of Fig. 3, and subtracting the values of this curve from those for the secondary rays, we obtain curves which should represent the ionization due to the softer secondary component. These curves for Echo Lake and sea level have been reduced to the same scale and plotted logarithmically in Fig. 4. The high altitude and sea level curves are similar in form, and after their initial rise, fall off linearly with a "decrease coefficient" of 0.3 cm^{-1} Pb. Woodward finds the coefficient 0.33 cm^{-1} Pb for the first peak of his shower curves. This indicates that the results of our analysis are in agreement with the assumption that the ionization arising from the transition effect is attributable to the radiation responsible for showers.

The research was supported by grants from the Carnegie Institution of Washington and the Harvard Milton Fund. Acknowledgment is made of the active cooperation given by the late Dr. J. L. Dunham in designing some of the apparatus, and by Professor H. R. Mimno, Dr. J. E. I. Cairns and the staff of the Huancayo Magnetic Observatory in securing the South American data. The authors are also under obligation to Professor J. C. Stearns and the University of Denver for affording laboratory facilities, to the City of Denver which furnished transportation of equipment through the courtesy of Mr. G. E. Cranmer and Mr. R. R. Vail, and to Dr. R. H. Woodward, who took some of the Cambridge data.

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Cosmic-Ray Measurements with a Small Ionization Chamber

II. Comparison of Small Bursts at Different Altitudes and Their Variations with Thickness of Shield*

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Measurements of bursts produced in a small ionization chamber (230 cc volume) have been carried out at several stations. A comparison is made between frequencies of occurrence of various sized bursts at Cambridge, Mass. (bar. 76), Echo Lake, Colo. (bar. 51.1), Mt. Evans, Colo. (bar. 44.7), and Cerro de Pasco, Peru (bar. 45.4). The altitude ratios for burst groups comprising 10–19 rays, 20–29, and 30 and greater, are: Echo Lake to Cambridge, 4.9, 8.0 and 8.8; Mt. Evans to Cambridge, 7.3, 14.3 and 20.0. The data for the smallest burst groups are in agreement with counter measurements on showers, while the ratios for the largest bursts agree with the ionization chamber data of others on large bursts. The maxima of the burst production absorption curves shift to greater lead thicknesses with increasing burst size.

1. Discussion of Apparatus

We have described in I[1] equipment which is suitable for the measurement of both total ionization and small bursts. The total ionization is determined from measurements of periodic jumps of a photographic trace when the grounding key is closed. While the key is open (see Fig. 1, I) the trace would be a straight line corresponding to the zero position of the galvanometer if the ionization were perfectly steady. However, if the radiation fluctuates within a period comparable to the time constant of the circuit, the trace will fluctuate. Instantaneous bursts of ionization will be recorded as sharp jumps. Fig. 1 is a reprint of a section of one of our records (6.66 cm Pb, Mt. Evans). The time scale is indicated by the large periodic jumps which correspond to the grounding of the collecting electrode every 70 seconds. Arrow 1 indicates a burst of 10 rays (the lower limit of

* Part of a dissertation presented to the Faculty of Arts and Sciences, Harvard University, in partial fulfillment of the requirements for the degree of Doctor of Philosophy. A preliminary report of this work was made at the New York meetings of the American Physical Society, February 21–22, 1936.

[1] R. T. Young, Jr., and J. C. Street, Phys. Rev. 50, 552 (1937).

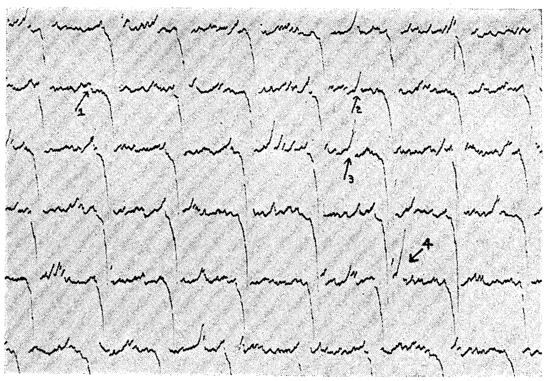

FIG. 1. Sample section of traces (Mt. Evans, 6.66 cm Pb)

our measurements), 2 one of 30, 3 one of 60, and 4 one of 100 rays passing simultaneously through the chamber.

The damped period of the galvanometer was $\frac{1}{2}$ second. The maximum time of collection of ions in a burst was calculated by assuming that the electric field inside the chamber could be approximated by that existing between infinite concentric cylinders. The radial electric field, R, at a distance r from the common axis of the cylinders is:

$$R=(E/r)\log(b/a) \quad a<r<b,$$

where a and b are the radii of the cylinders and E is the potential difference between them. The velocity, u, of an ion in this field is: $u=Rp/P$ where p is the mobility of the ion and P the pressure in the chamber. The time, T, required for an ion to traverse the space between the conductors is:

$$T=\int_a^b dr/u=\frac{P(b^2-a^2)}{2Ep}\log(b/a).$$

For our chamber $a=0.21$ cm, $b=3.85$ cm, $E=430$ volts, and $P=13.6$ atm. For argon $p=1.37$ cm per sec per volt per atm. Substituting these values into the above expression we find that $T=\frac{1}{2}$ sec. This is the time required by an ion formed at the wall of the chamber to reach the collecting electrode.

A point which must be checked is whether the recorded deflections are true representations of changes in the grid potential of the FP 54. Analyzing the circuit of Fig. 2 we have (neglecting the fixed e.m.f.'s of the supply batteries and the plate load drop) for the current, i, in R:

$$di/dt+i/RC=(1/R)d\epsilon/dt,$$

where ϵ is the variable potential difference produced in the plate load of the *FP* 54 by changes in its grid potential. Assume the ions produced in a burst are collected at a steady rate. ϵ is then a linear function of t and during the time of collection $\epsilon=Vt/t_0$, $0<t<t_0$, where t_0 is the time of collection and V the final change in potential of the plate of the FP 54. The potential of the grid of the 6–C–6 with respect to ground is:

$$iR=RCV(1-e^{-t/RC})/t_0.$$

For an instantaneous pulse giving rise to a potential V the initial current i' is V/R. We compare iR at $t=t_0$ and $i'R$.

$$\frac{(iR)_{t=t_0}}{(i'R)_{t=0}}=RC(1-e^{-t_0/RC})/t_0.$$

For $R=4\times10^6$ ohms, $C=1\mu f$ and $t_0=\frac{1}{2}$ second the ratio is .95. Hence we never record less than 95 percent of the full burst value.

Our burst data are presented in terms of the number of rays of average path length contributing to the burst. 80 ions per cm path in air at S.T.P. was taken as the value of the specific ionization.[2] Reduction to number of rays makes the size of burst recorded independent of the chamber dimensions. (Of course, a large chamber will in general record a larger fraction of the rays in a given burst.)

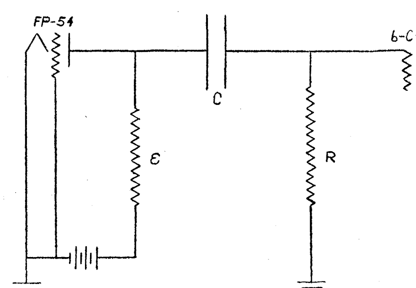

FIG. 2. Diagram for analysis of recording circuit.

[2] Swann (W. F. G. Swann, Phys. Rev. **44**, 961 (1933)) gives the value of 60 ions per cm as determined by direct measurement with a linear amplifier, while Street and Woodward (J. C. Street and R. H. Woodward, Phys. Rev. **46**, 1029 (1934)), by comparing counter data with Millikan's value for the total ionization of 2.48 ions per cc per sec in air at S.T.P., obtain 100 ions per cm path. The value used is the mean of these two.

The essential limitation on the smallness of burst size which can be measured arises from the statistical fluctuations which occur in the chamber. The probability of occurrence of fluctuations which will record as bursts is a rapidly increasing function of the average number of rays passing through the chamber during a period equal to the collecting interval. This number is proportional to r^2 and to the time of collection which, as we have seen above, is an increasing function of the radius, r, of the chamber, proportional to P and inversely proportional to E. Hence we desire P and r to be small and E large. The choice of these values must, however, be consistent with requirements of sensitivity and insulation. The smallest deflection on the traces which could be satisfactorily measured was 1 mm, which corresponded to about 10 rays. The rate of occurrence of kicks due to statistical fluctuations has been evaluated using Poisson's law. It was found that for a collecting time of $\frac{1}{2}$ sec. the contribution of this rate to the measured values for bursts of 10 rays or greater was negligible in all cases. For example, with a 1.27 cm lead shield at Mt. Evans the ionization is due to 3.5 rays per $\frac{1}{2}$ sec. interval. The chance of 13.5 rays passing through the chamber in this interval is 3.2×10^{-5}. The rate per min. due to statistical fluctuations would therefore be 0.0038 which is to be compared with the observed rate of 1.68 for measured kicks. The appearance of the kicks on the traces bears out the conclusion that they are due solely to bursts in that all jumps of 1 mm and greater are uniformly sharp. Some of the kicks below 1 mm are less sharp and are presumably attributable to statistical fluctuations. (See Fig. 1.)

Another source of false kicks is radioactive contamination of the walls of the chamber and consequent emission of alpha-particles. Counts were made of kicks appearing on records taken at reduced pressures; namely, 100 and 17 lb. (gauge). Ionization produced by cosmic rays would be reduced in proportion to the reduction in pressure, while the ionization produced by alpha-particles would increase, due to lessening of the effects of recombination. To evaluate the effective ionization of alpha-particles, a plaque coated with RaF was attached to the interior wall of a chamber similar to the one used in the rest of the work and a curve of average ionization *vs.* pressure obtained. The tests on kicks were made over five hour intervals, and during this time only one kick was observed on the "17 lb." trace which, after applying the alpha-particle ionization factors determined from the curve, would have registered as a 10 ray burst at 200 lb. For this same time interval, our cosmic-ray burst count for the "200 lb." trace was 50. The results obtained from the 100 lb. pressure run were not as conclusive. There were observed about as many kicks which would have recorded as 10–15 ray bursts at 200 lb. as were to be expected from cosmic rays. However, at 100 lb. the cosmic-ray bursts play a larger part, making it difficult to draw definite conclusions concerning the alpha-particle contamination. We consider the evidence from the trace at 17 lb. as quite conclusive that alpha-particle contamination plays little if any part in our results.

2. Results

In Table I are listed the number, N, of observed bursts and their rates of occurrence, R, per min. for each shielding condition.[3] The bursts were arbitrarily classified according to size into the groups listed. On the average a tenth mm deflection on the trace corresponded to the passage of a single ray through the chamber. Uncertainty of measurement amounted to 0.1 or 0.2 mm depending on the sharpness of the trace. (Some of the traces were remeasured after an interval of several months and checks to the above accuracy obtained.) The data were recorded to the nearest ray and then smoothed by dividing the total number of recorded bursts of a given size by three and assigning equal parts to bursts of the given size and to those directly above and below. Since measurements were actually made down to burst sizes of 9 rays the smoothing process could be applied to the 10 ray bursts. There may be some overlapping of the groups, but it is considered that the ranges taken are of sufficient extent that the burst

[3] Though ionization measurements were made at three South American stations (Lima, Huancayo and Cerro de Pasco, Peru), only the Cerro de Pasco burst data are included in the table. While the ionization data for Lima and Huancayo are reliable, the traces at these stations were of such nature that burst measurements were not considered sufficiently accurate to record.

TABLE I. *Frequency of occurrence of bursts.*

Thickness of Lead Above Chamber in cm		0.00	0.64	1.27	3.18	6.66	19.4
Cambridge: Sea Level; Bar. 76; Lat. 53° Mag. N.							
10–14	N	260	158	314	186	343	138
	R	.066	.081	.118	.096	.071	.059
15–19	N	115	97	124	79	237	97
	R	.036	.055	.052	.044	.062	.052
20–29	N	48	28	60	43	101	28
	R	.012	.016	.023	.023	.021	.015
30 and greater	N	22	11	19	13	32	12
	R	.0055	.006	.012	.0085	.0071	.0046
Total 10 and greater	N	485	294	527	321	713	275
	R	.114	.158	.205	.172	.161	.131
Echo Lake: 3250 m; Bar. 51.1; Lat. 49° Mag. N.							
10–14	N	372	277	337		275	149
	R	0.421	0.544	0.588		0.342	0.258
15–19	N	204	153	159		130	75
	R	.217	.249	.277		.162	.130
20–29	N	297	131	87		94	48
	R	.158	.147	.152		.118	.084
30 and greater	N	106	60	49		46	23
	R	.051	.063	.085		.058	.040
Total 10 and greater	N	979	621	632		545	295
	R	.847	1.00	1.10		.680	.512
Mt. Evans: 4350 m; Bar. 44.7; Lat. 49° Mag. N.							
10–14	N	393	601	282	301	312	310
	R	.602	.888	.875	.580	.398	.336
15–19	N	227	277	172	227	232	195
	R	.308	.408	.505	.437	.279	.200
20–29	N	584	284	116	159	192	174
	R	.222	.268	.340	.317	.226	.171
30 and greater	N	341	143	58	97	116	90
	R	.105	.114	.170	.187	.140	.086
Total 10 and greater	N	1545	1305	640	784	852	769
	R	1.24	1.68	1.93	1.51	1.04	.793

Cerro de Pasco: 4360 m; Bar. 45.4; Lat. 1° Mag. S.

Thickness of Lead Above Chamber in cm		0.00	.64	1.27	3.18	6.66	9.2	19.4
20–29	N	44	64	74	30	27	17	22
	R	.081	.280	.247	.105	.089	.045	.049
30 and greater	N	22	51	44	57	22	20	23
	R	.040	.218	.147	.153	.060	.034	.041
Total 10 and greater	N	66	115	118	87	49	37	45
	R	.121	.498	.394	.258	.139	.079	.090

groups represent within the probable error, $0.67/(N)^{\frac{1}{2}}$, the correct number within a given range.

To investigate the cause of the high zero rates, measurements were made with no side shields at Echo Lake. The results obtained were the same as with the side shields in place. This indicates there is a considerable contribution from the air above the chamber or from the shell of the sphere (1 mm steel). (See discussion in I[1] of the no shield values of the ionization contributed by secondary rays.)

Only the larger bursts could be measured on the South American traces. Taking the ratio of the average for all thicknesses of lead for bursts of 20 rays and greater, one obtains the latitude ratio: Mt. Evans : Cerro de Pasco, 1.34. This ratio agrees with the latitude ratio for the general ionization (1.30) at this altitude.[1] Since the South American and North American data were obtained from somewhat different types of records the ranges of burst sizes may not correspond, and we do not attach too great significance to this agreement.

In Table II the variations of burst frequency with elevation are tabulated in the form of ratios of rates of occurrence at the higher altitudes to those at Cambridge. It is seen that while the ratios fluctuate somewhat, if one considers the averages over all thicknesses, there is a definitely greater altitude effect for the bursts of larger size. Woodward[4] finds for the relative increase of shower rates between the same stations the ratios:

Echo Lake : Cambridge 5.0
Mt. Evans : Cambridge 8.5

These are in agreement with our ratios for the totals of bursts of all sizes and averaged over all

TABLE II. *Variations of burst frequencies with altitude.*

Thickness of Lead Above Chamber in cm	0.00	0.64	1.27	3.18	6.66	19.4	Average All Thickness
Echo Lake: Cambridge							
Size of burst in rays							
10–19	6.3	5.8	5.1		3.8	3.5	4.9
20–29	13.2	9.2	6.6		5.6	5.6	8.0
30 and greater	9.3	10.5	7.1		8.2	8.7	8.8
Total	7.4	6.3	5.4		4.2	3.9	5.4
Mt. Evans: Cambridge							
10–19	8.9	9.5	8.1	7.3	5.1	4.8	7.3
20–29	18.5	16.7	14.8	13.8	10.8	11.4	14.3
30 and greater	19.1	23.4	14.1	22.0	19.7	18.7	20.0
Total	10.7	10.6	9.4	8.8	8.5	6.1	8.7

[4] R. H. Woodward, Phys. Rev. **49**, 711 (1936).

thicknesses of lead The values for the frequency of occurrence of the net totals are controlled by the observed rates for the groups including only the smaller bursts. Counters record showers of all sizes, but cloud chamber photographs[5] indicate that the average shower tripping a counter set contains 4–6 rays, hence not much below the range of our smallest bursts. D. D. and C. G. Montgomery[6] have found for *large bursts* (40 rays and greater), observed in an ionization chamber shielded by 4 cm Pb, the following relative increases from Swarthmore, Pa. (61 m) to Glen Cove, Colo. (3500 m) and Pikes Peak (4300 m):

Glen Cove : Swarthmore	13.8
Pikes Peak : Swarthmore	26.6

These ratios are in rough agreement with those found by us for the larger bursts. The data in Table II cover the range between showers as observed with counter and large bursts recorded in chambers. One may conclude that counter and ionization chamber results are not contradictory when the size of burst is taken into consideration and that, in dealing with showers or bursts one has to do with a gradual change in the characteristics of the same phenomenon. This increase of altitude ratio with increasing burst (or shower)

[5] E. C. Stevenson and J. C. Street, Phys. Rev. **49**, 425 (1936).

[6] D. D. and C. G. Montgomery, Phys. Rev. **47**, 429 (1935).

TABLE III. *Contributions of bursts to measured ionizations (in ions per cc per sec).*

THICKNESS OF LEAD ABOVE CHAMBER IN CM	0.00	0.64	1.27	3.18	6.66	19.4
Mt. Evans						
Ionization	.67	.87	1.0	.94	.60	.48
Percent contribution	6.0	7.2	9.3	11.2	10.0	10.0
Echo Lake						
Ionization	.44	.54	.57		.38	.26
Percent contribution	6.0	6.8	7.8		8.8	7.0
Mt. Evans						
Ionization	.05	.07	.11	.09	.08	.06
Percent contribution	2.3	3.0	4.3	4.2	4.3	3.5

size is confirmed by the cloud chamber measurements of Anderson and Neddermeyer[7] who find the ratio between Pikes Peak and Pasadena of frequency of occurrence of photographs showing 2–4 tracks to be 8.6; 5–10 tracks, 21; and those showing 11–100 tracks to be 29.

The accuracy of the ratios of Table II is limited by that of the Cambridge data, which are based on a small number of recorded bursts, particularly those of large size. The above ratios can therefore only be considered as indicating general trends in the burst phenomena. There appears to be a definite decrease of the ratios

[7] C. D. Anderson and S. H. Neddermeyer, Phys. Rev. **50**, 263 (1936).

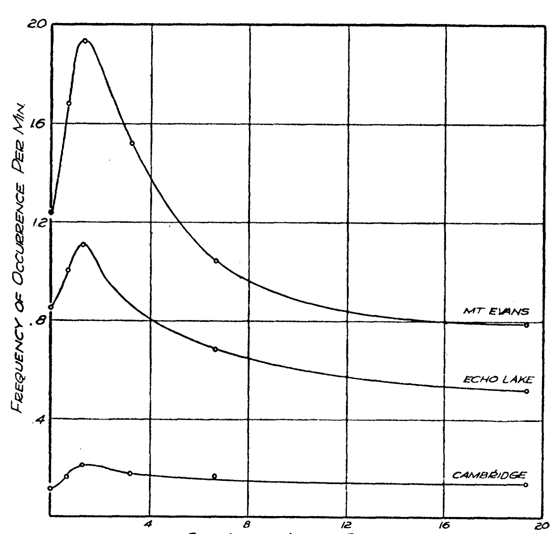

FIG. 3 (left). Curves of total burst frequencies at different altitudes.

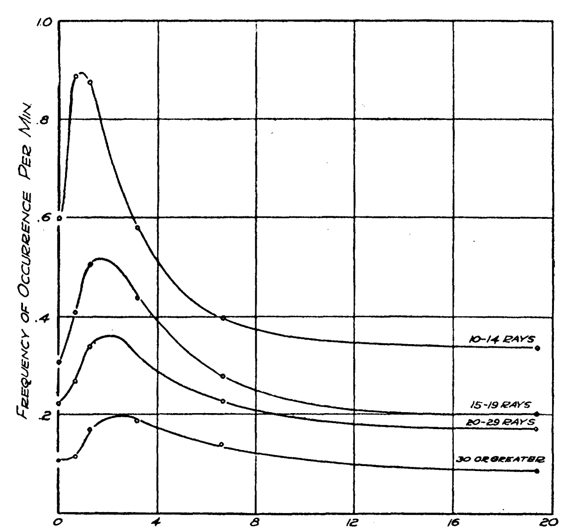

FIG. 4 (right). Curves of burst frequencies for different sized bursts (Mt. Evans).

for smaller bursts with increasing lead, while the ratios for the larger bursts remain sensibly constant.

The variation of total burst frequency with thickness of lead at different altitudes is shown by the curves of Fig. 3. They are of the form of transition curves, and in general shape similar to those for showers as obtained with counters. The set of curves of Fig. 4 are for the different burst groups at Mt. Evans. While the maxima cannot be located exactly, they definitely shift to greater lead thicknesses for the larger sized groups. This is in agreement with the findings of Carmichael,[8] and also with those of Nie[9] who observes the maxima for large bursts to occur at 5 cm lead. Carlson and Oppenheimer[10] show on the theory of multiplicative showers that this shift is to be expected. Bhabha and Heitler[11] also predict a similar shift.

[8] H. Carmichael, Proc. Roy. Soc. **A154**, 223 (1936).
[9] H. Nie, Zeits. f. Physik **99**, 776 (1936).
[10] J. F. Carlson and J. R. Oppenheimer, Phys. Rev. **51**, 220 (1937).
[11] H. J. Bhabha and W. Heitler, Proc. Roy. Soc. **A159**, 432 (1937).

In Table III are given the ionizations contributed by all bursts greater than 10 rays to the total observed ionizations given in I.[1] No extrapolation can be made to smaller showers and hence no estimate can be made of the total contribution of the shower phenomena to the total ionization. Since the values of the tables are lower limits, the actual ionization produced by showers is an appreciable fraction of the total.

The writer wishes to express his appreciation of the advice and encouragement of Professor J. C. Street, who suggested the problem. The writer is also under obligation to Dr. R. H. Woodward who took some of the Cambridge data, to Professor J. C. Stearns and the University of Denver in affording laboratory facilities, and to the City of Denver which furnished transportation of equipment through the courtesy of Mr. G. E. Cranmer and Mr. R. R. Vail. The research was supported in part by grants from the Carnegie Institute of Washington and the Harvard Milton Fund.

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Shower Production Under Thick Layers of Various Materials*

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(Received July 10, 1937)

Observations by G-M counters are reported on showers from lead and iron up to thicknesses of approximately 600 g/cm². As previously noted by others, the transition curves at large thicknesses have approximately the same slope as absorption curves obtained for the general cosmic radiation. Data are presented for the iron-lead and lead-iron transition curves beginning at a material thickness of 274 g/cm² in each case. For the iron-lead transition curve, the number of coincidences increases and attains a maximum in approximately 1 cm of lead, after which the counting rate decreases and finally falls on the air-lead transition curve. For the lead-iron transition curve, the number of coincidences decreases for the first increments of added iron, passes through a minimum and then increases to the air-iron transition curve. It is pointed out that the observations are consistent, in a qualitative way, with the multiplicative theory of the origin of cosmic-ray showers provided one assumes that additional shower producing radiation is generated in the lower layers of material.

THE application of the laws of radiation of high speed electrons and of the production of pairs by quanta has recently led to a fairly complete description of a portion of cosmic-ray shower phenomena. It has been shown[1, 2] that the multiplication theory is capable of accounting in a fairly satisfactory way for showers produced under relatively small thicknesses of material.

* A preliminary report on these experiments was presented at the Chapel Hill-Durham meeting of the American Physical Society, February 1937.

[1] J. F. Carlson and J. R. Oppenheimer, Phys. Rev. **51**, 220 (1937).
[2] H. J. Bhabha and W. Heitler, Proc. Roy. Soc. **159A**, 432 (1937).

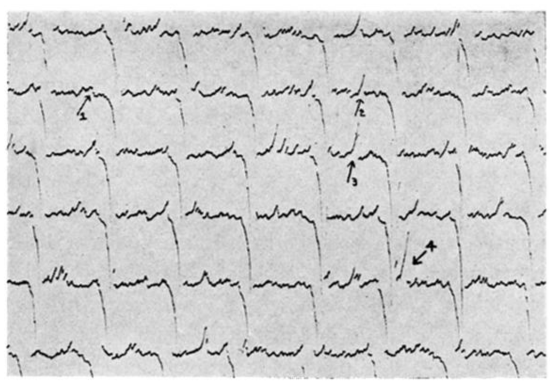

FIG. 1. Sample section of traces (Mt. Evans, 6.66 cm Pb)