These integrals are easily evaluated as power series in $\surd\mu(Hv_0/E)^2$, where $\frac{1}{2}mv_0^2=\lambda eE$, and give, in terms of the drift velocity without the magnetic field u_E

$$u_x=u_E[1-0.195(\mu)^{\frac{1}{2}}(Hv_0/E)^2+\cdots], \quad (35)$$

$$\frac{u_y}{u_x}=\frac{3}{2(2)^{\frac{1}{2}}}\frac{Hu_E}{E}=1.06\frac{Hu_E}{E}. \quad (36)$$

It is this last formula which Townsend used to measure u_E and it is seen that the numerical factor differs insignificantly from his value 1.

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Electron Temperatures and Mobilities in the Rare Gases

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The results of the theory developed by Allis and Allen are applied to the computation of electron "temperatures" and drift velocities in the rare gases; helium, neon and argon. Curves are given showing the effect of the variable cross sections on the distribution function in the three cases. A distribution function is derived to account for energy lost by inelastic impact at higher values of E/p. This distribution function depends on an adjustable parameter, ϵ_1 which has a value between the first resonance potential and the ionization potential, since it is assumed that the number of electrons with energies above ϵ_1 is negligible. Curves are given comparing the computed values of electron "temperatures" and drift velocities with experimental values. In most cases the check is good.

1. ELASTIC COLLISIONS ONLY

THE rare gases helium, neon, and argon offer an excellent opportunity to check the results of the theory developed in the previous paper by Allis and Allen, of the diffusion and drift velocity of electrons in these gases. The angular distribution of electrons scattered from these gases has been measured down to very low velocities by Ramsauer and Kollath[1] and by Normand[2] and these measurements allow the computation of cross sections for momentum transfer Q. It is found that helium has a falling curve, neon one which is practically flat, while argon has one which rises sharply (Fig. 1). The energy distribution in these three gases will consequently vary greatly on this account as well as because of the greatly differing masses. The argon curve cuts off more sharply on the high energy side. Furthermore the drift velocity u_E and the diffusion coefficient D correspond to the averages of $(\mu/\lambda E)v^3$ and λv, respectively; $\mu=3m/M$, three times the ratio of the masses of the electron and the gas molecule, λ is the mean free path, $1/NQ$. The quantity which is given by Townsend[3] as a result of his measurements is

$$T=eED/ku_E \quad (1)$$

and would be the temperature if the electrons had a Maxwell distribution. In the above expression e is the electron charge, E the field strength and k the Boltzmann constant. Formulae (9) of Allis and Allen show that for a constant cross section T should be proportional to $E\lambda/\mu^{\frac{1}{2}}$ but when the cross section varies it is

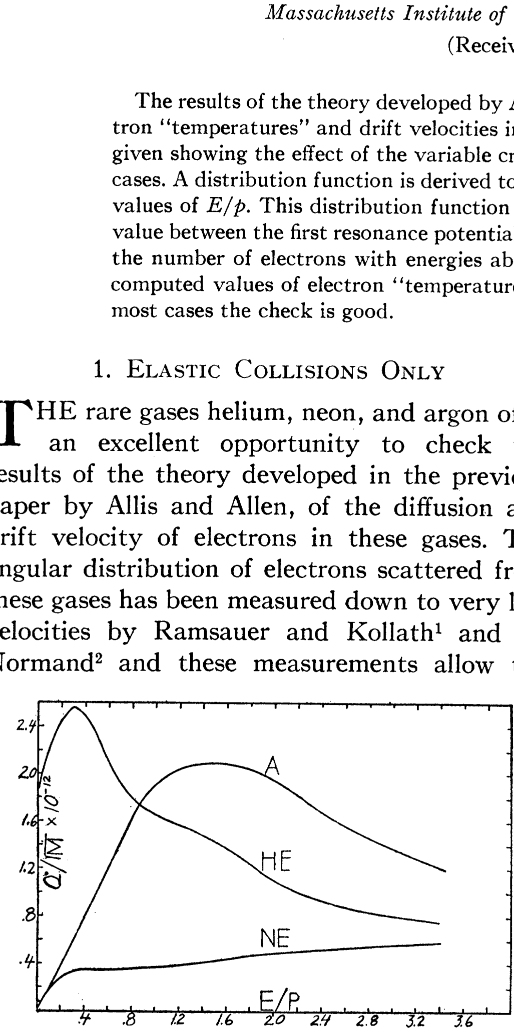

FIG. 1. Cross section in sq. A divided by the square root of the mass. This is the quantity which determines the mean energy $\bar{\epsilon}$ of the electrons.

[1] C. Ramsauer and R. Kollath, Ann. d. Physik (5) **12**, 529 (1932).
[2] C. E. Normand, Phys. Rev. **35**, 1217 (1930).
[3] J. S. Townsend, *Electricity in Gases* (Clarendon Press, Oxford, 1915).

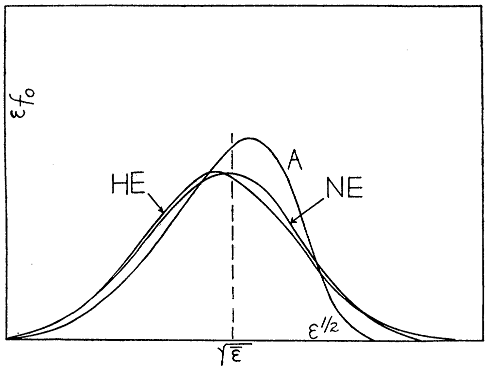

FIG. 2. Effect of variable cross section on the distribution function. The abscissae are chosen so as to give the same mean energy.

proportional to the more complicated function given by the ratio of the above averages (Fig. 2).

In the case of argon there are also probe measurements of "temperature" by Seeliger and Hirchert[4] and by Groos.[5] These are difficult to interpret since the probe measurement curves are not given. They should, if the theory is correct, be parabolic, and if the best representative straight were passed through the points at low currents where the scale is stretched the resulting "temperature" would tend to be lower than $2\bar{\epsilon}/3k$ where $\bar{\epsilon}$ is the average energy. The curve for the average energy as a function of E/p, expressed in volts per cm per mm of mercury gas pressure, is given on the figure for comparison.

The drift velocities agree fairly well with the experimental points for low E/p but are in all cases somewhat too low.

2. Inelastic Collisions

For higher values of E/p the experimental results depart markedly from the theory; "temperatures" approach a constant value and drift velocities increase more rapidly. It is evident from this that electrons with energies above the first resonance potential are undergoing inelastic collisions. This will modify the distribution function not only in the high energy range but also where collisions are purely elastic, as Smit[6] has shown. Indeed as electrons of high energy are being continually removed from the distribution and returned with very little energy, there must be an excess of energy all through the lower energy brackets. Eq. (10) of Morse, Allis and Lamar[7] representing energy unbalance can be written

$$8\pi eE\epsilon/3m^2[f_1-2\mu\epsilon f_0/\epsilon_0]=-j, \qquad (2)$$

where j measures the degree of unbalance. $-j$ represents the net number of electrons crossing a certain energy level per second, f_0 and f_1 determine the random distribution in velocity and the electron drift respectively and $\epsilon_0=\lambda eE$. Let us write F_0 for the equilibrium distribution,

$$\log F_0=-2\mu\int_0^\epsilon \frac{\epsilon}{\epsilon_0^2}d\epsilon.$$

Since $$f_1=-\epsilon_0\partial f_0/\partial\epsilon$$

the solution of Eq. (2) is

$$f_0=j\frac{3m^2}{8\pi eE}\frac{1}{F_0}\int_{\epsilon_1}^\epsilon \frac{\epsilon_0}{\epsilon}F_0 d\epsilon.$$

When the cross section is reasonably constant the integral is readily evaluated in terms of exponential integrals $\bar{E}i$ tabulated in Jahnke Emde[8]

$$f_0=B[\exp -2\mu\epsilon^2/\epsilon_0^2][\bar{E}i(2\mu\epsilon_1^2/\epsilon_0^2) \\ -\bar{E}i(2\mu\epsilon^2/\epsilon_0^2)].$$

This distribution function should hold up to the first resonance potential where it should be

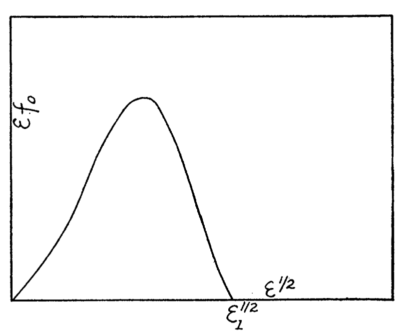

FIG. 3. Distribution with inelastic impacts. This distribution is probably correct over most of the range but should go rapidly to zero instead of cutting the axis at $(\epsilon_1)^{\frac{1}{2}}$.

[4] R. Seeliger and R. Hirchert, Ann. d. Physik (5) **11**, 817 (1931).
[5] O. Groos, Zeits. f. Physik **88**, 714 (1934).
[6] J. A. Smit, Physica **6**, 453 (1936).
[7] Morse, Allis and Lamar, Phys. Rev. **48**, 412 (1935).
[8] Jahnke and Emde, *Tables of Functions*, second edition (B. G. Teubner, Leipzig, 1923).

joined by some other function whose form will depend on the total inelastic cross section at all energies above the resonance potential. As this is an unknown quantity some simpler assumption must be made. The simplest is to keep the above distribution function up to the point where it crosses the axis for $\epsilon=\epsilon_1$ leaving ϵ_1 as an adjustable parameter which should, however, fall between the first resonance potential and the ionization potential. There is some justification for assuming that there are practically no electrons above a certain energy in the measurements of Kelly[9] who finds that in helium the distribution follows a smooth curve attaining the axis at about the second resonance potential (Fig. 3).

Another point should be mentioned. f_0 has a logarithmic infinity at the origin. This is because the electrons suffering inelastic collisions have been assumed to lose all their energy and therefore tend to accumulate somewhat at the origin. Better assumptions might be made but they would not change the drift velocities or "temperatures" as the slow electrons do not contribute appreciably to these quantities.

3. Results

Helium

The "temperature" curve starts with a slight positive curvature corresponding to a falling cross-section curve, but breaks away from this trend at $E/p=2$. The "temperatures" are always most sensitive to inelastic impacts and so were used to fix the parameter ϵ_1. In this case setting it at the first excitation potential, 19.7 volts, gives a calculated curve agreeing exactly with the data of Townsend and Bailey.[10]

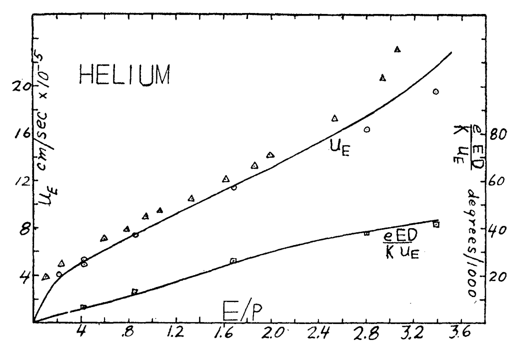

FIG. 4. Electron "temperatures" and drift velocities in helium. The experimental points □ are "temperatures" obtained by the Townsend method. The experimental points ⊙ and △ are drift velocities obtained by the Townsend method and by Neilsen, respectively. The same symbols are used in Figs. 5 and 6. The full line curves are computed using the distribution function corrected for ionization.

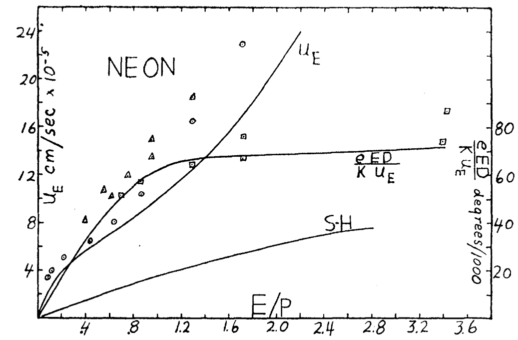

FIG. 5. Electron "temperatures" and drift velocities in neon. The curve marked *S–H* gives "temperatures" derived from the probe measurements of Seeliger and Hirchert. The theoretical curves were obtained using a constant cross section $Q=2.5$ sq. *A*.

The drift velocity curve starts as a parabola but soon curves upward since drift velocities vary as $\lambda^{\frac{1}{2}}$ and λ is increasing. The calculated curve is the same whether ϵ_1 is taken at 19.7 volts or at infinity. Inelastic collisions increase the drift velocity through decreasing the mean energy $\bar{\epsilon}$. In helium this also increases ϵ_0 thus tending to decrease the drift. The two effects seem to compensate exactly (Fig. 4).

Neon

The formulae for a constant cross section may here be used and give a straight line for the "temperature" and a parabola for the drift. The former agrees exactly with the results of the Townsend method as applied by Bailey[11] and the latter fairly well with Bailey[11] and with Neilsen[12] up to about $E/p=0.6$. Putting ϵ_1 at the ionization potential, 21.5 volts, brings exact agreement for the "temperatures" whereas the drift velocities would seem to require a lower value. The "temperatures" from the probe measurements of Seeliger and Hirchert[4] are also shown on the figure although they seem to have no relation to the other values (Fig. 5).

[9] H. C. Kelly, Doctor's Thesis, Massachusetts Institute of Technology, June 1936.

[10] J. S. Townsend and V. A. Bailey, Phil. Mag. **46**, 657 (1923).

[11] V. A. Bailey, Phil. Mag. **47**, 379 (1924).

[12] R. A. Neilsen, Phys. Rev. **50**, 950 (1936).

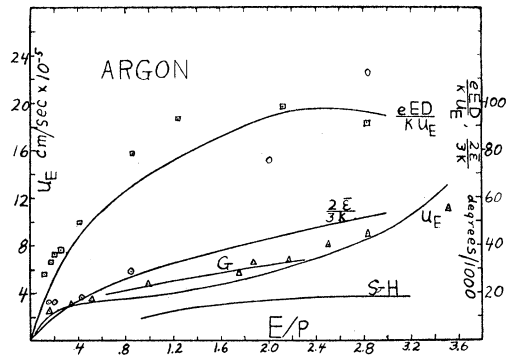

FIG. 6. Electron "temperatures" and drift velocities in argon. The curves marked *S–H* and *G* are "temperatures" derived from the probe measurements of Seeliger and Hirchert and of Groos, respectively.

Argon

Argon has the greatest Ramsauer effect of any gas. The result is that its "temperature" defined by formula (1) instead of being very nearly equal to $2\bar{\epsilon}/3k$ as it is when the cross section is constant, is very much greater. The two quantities are shown in Fig. 6 and it is seen that Townsend's[13] measurements agree fairly well with formula (1). On the other hand the probe measurements of Groos[5] and of Seeliger and Hirchert[4] are of the order of $2\bar{\epsilon}/3k$. Using $\epsilon_1 = 15.6$, the ionization potential, gives agreement at the top end of the curve.

Calculated drift velocities agree fairly well for low values of E/p though they are a bit low, as in the case of neon. At large E/p the use of 15.6 for ϵ_1 is quite inadequate to give the measured drifts, whereas $\epsilon_1 = 11.57$, the first critical potential gives excellent agreement with the measurements of Neilsen.[12] This value of ϵ_1, however, gives "temperatures" which are far too low. Townsend's measured velocities are far larger and are impossible of explanation on this theory.

In conclusion it may be said that "temperatures" and drift velocities may be calculated without any adjustable parameters and give excellent agreement with experiment for low values of E/p. The peculiarities of the "temperature" of argon are explained by its varying cross section. At large values of E/p where inelastic collisions are important their effect can in general be represented by choosing a single parameter, ϵ_1, which must lie in the range of the excitation potentials. The two measurements in argon cannot however be explained by the same parameters.

[13] J. S. Townsend and V. A. Bailey, Phil. Mag. **44**, 1033 (1922).

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Plasma Electron Drift in a Magnetic Field with a Velocity Distribution Function

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Previous calculations of the drift motion of electrons moving through a gas under the combined action of electric and magnetic fields and concentration gradient have been based upon inexact averaging methods. For electrons which, like arc plasma electrons, have a Maxwell distribution to a first approximation, the first-order correction to the distribution function which arises from drift motion is now calculated. This permits exact averaging of component velocities to be carried out. The resulting equations are compared with Townsend's earlier results. Tonks' theorem that "the drift speed is the same as would occur if the components of concentration gradient and electric field perpendicular to it, as well as the magnetic field itself did not exist" is found to hold within 12 percent.

THE detailed analysis in terms of a distribution function of the motion of electrons in a magnetic field given by W. P. Allis and H. W. Allen[1] makes it possible to replace the previously approximate expressions for the drift motion of electrons[2] in a magnetic field with exact expressions.

[1] W. P. Allis and H. W. Allen, Phys. Rev. **52**, 703 (1937).

[2] J. S. Townsend, *Electricity in Gases*, §§89–92. L. G. H. Huxley, Phil. Mag. **23**, 210 (1937).