4430, which has been extensively