Magnetic Scattering of Slow Neutrons

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According to well-known theoretical results, atomic or ionic magnetic moments should *per se* contribute to the cross section for the scattering of slow neutrons. It is suggested that this contribution may be separated from that due to nuclear scattering by a comparison of the scattering of a metal with the scattering from the corresponding ions of different valence. This magnetic scattering should occur in unmagnetized paramagnetic bodies and, in favorable cases, should be several times as great as the nuclear scattering. Accepting the present value for the magnetic moment of the neutron such experiments will yield information concerning the velocity distribution of the incident neutrons.

As first pointed out by Bloch,[1] the magnetic moments of atoms or ions should contribute appreciably to the scattering cross section of slow neutrons. To separate the magnetic from the nuclear scattering, he suggested magnetic polarization experiments on saturated ferromagnetic bodies. These experiments would allow the production of beams of neutrons whose spins were preferentially oriented; this partially "polarized" state could be detected by a second scattering or absorption in a ferromagnetic body with a properly chosen direction of magnetization.

The principle of Bloch's suggestion can be mathematically expressed as follows: Denoting by ψ_n and ψ_m the wave functions of a neutron scattered by nuclear or magnetic forces respectively, the total cross section is proportional to $|\psi_n \pm \psi_m|^2$, where the sign depends on the relative orientation of atomic and neutron magnetic moments. For an unpolarized beam of neutrons or an unmagnetized body one obtains by averaging an expression proportional to $|\psi_n|^2+|\psi_m|^2$ for the total scattering cross section. Whereas for a magnetized body the cross term $2\psi_n\psi_m$ will not disappear in the average, leading to the polarization effects mentioned above.

Bloch's suggestion has been given a more detailed mathematical treatment by Schwinger.[2] His result for the scattering cross section of an unpolarized beam in an unmagnetized body agrees with the expression of Bloch and can be written

$$\sigma=\sigma_n+\sigma_m=(\sigma_n+0.67\times 10^{-24}B^2)\ \text{cm}^2, \qquad (1)$$

where B denotes the number of Bohr magnetons of the scattering atom (ion). According to the derivation the expression is valid only if $\lambda/2\pi$ (λ = wave-length of incident neutron) is large compared to the linear dimensions of the atomic domain which gives rise to the magnetic moment. The second term of (1) is multiplied by a "magnetic form factor" which decreases rapidly from the value 1 when $\lambda/2\pi$ becomes smaller than the linear dimensions of the relevant atomic domain. It is further assumed in (1) that the magnetic moment of the neutron is equal to 2 nuclear Bohr magnetons.

Instead of experimenting with ferromagnetic bodies and producing "polarized" neutron beams, the magnetic scattering could also be detected and separated in nonmagnetized bodies as follows. Successive scattering observations with a metal and its salts will lead to different cross sections for the metal if the metallic salts show a varying paramagnetism depending on the valence with which the metal enters into the chemical combination. Since the nuclear cross sections are usually several times 10^{-24} cm^2, it is clear from (1) that the magnetic cross section is in general comparable to and may even be a multiple of the nuclear cross section.

Since (1) is only valid for sufficiently large values of $\lambda/2\pi$, discussion should be centered on substances with incomplete inner shells. Table I of the ions of the iron group shows the dependence of B on the valence. In Table I we have replaced the values for the "effective Bohr magneton numbers" given by Van Vleck[3] by the

[1] Bloch, Phys. Rev. **50**, 259 (1936).
[2] Schwinger, Phys. Rev. **51**, 544 (1937).
[3] J. H. Van Vleck, *Electric and Magnetic Susceptibilities* (Oxford University Press, 1932), Par. 72, 73.

actual Bohr magneton numbers which are of importance for the neutron scattering. With the exception of the ferromagnetic substances Fe, Co and Ni, the metals show comparatively small paramagnetism. But even in the last mentioned cases the difference between the magneton numbers of the ions in metallic and salt form are very appreciable, the maximum magneton number for the ferromagnetic metals being about 2.

It deserves mentioning that the magneton numbers given in Table I are very nearly the same for solid salts and solutions so that they can be ascribed with certainty to the metallic ion and not to any molecular complex. Under this same assumption the experimental paramagnetism may be theoretically understood (cf. Van Vleck, reference 3).

The elements of the Pd, Pt and Ur group, except Mo^{+++}, do not seem to show sufficiently large values of B to be of interest. Among the trivalent ions of the rare earths, however, the elements Tb, Er, Ds, Ho, seem to be most promising. The first two having $B=9$, the last two $B=10$. For this group, on the other hand, Gd shows a *capture* cross section of the order 2×10^{-20} cm^2. Hence any possibility of using these elements rests on a quite complete chemical separation of the rare earths. Assuming that any one of these elements has a capture cross section less than 50×10^{-24} cm^2 and that it is free of Gd to a tenth of a percent, the capture and magnetic scattering cross section should become of equal order of magnitude and thus the magnetic effect easily demonstrable.

The principal difficulty, in our opinion in obtaining a result of the estimated magnitude in these experiments will lie in the wave-length of the so-called thermal neutrons. The wave-length of neutrons in thermal equilibrium at room temperature is approximately 1.5A. Since the average velocity of a beam of thermal neutrons in our opinion will lie considerably above the so-called thermal velocity, it will in general be impossible to satisfy the condition that $\lambda/2\pi$ is larger than the relevant atomic domain. Rough considerations have led us to believe that this atomic domain is not much smaller than a Bohr radius (0.53×10^{-8} cm) for the elements of the iron group. For this reason the rare earths would be preferable if the above-mentioned obstacles can be overcome. This should lead to a pronounced effect from the form factor and in particular, should result in preferential forward scattering as well as a decrease in the total scattering cross sections.

TABLE I. *Number of Bohr magnetons.*

Ion	True Magnetic Number	Ion	True Magnetic Number
V^{++++}	1	Mn^{++}	5
V^{+++}	2	Fe^{+++}	5
V^{++}	3	Fe^{++}	>4
Cr^{+++}	3	Co^{++}	4
Mn^{++++}	>3	Ni^{++}	>2
Cr^{++}	4	Cu^{++}	1
Mn^{+++}	>4		

The polarization experiments so far carried out have led to results only a small fraction of the anticipated effect. This might be due either to high velocities (small wave-length) of the neutrons, as seems likely, or due to a smaller magnetic moment of the neutron. The scattering experiments here suggested should show an increase in the cross section by many hundred percent. If the observation should not agree with this theoretical prediction, then further experiments on the angular distribution of scattering should distinguish between the two possible causes for the discrepency. Since for scattering under small angles, the influence of the form factor (shortness of wave-length) can be neglected, too small an effect in such experiments would have to be ascribed to an incorrect assumption as to the magnitude of the neutron's magnetic moment. On the other hand if the small angle scattering is large, these experiments would present some information about the velocity distribution of the incoming neutrons.

Our colleague, Dr. M. D. Whitaker, has begun experiments of the kind suggested in this note.*

* *Footnote added in proof:* For scattering in paramagnetic media the second term of (1) should be multiplied by $(j+1)/j$ where j is the angular momentum of the scattering ion in units $\hbar$. See Phys. Rev. **51**, 992 (1937).