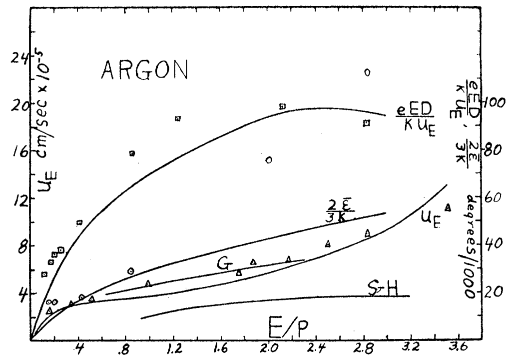

FIG. 6. Electron "temperatures" and drift velocities in argon. The curves marked *S–H* and *G* are "temperatures" derived from the probe measurements of Seeliger and Hirchert and of Groos, respectively.

Argon

Argon has the greatest Ramsauer effect of any gas. The result is that its "temperature" defined by formula (1) instead of being very nearly equal to $2\bar{\epsilon}/3k$ as it is when the cross section is constant, is very much greater. The two quantities are shown in Fig. 6 and it is seen that Townsend's[13] measurements agree fairly well with formula (1). On the other hand the probe measurements of Groos[5] and of Seeliger and Hirchert[4] are of the order of $2\bar{\epsilon}/3k$. Using $\epsilon_1 = 15.6$, the ionization potential, gives agreement at the top end of the curve.

Calculated drift velocities agree fairly well for low values of E/p though they are a bit low, as in the case of neon. At large E/p the use of 15.6 for ϵ_1 is quite inadequate to give the measured drifts, whereas $\epsilon_1 = 11.57$, the first critical potential gives excellent agreement with the measurements of Neilsen.[12] This value of ϵ_1, however, gives "temperatures" which are far too low. Townsend's measured velocities are far larger and are impossible of explanation on this theory.

In conclusion it may be said that "temperatures" and drift velocities may be calculated without any adjustable parameters and give excellent agreement with experiment for low values of E/p. The peculiarities of the "temperature" of argon are explained by its varying cross section. At large values of E/p where inelastic collisions are important their effect can in general be represented by choosing a single parameter, ϵ_1, which must lie in the range of the excitation potentials. The two measurements in argon cannot however be explained by the same parameters.

[13] J. S. Townsend and V. A. Bailey, Phil. Mag. **44**, 1033 (1922).

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Plasma Electron Drift in a Magnetic Field with a Velocity Distribution Function

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Previous calculations of the drift motion of electrons moving through a gas under the combined action of electric and magnetic fields and concentration gradient have been based upon inexact averaging methods. For electrons which, like arc plasma electrons, have a Maxwell distribution to a first approximation, the first-order correction to the distribution function which arises from drift motion is now calculated. This permits exact averaging of component velocities to be carried out. The resulting equations are compared with Townsend's earlier results. Tonks' theorem that "the drift speed is the same as would occur if the components of concentration gradient and electric field perpendicular to it, as well as the magnetic field itself did not exist" is found to hold within 12 percent.

THE detailed analysis in terms of a distribution function of the motion of electrons in a magnetic field given by W. P. Allis and H. W. Allen[1] makes it possible to replace the previously approximate expressions for the drift motion of electrons[2] in a magnetic field with exact expressions.

[1] W. P. Allis and H. W. Allen, Phys. Rev. **52**, 703 (1937).

[2] J. S. Townsend, *Electricity in Gases*, §§89–92. L. G. H. Huxley, Phil. Mag. **23**, 210 (1937).

The case to be treated is that in which the magnetic field is in the z direction. The electric field and concentration gradient components in the z direction are assumed to be zero, since motion in this direction is independent of H and takes place according to well-known laws. With Allis and Allen we assume that the distribution function f giving the number of electrons per unit volume of ξ, η, ζ, x, y, z (velocity-displacement) phase space is

$$f=f_0(v, x, y)+(\xi/v)f_1(v, x, y) +(\eta/v)f_2(v, x, y), \quad (1)$$

where $$v^2=\xi^2+\eta^2+\zeta^2.$$

This is in accordance with the reasoning of Morse, Allis, and Lamar.[3] The analysis leads to three equations between f_0, f_1 and f_2 (Allis and Allen's Eqs. (28), (29), and (30)), of which the first gives the energy balance and the second and third the momentum balance in the x and y directions respectively.

The first equation is of importance in determining f_0 from the energy transfers which involve the electrons. The equation is based on two types of interchange. The first is the work done on the electrons by the electric field and the second is the energy elastically transferred to atoms on impact. In an arc plasma, however, other transfer mechanisms operate, namely loss of energy in inelastic impacts and probably direct interchange of energy between electrons. We must, therefore, abandon this equation and in its place use for f_0 the Maxwell distribution

$$f_0=\pi^{-\frac{3}{2}}w^{-3}n\epsilon^{-v^2/w^2}, \quad (2)$$

to which experiment shows that electrons conform in many cases. Here n is the electron density and w^2 is 2/3 of the mean square velocity C^2. In terms of electron temperature T and average electron speed $\bar{c}$

$$w^2=2kT/m=(\pi/4)\bar{c}^2. \quad (3)$$

The second and third equations have to be filled out by the inclusion of a y component of electric field E_y and also by concentration gradient terms which can obviously be added on the basis of the Morse-Allis-Lamar Eq. (7). We then have

$$\begin{aligned} eE_x\partial f_0/\partial v+mv\partial f_0/\partial x-Hef_2&=-mvf_1/\lambda,\\ eE_y\partial f_0/\partial v+mv\partial f_0/\partial y+Hef_1&=-mvf_2/\lambda, \end{aligned} \quad (4)$$

from which to determine f_1 and f_2. We shall assume that λ is independent of v, although it will appear that theoretically its variation with v can be included in the treatment.

Substitution in Eqs. (4) from (2) gives

$$\begin{aligned} -\pi^{-\frac{3}{2}}w^{-3}\epsilon^{-v^2/w^2}n\lambda b_x-(v_h/v)f_2+f_1&=0,\\ -\pi^{-\frac{3}{2}}w^{-3}\epsilon^{-v^2/w^2}n\lambda b_y+(v_h/v)f_1+f_2&=0, \end{aligned} \quad (5)$$

where

$$\begin{aligned} b_x&=eE_x/(kT)-\partial \ln n/\partial x,\\ b_y&=eE_y/(kT)-\partial \ln n/\partial y, \end{aligned} \quad (6)$$

$$v_h=\lambda eH/m. \quad (7)$$

It is worth noting that the electric field and density gradient components enter Eqs. (5) through the b's only and that the b's are themselves gradients of the function

$$B=-eV/(kT)-\ln n,$$

which is directly related to the Boltzmann equation. In the absence of magnetic field $(\lambda\bar{c}/3)B$ is the velocity potential for electron drift. Its space derivatives are still fundamental in the presence of a magnetic field. That the equations develop in this way is a consequence of the fact that a magnetic field does not destroy the validity of Boltzmann's equation.[4]

It is evident that v must enter f_1 and f_2 in Eqs. (5) in the exponential form. Accordingly, we set

$$f_1=\pi^{-\frac{3}{2}}w^{-3}nA_1\epsilon^{-v^2/w^2}, \quad f_2=\pi^{-\frac{3}{2}}w^{-3}nA_2\epsilon^{-v^2/w^2} \quad (8)$$

with A_1 and A_2 to be determined by substitution in Eqs. (5). The solution of the resulting simultaneous equations gives

$$\begin{aligned} A_1&=\lambda[b_x+(v_h/v)b_y]/[1+(v_h/v)^2],\\ A_2&=\lambda[b_y-(v_h/v)b_x]/[1+(v_h/v)^2], \end{aligned} \quad (9)$$

thus completing the specification of f_1 and f_2.

The flux of electrons in the x direction is

$$n\bar{\xi}=4\pi\int_0^\infty \xi v^2 f dv=4\pi\int_0^\infty v\xi^2 f_1 dv$$

[3] P. M. Morse, W. P. Allis, and E. S. Lamar, Phys. Rev. **48**, 412 (1935).

[4] L. Tonks, Phys. Rev. **51**, 744 (1937).

and since ξ^2 can be replaced[3] by $v^2/3$,

$$n\bar{\xi}=(4\pi/3)\int_0^\infty v^3 f_1 dv.$$

In the course of the indicated integration we require the value of

$$J=\int_0^\infty \frac{\epsilon^{-px^2}}{1+x^2}dx=\pi^{\frac{1}{2}}\epsilon^p\int_{p^{\frac{1}{2}}}^\infty \epsilon^{-z^2}dz,$$

which does not appear in the tables. The value given was found by the process

$$\epsilon^p\int_\infty^p \frac{d(J\epsilon^{-p})}{dp}dp,$$

the x integration being carried out after taking the derivative with respect to p.

The result of the integration is

$$\bar{\xi}=(\lambda\bar{c}/3)(\alpha b_x+\beta b_y), \tag{10}$$

where
$$\alpha=1-h^2+h^4\epsilon^{h^2}Ei(h^2), \tag{11}$$

$$\beta=(\pi^{\frac{1}{2}}/2)h[1-2h^2+2\pi^{\frac{1}{2}}h^3\epsilon^{h^2}\text{erf }(h)], \tag{12}$$

$$h=v_h/w,\quad Ei(x)=\int_x^\infty(\epsilon^{-x}/x)dx,^5$$

$$\text{erf }(x)=2\pi^{-\frac{1}{2}}\int_x^\infty \epsilon^{-x^2}dx. \tag{13}$$

In terms of the Larmor precession, $\omega=eH/m$

$$h=\omega\lambda/w. \tag{14}$$

For the y drift

$$\bar{\eta}=(\lambda\bar{c}/3)(\alpha b_y-\beta b_x). \tag{15}$$

For small values of h

$$\alpha=1-h^2+h^4(\ln h^2+0.5772+\cdots), \tag{16}$$

$$\beta=(\pi^{\frac{1}{2}}/2)h[1-2h^2+2\pi^{\frac{1}{2}}h^3+\cdots]. \tag{17}$$

For large values of h

$$\alpha=2h^{-2}[1-3h^{-2}+12h^{-4}+\cdots]. \tag{18}$$

$$\beta=(3/4)\pi^{\frac{1}{2}}h^{-1}[1-(5/2)h^{-2}+(35/4)h^{-4}+\cdots]. \tag{19}$$

[5] This is in accord with G. A. Campbell and R. M. Foster, "Fourier Integrals for Practical Application," Bell Telephone System Monograph B-584 (1931). Jahnke-Emde, *Tables of Functions* (1933) on p. 78 denote this integral as $-Ei(-x)$.

The table gives values for α and β for a range of values of h.

h	0	0.2	0.4	1.0	2.0	5.0	10.0
α	1.0	0.964	0.882	0.595	0.300	0.0716	0.0194
β	0.0	0.1671	0.295	0.457	0.44	0.243	0.130

In the present notation Townsend's result (amended[4]) is

$$\bar{\xi}=(\lambda\bar{c}/3)\frac{b_x+(\pi^{\frac{1}{2}}/2)hb_y}{1+(\pi/4)h^2}. \tag{20}$$

For small h this gives both α and β correctly to the lowest order in each. For large h, and again to the lowest order, his α is $2/\pi$ and his β is $8/(3\pi)$ of the proper value. Thus the agreement is surprisingly good. Considering ease of mathematical manipulation Eq. (20) is probably as good a simple approximation over the whole range of h as is possible where the basic distribution is Maxwellian.

Of particular interest is the modification which may be necessary to the theorem that the motion in the direction of drift is that which would be caused by the potential and concentration gradients in that direction alone with all other components and also the magnetic field eliminated.[4]

Since any deviations will be most marked for a drift perpendicular to the magnetic field, we shall assume that the drift lies in the xy plane and shall orient the x and y axes so that the flow is in the x direction. Then $\bar{\eta}$ is zero and

$$b_y=(\beta/\alpha)b_x,$$

whence
$$\bar{\xi}=(\lambda\bar{c}/3)(\alpha+\beta^2/\alpha)b_x. \tag{21}$$

Accordingly, for small h, using Eqs. (17) and (18)

$$\bar{\xi}=(\lambda\bar{c}/3)[1-(1-\pi/4)h^2]b_x$$
$$=(\lambda\bar{c}/3)[1-0.2146h^2]b_x \tag{22}$$

and for large h, using Eqs. (19) and (20),

$$\bar{\xi}=(\lambda\bar{c}/3)(9\pi/32)[1+(64/9\pi-2)h^{-2}]b_x, \tag{23}$$
$$=0.883(\lambda\bar{c}/3)[1+0.2635h^{-2}]b_x.$$

Thus the theorem is accurate to within 12 percent over the whole range of field strength.

Note added July 11, 1937. Expressions corresponding to our α and β are given without references in Knoll, Ollendorf and Rompe, Gasentladungstabellen, Sections $f7$ and $f12$. In the former, on page 47, the ratio of transverse diffusion coefficient with magnetic field to that without magnetic field is given. This agrees, as it should, with our α. In the latter, on page 51, the ratio of drift parallel to the electric field with magnetic field to that without is given and also the ratio of transverse drift with magnetic field to parallel drift without. These should, but do not, agree with our α and β, respectively. In the absence of references it has not been possible to determine the source of the differences, but it probably lies either in the method used in averaging over the velocity distribution or in the assumptions, perhaps tacit, regarding that distribution.

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The Coupling of p Electron Configurations

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The structure and type of coupling of the observed p^2, p^3, and p^4 configurations is considered in detail in terms of the intermediate-coupling theory, in particular for the long isoelectronic sequences which have recently been analyzed. The theoretical values of the ratios of various intervals in these configurations are plotted as functions of the single parameter which specifies the type of coupling. The experimental values have been fitted to these curves and the parameters evaluated. The points fit the theoretical curves fairly well; and the departures of the experimental points of an isoelectronic sequence from these curves are sufficiently regular to enable accurate prediction of unknown levels. This method has yielded new classifications in K IV and Ca VII. For the configurations p^3 and p^3s, interval ratios are found which are predicted to be entirely independent of coupling. The parameters are plotted for the isoelectronic sequences of sp, p, p^2, p^3, p^4, p^5, and p^5s configurations. The electrostatic interaction parameter F_2 is found to be a linear function of Z to a good approximation; the spin-orbit parameter ζ_p is accurately proportional to $(Z-S)^4$ for all but the first few members of each sequence. The screening constants (S) for ζ_p are much smaller than the corresponding screening constants for F_2. A complete bibliography of data for those atoms with p electrons in the normal configuration is appended.

Introduction

Recent progress[1] in the analysis of isoelectronic spectra has now made possible a thorough comparison of the intermediate-coupling theories with experiment for those elements containing equivalent p electron groups. The manner in which the energy of the various levels is predicted to vary with the coupling is shown by Fig. 1. These diagrams, but with only a few data superposed, were given by Condon and Shortley.[2] The size of the parameter χ measures essentially the departure from Russell-Saunders coupling ($\chi=0$). For pure jj coupling $\chi=\infty$ ($1/\chi=0$). χ is defined as the ratio of the spin-orbit interaction integral ζ_p (C-S 4^54) to the electrostatic integral $5F_2$ (C-S page 177). Diagrams of this type are very compact and give a clear picture of the manner in which the relative values of the various energy intervals vary with the coupling. In actual practice, however, they are inconvenient to apply to experimental results because of the various scale factors involved and are inadequate for either extreme, $\chi \ll 1$ or $\chi \gg 1$, where certain of the intervals become very small compared to the rest. These difficulties are obviated in what follows by using ratios between directly observed energy intervals; these are calculated from theory and plotted directly against χ.

In the theory for the p^2, p^3, and p^4 configurations, the ratio of any two intervals (e.g. ${}^1S_0-{}^1D_2$ and ${}^3P_2-{}^3P_1$) is completely determined by χ alone. Thus in these configurations, if *any* three levels are known, χ may be predicted and the positions of the other levels as well. For example, a knowledge of the Landé ratio for a triplet term should predict absolutely the position of the two singlet terms arising from a p^2 or p^4

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[1] For a complete bibliography see the last section of this paper.

[2] Condon and Shortley, *Theory of Atomic Spectra* (Cambridge, 1935). For the general theory and notation used in this paper see chapters XI and XIII of this book. In what follows this reference will be denoted as C-S.