TABLE II. *Percent of tracks transmitted through lead and carbon as a function of incident energy.*

	2 to 4 Mev	4 to 6 Mev	6 to 8 Mev	8 to 11 Mev
Lead	20	40	58	75
Carbon	36	64		

trons, the effect is in the direction of causing discrimination against those electrons which have suffered large energy losses. This would mean that the true values for the losses are even greater than those obtained.

Comparisons of the numbers of tracks incident upon the absorber with the numbers which pass through and are measurable on the opposite side have been made, by using about 1000 tracks, and the percent transmitted as a function of energy is given in Table II. This tends to indicate that scattering is large (unless some new mechanism is postulated to account for the stopping), and shows that the results would be radically changed if those electrons which appear to be stopped completely were included in the data.

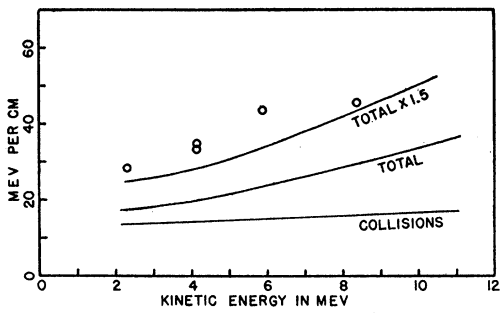

FIG. 7. Comparison of experimental points with theoretical curves, in terms of differential energy loss, Mev per cm of lead. The highest curve is that obtained by using 1.5 for the path length-thickness ratio.

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Energy of Lattice Distortion in Cold Worked Permalloy

F. E. HAWORTH
Bell Telephone Laboratories, New York, New York

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The lattice distortion produced by severe cold working of permalloy of 70 percent Ni content has been studied by measuring the broadening of the reflection of the Fe $K\alpha$ doublet by the (311) planes, with a focusing camera. The broadening decreases upon annealing and recovery is complete at 650°C, when the breadth of the x-ray intensity curve at half-maximum is as small as that obtained by use of the two-crystal spectrometer. The mean square distortion in the lattice spacing due to cold work is derived from the x-ray measurements after photometering the x-ray film, converting the curve into an x-ray intensity curve, fitting the latter with an empirical equation and using an analysis worked out by S. O. Rice. The energy of the distortion is then calculated by using an equation derived by G. R. Stibitz. The root-mean-square distortion was found to be 0.31 percent of the lattice spacing after the material had been reduced 96 percent in cross-sectional area by cold working. The energy of distortion in the hard worked condition is thus found to be 23×10^6 ergs/cm^3 or 0.065 calorie/gram.

ONE of the interesting problems in the physics of metals is to explain the increase in internal energy which takes place when a metal is severely cold worked. This increase has been measured by Taylor and Quinney[1] in copper and steel, and Cagliotti and Sachs[2] have shown on the basis of x-ray measurements on copper that the lattice strain as measured by the broadening of the x-ray lines does not nearly account for the energy increase.

The purpose of this work is to study the internal strain in more detail, using a material on which parallel measurements[3] of electrical

[1] G. I. Taylor and H. Quinney, Proc. Roy. Soc. London **A143**, 307 (1934).
[2] V. Cagliotti and G. Sachs, Zeits. f. Physik **74**, 647 (1932).
[3] J. F. Dillinger, Phys. Rev. **49**, 863 (1936).

resistivity, mechanical hardness, and various magnetic properties have been made for various reductions in area by hard rolling or drawing and for various times and temperatures of annealing after such reduction. The state of strain is

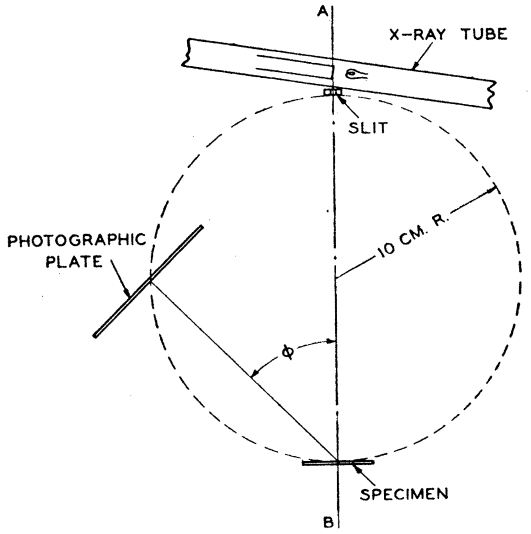


FIG. 1. Arrangement of apparatus for focusing and photographing the x-ray reflections.

derived in a new way from the broadening of x-ray diffraction maxima.

The derivation of the internal energy increase proceeds in four steps, as follows: (1) measurement of the distribution of reflection angles by photographic densitometry; (2) separation of overlapping α_1 and α_2 components of the K radiation by an empirical method; (3) derivation of the strain distribution from the distribution of α_1 radiation so obtained; (4) computation of the increase in potential energy density from the derived strain distribution.

The material chosen was an alloy containing 70 percent nickel and 30 percent iron. Bozorth and Dillinger[4] obtained a maximum permeability of over 500,000 for this alloy by heat treatment in a magnetic field, and measurements had been made of the properties listed above. Two series of specimens were prepared from this material, which was received from the manufacturer (Driver-Harris) in quarter-inch rods. For a study of the recovery of the material from cold rolling, rods were rolled to tape 0.006 inch (0.015 cm) thick—this involving a reduction in cross-sectional area of 96 percent—and then annealed in hydrogen for two hours at selected temperatures up to 1090°C. In the other series, to test for a possible grain size effect, specimens were annealed at a temperature of 1000°C at various stages in the rolling schedule, but all were finally cold rolled to tape of the standard thickness, thus giving various degrees of reduction since the last anneal, in otherwise similar pieces.

[4] Richard M. Bozorth and Joy F. Dillinger, Physics **6**, 279–291 (1935).

APPARATUS

The apparatus used for photographing the x-ray reflections is shown schematically in Fig. 1. The x-ray source is a Philips Metalix tube with an iron target, and the slit, specimen, and plate are arranged on the circumference of a circle to focus the (311) reflections of the $K\alpha$ doublet sharply on the plate. The slit was 0.1 mm wide and 5 mm high, made in a quarter-inch brass plate. The specimens were mounted with a flat surface perpendicular to the axis AB and were rotated about this axis in order to produce smooth lines. The angular divergence of the beam was such that at the specimen it was a millimeter wide. The reflection angle θ, was about 64.5°, making $\varphi = 51°$. The photographic plate was placed at right angles to the reflection to eliminate any broadening due to emulsion thickness. To obtain the density distribution of the reflections a Moll microphotometer was used with its optical system so arranged that the area scanned on the plate was 0.014 mm by 1.0 mm. It was difficult to obtain good microphotometer curves with x-ray plates on account of their large grain size and Eastman 33 plates were used instead.

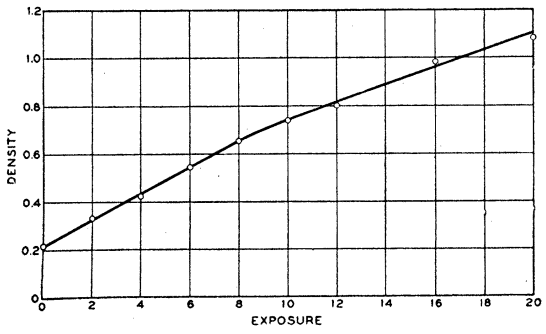
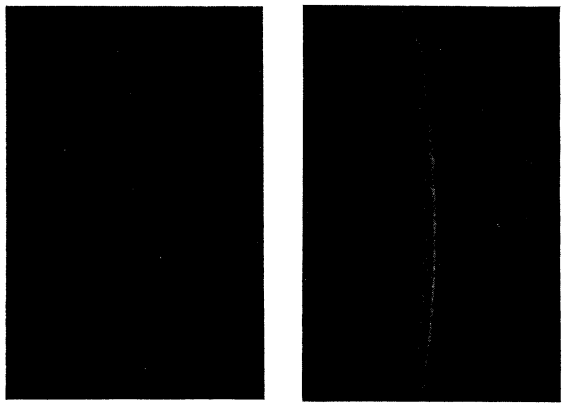


FIG. 2. Curve of exposure *vs.* density for the photographic plates used (Eastman 33).



(a) (b)

FIG. 3. Reflection of the Fe $K\alpha_1$, α_2 lines by the (311) planes: (a) in the hard rolled tape; and (b) in the tape annealed at 650°C. The exposure time for (a) was twenty times that for (b).

For accurate measurements of the lattice spacing a symmetrical focusing camera of the Seeman-Bohlin type was used. The distances between lines on the film were measured with a comparator to about 0.01 mm, the calculated spacings plotted against $\varphi \tan \varphi/2$, and the resultant curve extrapolated to $\varphi=0$. Repeated measurements indicate that spacings determined in this manner are accurate to within 0.01 percent.

MEASUREMENT OF THE LINE BROADENING

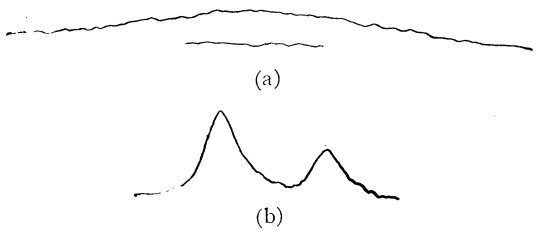
Photographs of the (311) reflections from the proposed specimens were made using the arrangement as described above. Exposure times varied from one to twenty hours and care was taken that the maximum density on the plate did not exceed that at the knee of the exposure-density curve. This curve had been determined previously and is shown in Fig. 2. Figs. 3a and b are reproductions of the $K\alpha$ doublet from the specimens hard rolled from a $\frac{1}{4}$ inch rod to 0.006 inch tape and from the same tape annealed at 650°C, respectively. The microphotometer records for these photographs are shown in Figs. 4a and b. From the relation between plate density and transmission of light it is easily shown that if δ_B is the microphotometer deflection for the background, δ_M that for maximum density, and $\delta_{\frac{1}{2}}$ that for half-maximum then

$$\delta_{\frac{1}{2}}=(\delta_B\delta_M)^{\frac{1}{2}},$$

and if all three points lie on the linear part of the exposure-density curve, this will also be the deflection for the half-maximum x-ray intensity.

The curve of the specimen annealed at 650°C, Fig. 4b, shows the $K\alpha$ doublet well resolved. Comparing this with published curves obtained by the two crystal spectrometer it not only shows as good resolution, but has the same asymmetry and breadth at half-maximum intensity. Correcting for the effect of the finite slit width the breadth of the $K\alpha_1$ line is here 1.06 X.U., to be compared with Allison's value of 1.00 X.U. and Parratt's of 1.02 X.U.[5] From this it appears that after annealing at 650°C the mean lattice distortion has fallen to zero, as nearly as can be detected. A mean distortion of 0.01 percent of the lattice spacing would increase the breadth of the $K\alpha_1$ line to about 1.25 X.U. and hence could readily be observed. Fig. 5 shows the breadths at half-maximum of the whole series as a function of annealing temperature.

Since these data were obtained from reflections by (311) planes only in the surface of the specimen, there was some question as to whether they were representative of the whole specimen. Measurements were therefore made of the broadening of the (222) reflections, and of the broadening for (311) planes after etching the specimen



(a)

(b)

FIG. 4. Microphotometer curves of the reflections shown in Fig. 3: (a) for the hard rolled tape; and (b) for the tape annealed at 650°C.

to about one-third of its thickness. In both cases the broadening, corrected for resolving power, was the same as previously measured.

To see how much of the observed broadening is due to strain it is necessary to estimate the broadening due to grain size. Wood[6] concludes

[5] Compton and Allison, *X-Rays In Theory and Experiment*, p. 745 (D. Van Nostrand Co., 1935).

[6] W. A. Wood, Phil. Mag. **20**, 964 (1935) and Phil. Mag. **15**, 555 (1933).

from comparisons of hard drawn and electrodeposited metals that line broadening in the former is to be attributed to lattice distortion. If the broadening is measured for various reductions by cold working he found that it quickly reached a maximum and then changed very little. This finding agrees with the present measurements on the series with varying amounts of reduction after annealing as shown in Fig. 6. The greatest breadth is very much less than that for particles which are known to be less than 10^{-5} cm in diameter such as occur in colloids and electrodeposits, and hence the indication is that hard working has not reduced the crystal size sufficiently to account for the broadening observed. An actual estimate of the size of the crystals can be made from the photomicrographs of rolled permalloy published by F. F. Lucas.[7] In these the slip planes are shown to be from 3 to 5×10^{-5} cm apart, and it is reasonable to suppose that the crystal fragments extend from one slip plane to the next. Substituting in Scherrer's formula for line broadening shows that (311) planes in crystals of this size would have a line broadening at half-maximum of about 5×10^{-4} radian, approximately one-tenth the total broadening observed. Thus all available evidence indicates that lattice distortion, and not crystal size, is the primary cause of line broadening produced by cold working, though not necessarily responsible for all of it.

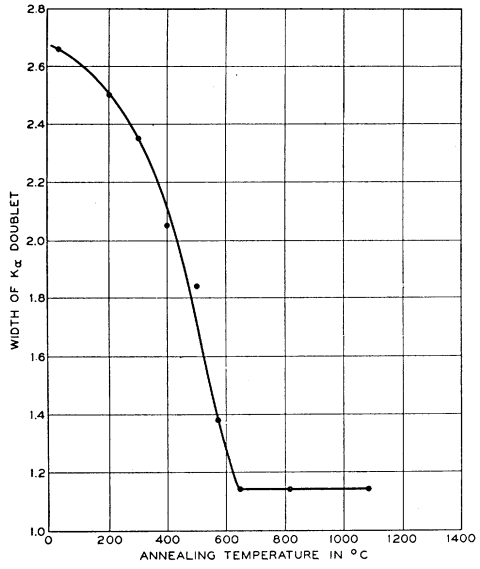


FIG. 5. Breadth at half-maximum x-ray intensity of the reflected $K\alpha$ doublet after annealing for two hours at the indicated temperatures.

[7] F. F. Lucas, J. Frank. Inst. **201**, 177 (1926).

SEPARATION OF THE α-COMPONENTS

Assuming that the substance is elastically isotropic and that the stresses are independent and oriented at random with zero mean values, G. R. Stibitz has derived the following expression for the potential energy density V of the distortion,

$$V=3E[(\Delta d^2)_{Av}/d^2]/[2(1+2v^2)]. \quad (1)$$

E is Young's modulus, v Poisson's ratio, and $(\Delta d^2)_{Av}$ the mean square increment of the plane spacing d. The derivation of this equation is given in Appendix A. An evaluation of the last term requires a knowledge of the distribution curve of Δd. To obtain this the first step was to fit an empirical equation to the observed intensity curves. The microphotometer curves were first reduced to curves of x-ray intensity *vs.* angle of reflection, and then various distribution curves fitted to them. As has been shown by others the normal Gauss distribution falls off too rapidly at the base, and Hoyt's[8] curve, of the form $y=a/(1+bx^2)$, does not fit at the base of the curve, and in addition yields an infinite value for $(x^2)_{Av}$. As has been pointed out by R. C. Spencer[9] if a fourth power term is added to Hoyt's equation a much better fit is obtainable and the following expression was found to fit the curves for both the hard worked and annealed specimens:

$$I=\frac{A}{1+0.828(x/B)^2+0.172(x/B)^4}+\frac{A/2}{1+0.828[(x-x_0)^2/B]+0.172[(x-x_0)^4/B]}, \quad (2)$$

where the origin is chosen as shown in Fig. 7, and x_0 is the separation of the α_1 and α_2 components, one unit in x corresponding to $\Delta\theta=10^{-3}$ radian. This equation has only two adjustable constants, A being the maximum ordinate of the α_1 component and B being the half-breadth of

[8] A. Hoyt, Phys. Rev. **40**, 477 (1932).
[9] R. C. Spencer, Phys. Rev. **46**, 1108 (1934).

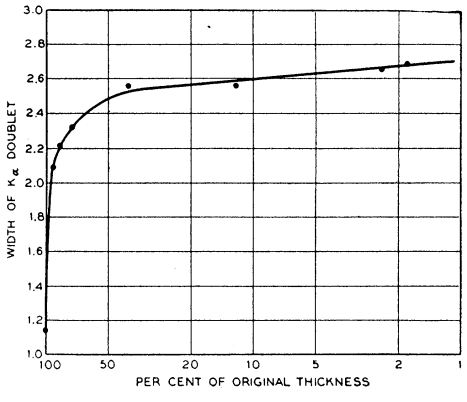


FIG. 6. Breadth at half-maximum x-ray intensity of the reflected $K\alpha$ doublet for various amounts of reduction.

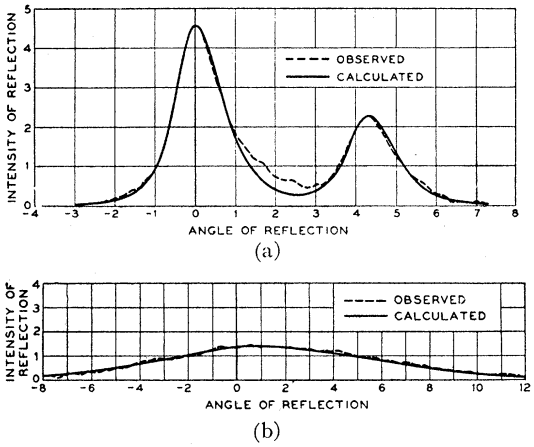


FIG. 7. Comparison of calculated and observed x-ray intensities for the $K\alpha$ doublet reflected (a) from the annealed specimen, and (b) from the hard rolled specimen. The calculated curves are defined by Eq. (2).

either single component at half-maximum intensity. On account of the asymmetry of the reflection from the annealed specimen, two values of B had to be used, and Fig. 7a shows the accurate fit obtained. For the right side of the components B is 0.8, and for the left it is 0.6. Fig. 7b shows the equation fitted to the curve for the unannealed specimen with $B=4.3$. No asymmetry was here assumed.

Determination of the Strain Distribution and Calculation of the Energy

Since the intensity curve for the annealed specimen shows no evidence of lattice distortion the assumptions are made that the equation fitted to it represents the distribution of $\Delta\theta$ caused by the variation $\Delta\lambda$ in the wave-lengths of the α components and that the equation fitted to any intensity curve in which distortion is present represents a distribution of $\Delta\theta$ caused by $\Delta\lambda$ and Δd. The next problem is to evaluate the distribution for Δd from the distribution for the combination of $\Delta\lambda$ and Δd. This involves the solution of an integral equation, solved by S. O. Rice, as given in Appendix B, where functions and symbols used in the process are defined. The function $f_d(u)$, which gives the distribution of Δd, has been evaluated by graphical integration for the values of M and m equal to 6.0 and 1.0, respectively, the former being about the value obtained from the unannealed specimen, and the latter being that from the annealed specimen. The resulting distribution curve of Δd is shown in Fig. 8.

The mean square values of the functions $F(\theta)$, $f_\lambda(\theta)$, and $f_d(u)$ are M^2, m^2, and M^2-m^2, respectively, and from Eqs. (2) and (2a) of Appendix B, and Eq. (2) given above, it follows that $M=1.55B$, giving $M^2-m^2=(2.4B^2-1)$ for B in thousandths of a radian. From the camera dimensions 10^{-3} radian corresponds to a change in d of 0.048 percent. Therefore

$$(\Delta d^2)_{Av}/d^2=0.23(2.4B^2-1)10^{-6}.$$

The values of $(\Delta d^2)_{Av}/d^2$ have been calculated for the specimens from which Fig. 5 was plotted. For the hard worked specimen, the root mean square strain is 0.3 percent. It is interesting that this is about the value obtained for the maximum strain in iron or nickel if the breaking stress of the hard worked metal is divided by Young's modulus.

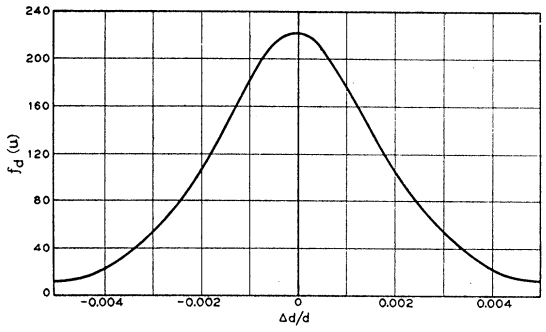


FIG. 8. Derived distribution of the distortion in the hard rolled specimen. Here Δd is the change in the lattice spacing d.

The stored specific energy[9a] of the distortion, obtained by substituting $(\Delta d^2)_{Av}/d^2$ in Stibitz's equation, is shown in Fig. 9.

Discussion of the Results

It is of interest to compare these energy densities with others reported in the literature. Taylor and Quinney[1] measured calorimetrically energies stores in the hard working of wires by twisting, and obtained maximum values of 1.15 cal./g for copper and 1.27 cal./g for steel. W. A. Wood,[10] measuring a shift in lattice parameter of copper on cold working calculated energies of the same order of magnitude but C. G. Maier[11] has stated that the assumptions on which those calculations were based are doubtful. Wood obtained a shift of the (420) reflection in cold worked copper corresponding to $\Delta d/d = 2.8\times10^{-4}$ at the peak of the line, 16.7×10^{-4} at the base. For cold rolled α-brass he calculated from the

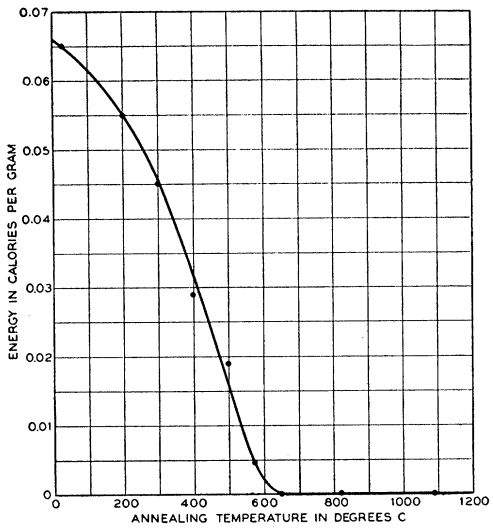


FIG. 9. The latent energy of distortion which produced the x-ray line broadening for the tape which had been hard rolled and then annealed at the temperatures indicated.

line broadening for (331), $\Delta d/d = 3\times10^{-3}$, and from that for (420), $\Delta d/d = 6\times10^{-3}$. These values are all of the same order of magnitude as the distortions derived from the present measurements or are less, and according to the analysis here offered indicate energy changes not greater than those reported here.

In studies of the nature of cold working of copper and iron, Maier[11] made some calculations of the potential energy due to observed changes of density. These energies are of the same order of magnitude as the ones obtained here, but since a density change would affect x-ray reflections by shifting the reflected line instead of merely broadening it, the effect he measures is of a different nature, and the agreement has no significance. To see if density changes occurred in the regions where $\Delta d/d$ was here estimated, the lattice constants of the annealed and cold worked specimens were measured by use of the symmetrical focusing camera and no difference was found, though a change as little as 0.02 percent should have been detectable. Maier's theory, however, would permit density changes by the formation at crystal boundaries of regions of higher than average density, the material in such regions having no definite crystal structure, and hence not affecting a measurement of average lattice spacing.

Brindley and Spiers[12] in measurements of the atomic scattering of copper and nickel powders found it to be lowered by lattice distortions resulting from filing. Assuming an exponential factor similar to that used in calculating atomic displacements due to heat motions they calculated $(\Delta d^2)_{Av}^{\frac{1}{2}} = 0.106\times10^{-8}$ cm for filed copper, 0.083×10^{-8} cm for nickel. From these values they calculated the thermal energy necessary to give a corresponding change in lattice dimensions and obtained 7.4 cal./g for copper and 6.4 cal./g for Ni. Boas, in a recent paper,[13] considers these too high by a factor of about thirty, and sets the upper limit for the latent energy for copper at about 0.23 cal./g, as measured by Rosenhain and Stott.[14] From this he concludes that the cause of the intensity changes must be

[9a] A sample of the material was cold rolled to 0.035 inch (0.089 cm) and sent to Professor P. W. Bridgman, who kindly reduced and sheared it to approximately 0.006 inch (0.015 cm) under tremendous pressure. Measurement of the line breadth at half-maximum x-ray intensity of the reflection from the (311) planes of this specimen gave a value of 3.5 units on the same scale as in Fig. 5. Provided there is no grain-size effect, this broadening corresponds to a root-mean-square distortion of 0.48 percent and an energy of 0.17 cal./g.

[10] W. A. Wood, Phil. Mag. **18**, 495 (1934).

[11] C. G. Maier, A. I. M. M. Technical Publication No. 701 (1936).

[12] G. W. Brindley and F. W. Spiers, Phil. Mag. **20**, 882 (1935).

[13] W. Boas, Zeits. f. Krist. **96**, 214 (1937).

[14] Rosenhain and Stott, Proc. Roy. Soc. London **140**, 9 (1933).

looked for elsewhere, ignoring the value of 1.15 cal./g for copper obtained by Taylor and Quinney as mentioned above. If the atomic displacements calculated by Brindley and Spiers are substituted in Stibitz's equation the latent energy is 3.0 cal./g for copper and 3.7 cal./g for nickel, values which differ from Taylor and Quinney's results by a factor of only three. From this it is evident that a great amount of the energy of cold rolling may be absorbed by lattice distortions extending over volumes too small to cause broadening of the reflected x-ray lines. To measure this in cold rolled materials it would be necessary to compare the integrated intensities of the reflections from the unannealed and annealed specimens, but unfortunately in removing the distortions by annealing the specimen recrystallizes with a change of special orientation, so that the usual intensity comparisons become meaningless. Nevertheless, experiments are being continued in hope of finding a suitable heat treatment which relieves strains without affecting crystal orientation or grain size.

Appendix

A. Energy of lattice distortion[15]

Assuming that the substance is elastically isotropic, and that the magnitudes of the principal stresses at a point are independent of each other and of the crystallographic indices of the principal directions, we choose the coordinate axes in the directions of the principal stresses, which then are F_x, F_y and F_z. It is necessary to find the elongation of the material in the direction of the normal to the reflecting planes.

The elongation in the x-direction in the neighborhood of the given point when $F_y=F_z=0$ is F_x/E, where E is Young's modulus. This is accompanied by negative elongations in the y and z directions of amount $-\nu F_x/E$, where ν is Poisson's ratio. Similar effects follow when $F_x=F_z=0$ and $F_y\neq 0$, and likewise when $F_z\neq 0$. Summing them we have the total elongations

$$A_x=(F_x-\nu F_y-\nu F_z)/E,$$
$$A_y=(F_y-\nu F_z-\nu F_x)/E,$$
$$A_z=(F_z-\nu F_x-\nu F_y)/E.$$

Next consider the change in the distance between parallel planes, the normal N of which makes an angle ψ with the z-axis in a plane zN at an angle θ with the x-z plane. The elongation in the direction of N is the sum of the projections on N of the elongations in the x, y, z-directions. It is easily seen that the angles $[xN]$, $[yN]$, $[zN]$ are given by

$$\cos[zN]=\cos\psi,$$
$$\cos[xN]=\sin\psi\cos\theta,$$
$$\cos[yN]=\sin\psi\sin\theta.$$

The required elongation is

$$\begin{aligned}\Delta d/d&=A_x\cos[xN]+A_y\cos[yN]+A_z\cos[zN]\\&=(1/E)[(F_x-\nu F_y-\nu F_z)\sin\psi\cos\theta\\&\quad+(F_y-\nu F_z-\nu F_x)\sin\psi\sin\theta\\&\quad+(F_z-\nu F_x-\nu F_y)\cos\psi].\end{aligned}$$

Now let a beam of x-rays fall normally on the surface of the material, the reflected rays being received at a specified angular distance from the normal. In general, all such reflected rays arise from crystal planes of which the normals make a specified angle ω with the surface normal. The small deviations of the reflected rays from various points on the material will be proportional to the values of $\Delta d/d$ for those points, and will thus depend on ψ, θ, F_x, F_y, F_z. The intensity of the reflected ray corresponding to a certain value D of $\Delta d/d$ is proportional to the number of elementary areas for which ψ, θ, F_x, F_y, F_z are such that $\Delta d/d=D$. In particular if we wish the root mean square value of $\Delta d/d$, we multiply $(\Delta d/d)^2$ for each value of ψ, θ, F_x, F_y, F_z by the distributions of ψ, θ, F_x, F_y, F_z and average the product. Averaging with respect to θ, which is unrestricted, we have

$$\begin{aligned}\text{av}\,(\Delta d/d)^2=(1/E^2)\,\text{av}[&\tfrac{1}{2}(F_x-\nu F_y-\nu F_z)^2\sin^2\psi\\&+\tfrac{1}{2}(F_y-\nu F_z-\nu F_x)^2\sin^2\psi\\&+(F_z-\nu F_x-\nu F_y)^2\cos^2\psi].\end{aligned}$$

The assumptions regarding F_x, F_y and F_x imply that, for average values $F_x^2=F_y^2=F_z^2=F^2$, $F_xF_y=F_yF_z=F_zF_x=0$, in which case the average is independent of ψ, and

$$[(\Delta d)_{\text{Av}}/d]^2=[(F^2)_{\text{Av}}/E^2](1+2\nu^2).$$

It may be emphasized at this point that the median value of $\Delta d/d$ could not be obtained without an additional assumption giving the form of the distribution functions for the F's, whereas the present computation required only qualitative hypotheses about that function.

The potential energy V, stored in the metal by the residual stresses is easily obtained as

$$V=(1/2E)(F_x^2+F_y^2+F_z^2)-(\nu/E)(F_xF_y+F_yF_z+F_zF_x).$$

Averaging for the type of distribution assumed for F_x, F_y and F_z, we have

$$V=3(F^2)_{\text{Av}}/2E.$$

From this and the equation for strain it follows that

$$V=\frac{3E}{2(1+2\nu^2)}(\Delta d/d)^2_{\text{Av}}.$$

B. Analysis of line broadening due to strain[16]

By solving the equation

$$F(\theta)=\int_{-\infty}^{+\infty}f_\lambda(\theta-u)f_d(u)du \qquad (1)$$

[15] This analysis was supplied by G. R. Stibitz. It is abstracted in Phys. Rev. **49**, 862 (1936).

[16] This analysis was supplied by S. O. Rice. It is abstracted in Phys. Rev. **49**, 862 (1936).

for the Δd distribution $f_d(u)$, when the $\Delta\lambda$ distribution $f_\lambda(\theta)$, and the combined distribution $F(\theta)$ for a single α component are given by the expressions

$$f_\lambda(\theta)=q/(1+r\theta^2+s\theta^4) \quad \text{and} \quad F(\theta)=Q/(1+R\theta^2+S\theta^4) \tag{2}$$

Rice[16] derives a series giving $f_d(u)$. This series proved to be difficult to handle with the values of the constants derived from the experimental curves, and consequently one of the preceding integral equations was evaluated graphically. The following derivation has been provided by Rice.

Equation (1) may be solved by using the multiplication theorem for Fourier integrals.[17] The result is

$$f_d(u)=\frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{iug}\frac{S_1(g)}{S_2(g)}dg, \tag{3}$$

where

$$S_1(g)=\frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{-i\theta g}F(\theta)d\theta, \tag{4}$$

$$S_2(g)=\frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{-i\theta g}f_\lambda(\theta)d\theta. \tag{5}$$

Since Eqs. (2) are distribution functions, Q, R, and S are not independent, and neither are q, r, and s. By using the necessary relations between these coefficients (2) may be written

$$F(\theta)=\frac{2M(M^2-N^2)/\pi}{\theta^4+2(M^2+N^2)\theta^2+(M^2-N^2)^2} \tag{2a}$$

and

$$f_\lambda(\theta)=\frac{2m(m^2-n^2)/\pi}{\theta^4+2(m^2+n^2)\theta^2+(m^2-n^2)^2}. \tag{2b}$$

It is now convenient to express $F(\theta)$ in the form

$$F(\theta)=\frac{M^2-N^2}{2\pi N}\left[\frac{1}{\theta^2+(M-n)^2}-\frac{1}{\theta^2+(M+N)^2}\right],$$

which may be readily verified. Then upon using the result

$$\int_{-\infty}^{+\infty}\frac{e^{-i\theta g}}{\theta^2+\alpha^2}d\theta=\pi/\alpha\, e^{-|g|\alpha} \qquad \alpha>0$$

we have from (4)

$$S_1(g)=\frac{M^2-N^2}{4\pi N}\left[\frac{e^{-|g|(M-N)}}{M-N}-\frac{e^{-|g|(M+N)}}{M+N}\right]$$
$$=\frac{M+N}{4\pi N}e^{-|g|(M-N)}[1-Ze^{-2|g|N}],$$

where $Z=(M-N)/(M+N)$.

In a similar manner, writing $z=(m-n)/(m+n)$, we find from (5)

$$S_2(g)=\frac{m+n}{4\pi n}e^{-|g|(m-n)}[1-ze^{-2|g|n}].$$

When these values of $S_1(g)$ and $S_2(g)$ are inserted in Eq. (3) we have

$$f_d(u)=\frac{1}{2\pi}\frac{M+N}{m+n}\frac{n}{N}\int_{-\infty}^{+\infty}dge^{iug}\left[\frac{1-Ze^{-2|g|N}}{1-ze^{-2|g|n}}\right]e^{-|g|p}$$

or
$$f_d(u)=\frac{1}{\pi}\frac{M+N}{m+n}\frac{n}{N}\int_0^\infty dg\cos ug\frac{1-Ze^{-2gN}}{1-ze^{-2gn}}(e^{-gp}), \tag{6}$$

where $p=M-N-m+n$, which is assumed positive so that the integral converges.

This may be further simplified if we let N and n equal zero, which will be true if $R^2=4S$ and $r^2=4s$. This adjustment of the constants was made in the equations for the experimental curve given above. Therefore evaluating (6) for the limit as N and n approach zero we have:

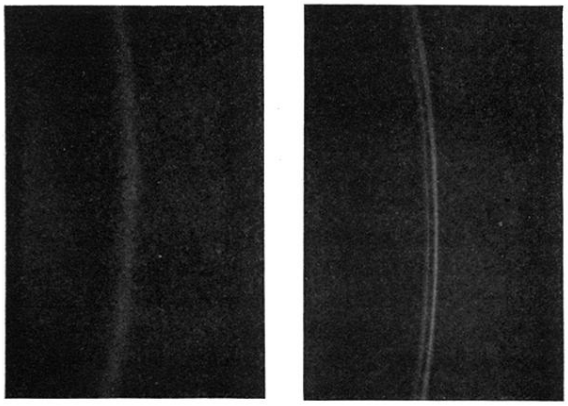
$$f_d(u)=\frac{1}{\pi}\int_0^\infty dg\cos uge^{-g(M-m)}\frac{1+Mg}{1+mg}$$
$$=-\frac{M-m}{\pi m}\int_0^\infty\frac{e^{-g(M-n)}\cos ugdg}{1+mg}$$
$$+\frac{M}{\pi m}\int_0^\infty e^{-g(M-m)}\cos ugdg \tag{7}$$
$$=-\frac{M-m}{\pi m}\int_0^\infty\frac{e^{-g(M-m)}\cos ugdg}{1+mg}+\frac{M}{\pi m}\frac{M-m}{(M-m)^2+u^2}.$$

Finally,

$$f_d(u)=\frac{M-m}{\pi m}\left[\frac{M}{(M-m)^2+u^2}-\int_0^\infty\frac{e^{-g(M-m)}\cos ugdg}{1+mg}\right]. \tag{8}$$

The mean square values of the functions $F(\theta)$, $f_\lambda(\theta)$, and $f_d(u)$ when N and n are zero, are M^2, m^2 and M^2-m^2, respectively.

[17] Campbell and Foster, *Fourier Integrals for Practical Application*, Bell System Monograph B-584 (1931). Pair 202.



(a) (b)

FIG. 3. Reflection of the Fe $K\alpha_1$, α_2 lines by the (311) planes: (a) in the hard rolled tape; and (b) in the tape annealed at 650°C. The exposure time for (a) was twenty times that for (b).