A Determination of e/m for an Electron by a New Deflection Method. II

FRANK G. DUNNINGTON*
Norman Bridge Laboratory of Physics, California Institute of Technology, Pasadena, California

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A final report is here made of a deflection determination using a new method described in a preliminary report (Part I). There are two fundamental advantages of this method over other free electron methods: (1) the accelerating voltage is not measured. In its place, the determination of the frequency of a radiofrequency oscillator is made, and this can be done with far greater precision. (2) Errors from *all* contact potentials are either entirely eliminated or made negligibly small, as is shown in detail. The ever prevalent error arising from surface changes in all free electron determinations was at least greatly reduced by the use of evaporated gold surfaces throughout the deflecting chamber, said surfaces being frequently renewed to cover over the insulating layers formed. The error remaining was eliminated by extrapolation of results to infinite electron energy. The magnetic field was most carefully calibrated in terms of a specially designed standard solenoid. The frequency calibration was based on government standards through the cooperation of the Federal Radio Commission. Standards of voltage, resistance and lengths were calibrated at the National Bureau of Standards. The object throughout the work has been to produce a value as free as possible from errors both experimental and subjective. The result obtained is

$$e/m_0=(1.7597\pm0.0004)\times10^7 \text{ e.m.u.}$$

The stated probable error is based on allowances for unknown constant errors, the observational probable errors being negligible. This value of e/m_0 is about 1.25 parts in 10^3 higher than that found in recent spectroscopic determinations, a definite discrepancy being indicated. With the inclusion of the final values obtained in the last ten years by nine experimenters using both free electron and spectroscopic methods, the present most probable value is found to be

$$e/m_0=(1.7584\pm0.0003)\times10^7 \text{ e.m.u.}$$

but in view of the discrepancy this is at best only a tentative value.

INTRODUCTION

In an earlier paper[1] a preliminary report was made on the determination of e/m by a new deflection method. Work has been in progress since then on the perfection of the method and on the establishment of the necessary standards. The object throughout this work has been to produce a result as free as possible from errors, both experimental and subjective, so that the result would be good as near as possible to one part in 10,000. As a consequence many sources of error were discovered, the existence of some of which was not even suspected at the end of the year's preliminary work. Insofar as possible such errors have been corrected. The precision of observation in all the factors entering into e/m has been refined until the observational probable errors are negligible, leaving only basic uncertainties in the method and in the interpretation of the physical phenomena observed.

No free electron determinations of e/m have appeared since the preliminary paper[1] (hereafter called Part I) in which the value of $e/m_0=(1.7571\pm0.0015)\times10^7$ e.m.u. was tentatively set. Three new spectroscopic determinations[2–4] have given consistent values ranging from 1.7570 to 1.7579. The currently accepted figure has therefore been about 1.7576. The present work indicates an appreciably higher value.

METHOD

The method used has been described in Part I. Reference should be made to the section "Simplified Description of Action in the Tube" on pages 406–7. To summarize in brief: electrons from the filament F, Fig. 1, are accelerated across the gap A during half of each cycle of the radio frequency voltage impressed on the lead-in L. The velocity attained by an electron depends on the part of the cycle during which it crosses A.

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[1] F. G. Dunnington, Phys. Rev. **43**, 404 (1933).

[2] L. E. Kinsler and W. V. Houston, Phys. Rev. **45**, 104 (1934); **46**, 533 (1934).

[3] C. D. Shane and F. H. Spedding, Phys. Rev. **47**, 33 (1935).

[4] R. C. Gibbs and R. C. Williams, Phys. Rev. **48**, 971 (1935).

A magnetic field perpendicular to the figure bends the electrons in circles with radii proportional to their velocity. With the frequency constant, any given magnetic field allows electrons of one velocity to pass around through the slits and arrive at the gap D. If the magnetic field passes a velocity such that the electrons require exactly one cycle to travel from A to D, then the electrons are completely stopped by the retarding field at D and do not reach the shielded collector at C. For any other magnetic field half the electrons reach the collector. Observations then consist in noting the magnetic field H_r at which this current minimum or "resonance" condition occurs, together with the frequency ν of the impressed voltage and the angle θ in radians subtended by the electron path. e/m is then given by (Eq. (3), Part I)

$$e/m=\theta\nu/H_r\cdots\text{e.m.u.},\qquad (1)$$

in which m is the relativistic mass.

It should be noted particularly that, in contrast to Part I, no d.c. voltage was applied between the filament and the first slit. This results in considerable simplification of the phenomena involved and makes unnecessary the extra d.c. retarding field in front of the collector for which it has been found impossible to calculate the proper correction.

There are two fundamental advantages of the present method over other free electron methods: (1) the electron accelerating voltage is not measured. In its place, the determination of the frequency of a radiofrequency oscillator is made, and this determination can be done with far greater precision. (2) Errors from *all* contact potentials are either entirely eliminated or made negligibly small. This is shown in detail in the next section. (Errors from surface charges[5, 6, 6a] in the supposedly field free regions 1, 2 and 3, Fig. 1, are not eliminated but are minimized by the technique used.) (3) A third advantage of the method is the high observational precision possible. Thus a series of ten readings have an observational probable error of the order of 3 parts in a million.

[5] R. Lariviere Stewart, Phys. Rev. **45**, 488 (1934).
[6] A. E. Shaw, Phys. Rev. **44**, 1009 (1933).
[6a] Bainbridge and Jordan, Phys. Rev. **50**, 290 (1936).

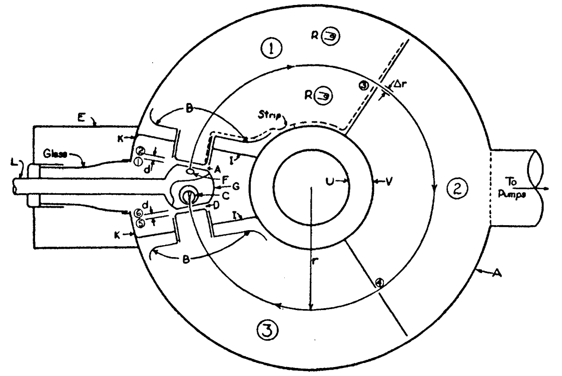

FIG. 1. Schematic diagram of measuring chamber, approximately to scale. Chamber diameter = 32 cm. Electron radius = 9.9 cm. The "strip" indicated by the dotted line was inserted only in a test experiment.

Effective Angle θ

In Eq. (1) above, the quantities ν and H_r are determined by direct measurement, but θ does not correspond exactly to the angle between any physical points. By definition in the derivative of Eq. (1), θ is defined as "the angle which would be subtended by the path of an electron if an electron traveled for one cycle at the constant velocity v_f which it has in the electric field free space" (regions 1, 2 and e, Fig. 1). Since finite times are required for acceleration and deceleration, an electron does not travel for quite a full cycle at its maximum speed. An accurate expression for θ in terms of measurable quantities is needed.[7]

The obtaining of an expression for θ is complicated by the penetration of the radiofrequency field through the slits at A and D, Fig. 1, since this alters the distance over which acceleration and deceleration occurs. With the geometry necessary the penetration was large. Thus with a gap $d=1.6$ mm and a slit width $\Delta r=0.25$ mm the field at $1/10d$ either way from the slits still amounts to 10 percent of the maximum. An attempt was made to obtain the expression for θ, but even rough approximations could not be handled due to the fact that with the slit width not zero the acceleration is a function of both time and position. It was found, however, that the function giving θ could be obtained if Δr were assumed to be zero. To utilize this, experimental

[7] The expression obtained in Part I was based on such an oversimplified picture of conditions that it is considerably in error.

observations were taken as follows: The value of the magnetic field H_r at "resonance" was observed for several values of slit width. A curve was then plotted of H_r against Δr and the extrapolated value at $\Delta r=0$ was used with the θ computed for the same condition to obtain e/m.

Derivation of expression for θ with $\Delta r=0$

The path of an electron is conveniently divided into three parts: (1) an acceleration distance of length d between slits 1 and 2 (see Fig. 1); a distance d_f of constant speed between slits 2 to 5; and a decelerating distance, also equal to d, between slits 5 and 6. Since an angle is equal to its arc divided by the radius Eq. (1) may be rewritten as

$$e/m=[d_f+f(d)](\nu/rH_r), \tag{2}$$

where the function of d, namely $f(d)$, is the expression to be found.

Five simplifying assumptions are necessary besides the condition that $\Delta r=0$: (1) there exists a uniform electric field in both gaps A and D, but the field is zero elsewhere. The extent and symmetry of the slits on either side of the region crossed by the electron beam assures uniformity. Its absence elsewhere was assured insofar as it is possible by completely coating regions 1, 2 and 3, Fig. 1, with gold (evaporated on). Surface charges will be discussed in the section "Results." (2) The electric field E between the slits varies linearly with time during the acceleration or deceleration of an electron (i.e., curvature of the sine wave neglected); that is:

$$E=E_i+st \text{ (e.m.u.)}, \tag{3}$$

where $s=dE/dt$, t is time measured from entrance of electron into the field and E_i is the field at $t=0$. Since the time spent by an electron in crossing either gap is about 1/200 of a period and since E_i is about 80 percent of the maximum field, it is easily found that the deviation from linearity amounts to only about 1.2 parts in 10,000 so that the assumption is well justified. (3) The mass m of the electron is constant and equal to that at its maximum velocity, as determined by the radiofrequency ν (see straight line $m=m_\nu$, Fig. 2). The actual variation (of the order of 0.3 to 0.6 percent) is indicated by the dotted curve. To test the assumption, a solution B was also obtained with the other extreme of mass in the gaps 1–2 and 5–6, namely $m=m_0$. This changed the calculated e/m_0 by the utterly negligible amount of 6 parts in 10^{10}. (4) The initial velocity (at slit (1)) is zero. This neglects thermal velocities as well as contact potentials between the filament and slit (1). (As previously stated no d.c. potential was applied.) (5) No contact potentials exist around the path between slits (1) and (6). Assumptions (4) and (5) are tested by special solutions given after the main derivation and are shown not to effect appreciably the results.

The method of solution can be most easily understood by referring to Fig. 3 in which two segments of the sine wave of field variation are shown, and here drawn as straight lines of slope s in view of assumption (2). Equal ordinates of the two segments are, of course, separated by a time interval equal to one period or $1/\nu$. E_a and E_d are the fields at the beginning of acceleration and deceleration respectively, while t_a and t_d are the total times of acceleration and deceleration. The constant velocity v_f of an electron between slits (2) and (5) is directly obtained from the radial force equation in terms of the magnetic field H_r as

$$v_f=H_r re/m \text{ (e.m.u.)}. \tag{4}$$

Hence the time spent between slits (2) and (5) is simply the distance divided by the velocity as given in Fig. 3. In view of the periodicity of E we can write

$$E_d=E_a+s\left(t_a+\frac{d_f}{H_r re/m}-\frac{1}{\nu}\right). \tag{5}$$

The tangential force equation for acceleration (tangential motion being taken as the y direction) is

$$d^2y/dt^2=(e/m)(E_a+st). \tag{6}$$

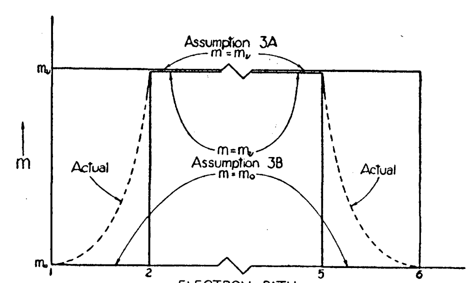

FIG. 2. Assumed and actual variation of electron mass.

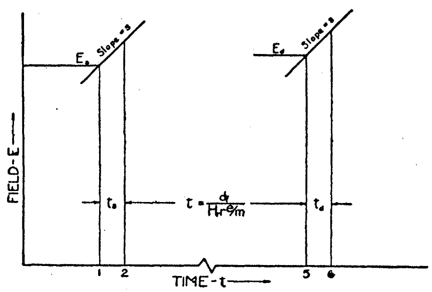

FIG. 3. Field and time terminology in motion of electron from slits (1) to (6). Lines with slope s are segments of successive sine waves.

The value of E_a can be obtained from this upon integration once and setting equal to Eq. (4) with $t=t_a$, giving

$$E_a=(H_r r/t_a)-(st_a/2). \tag{7}$$

Eq. (7) substituted in a second integration of (6), again with $t=t_a$, yields a cubic in t_a. The correct root after lengthy algebraic manipulation gives a power series expression for t_a

$$t_a=\frac{2d}{H_r r(e/m)}\left[1+\frac{2}{3}\left(\frac{d^2 s}{H_r^3 r^3 (e/m)^2}\right)+\frac{4}{3}\left(\frac{d^2 s}{H_r^3 r^3 (e/m)^2}\right)^2\right]. \tag{8}$$

In a similar manner E_d is found identical with Eq. (7) when the subscripts are changed from a to d, and t_d is identical with Eq. (8) except for a negative second term in the bracket.

These results placed in Eq. (5) give

$$\frac{e}{m}=\left\{d_f+d\left[\frac{4}{3}+\frac{8}{3}\left(\frac{d^2 s}{H_r^3 r^3 (e/m)^2}\right)^2\right]\right\}\frac{v}{rH_r}\ \text{e.m.u.} \tag{9}$$

The last term in the bracket [ ] affects e/m only to the extent of two parts in 10^7 and so is quite negligible. With sufficient accuracy we therefore have

$$e/m=\{d_f+(4/3)d\}v/rH_r\ \text{e.m.u.} \tag{10}$$

Comparison with Eq. (2) shows that $f(d)$ is simply $4/3d$ and the effective angle θ is then

$$\theta=(d_f+(4/3)d)1/r=\theta_f+(4/3)\theta_d. \tag{11}$$

We have so far considered only the group R electrons (see Part I, page 407) accelerated and decelerated on the rising or positive slope part of the voltage wave. A similar treatment of the group F on the falling or negative slope of the voltage wave yields the same relation as Eq. (9), and hence the two groups behave identically in spite of the asymmetry in acceleration.

Test of assumption (4)

The initial and final velocities in the above deviation were assumed zero (assumptions 4 and 5). Let us now assume the initial velocity is not zero but say equal to v_0 due to thermal velocities and a contact potential between filament and slit (1). Then if no further contact potentials exists (assumption 5) the final velocity will also be v_0. If a new solution is made with these limits, the result can be put into the form of Eq. (10) but with an added factor multiplying d, *viz.*

$$e/m=\left\{d_f+\frac{4}{3}d\left[\frac{1+2(v_0/v_f)+\frac{3}{4}(1+v_0/v_f)(8/3)K}{1+2(v_0/v_f)+(v_0/v_f)^2}\right]\right\}\frac{v}{rH_r}\ \text{(e.m.u.)}, \tag{12}$$

in which $(8/3)K$ is the negligible term in Eq. (9), and v_f is defined in Eq. (4). Since v_f involves e/m the equation is not completely solved for e/m, but due to the smallness of the [ ] term, this is immaterial. The effect of v on the calculated e/m can be determined by substituting in Eq. (12) the following typical values of the quantities in the bracket { }: $d_f=58.4$ cm, $d=0.1594$ cm, $(8/3)K=7.9\times10^{-5}$, $v_0=$ velocity of a one volt electron, $v_f=$ velocity of a 2000 volt electron. The change in the calculated e/m is found to be only -1.5 parts in 10^6. For an initial velocity corresponding to two volts, it is -3.2 parts in 10^6. Thus no reasonable assumption for the initial velocity results in an appreciable change in the calculated e/m. In passing, it should be noted that if the direction of the contact potential is such as to oppose the electron motion from filament to slit (1) then no error results from the contact potential as long as it is constant during a series of measurements.

Test of assumption (5)

In all of the foregoing, it has been assumed there were no contact potentials between slits

(1) and (6) (assumption 5). This is equivalent to saying that the initial fields E_a and E_d (see Fig. 3) are produced entirely by the impressed radiofrequency voltage. If a contact potential V_a exists between slits (1) and (2) and a potential V_d between slits (5) and (6), then the following substitutions must be made in place of the initial fields:

$$E_a \rightarrow E_a + V_a/d, \quad E_d \rightarrow E_d + V_d/d. \qquad (13)$$

The potentials are taken as positive when they produce a field in the same direction as E_a or E_d. If with these substitutions a new solution is made for each electron group there results in place of Eq. (10)

$$e/m = \frac{d_f + (4/3)d}{rH_r((1/\nu) \pm (V_a - V_d)/sd)} \text{ (e.m.u.)}, \qquad (14)$$

where the (+) sign is for the (R) group and the (−) sign for the (F) group of electrons. This result shows that if $V_a \neq V_d$ the two groups are not in resonance (i.e., will not be stopped) at the same values of the magnetic field. Since the shift for the two groups is equal and opposite, the effect (see Fig. 4) is to make the collector current minimum at resonance shallow if the potential difference is positive and broad if negative. Thus contact potentials in the two gaps can cause poor experimental conditions but *do not produce any shift of the observed e/m*. The existence of any contact potentials in the region between slits (2) and (5) is highly improbable since all the surfaces there were coated with gold by evaporation. Surface charges[5, 6] will be considered in "Results."

Evaluated for typical conditions, the magnitude of the shift indicated in Eq. (14) can be seen from the expression

$$\frac{(V_a - V_d)/sd}{1/\nu} \doteq \pm 8 \times 10^{-4}(V_a - V_d), \qquad (15)$$

where the potentials are now expressed in volts. Thus a *net* contact potential difference of one volt results in a shift in the magnetic field H of ± 8 parts in 10,000. Judging from the depth and sharpness of the minimum existing during the recording of all final data of this work, and making allowance for the actual slit width used, it is probable that $(V_a - V_d) \leq 0.05$ volt.

To summarize this section: an accurate expression giving θ in terms of measurable quantities has been found in Eq. (11), and contact potentials as well as initial thermal velocities have been shown to produce a negligible effect on the result. Eq. (1) when multiplied by m/m_0 gives the formula used in all computations, namely

$$e/m_0 = (m/m_0)(\theta\nu/H_r) \text{ (e.m.u.)}, \qquad (16)$$

where we now have

$$\theta = \theta_f + (4/3)\theta_d \text{ (radians)}. \qquad (11)$$

θ_f is the angle subtended by the path from slit (2) to (5) and θ_d is the angle subtended by the path from slit (1) to (2) (equal to that from (5) to (6)). The mass ratio is

$$m/m_0 = [1 - (r\theta\nu/c)^2]^{-\frac{1}{2}} \qquad (18)$$

since $v_f = r\theta\nu$ (i.e., distance ÷ time). c is the velocity of light, H_r is the magnetic field at resonance and ν is the radiofrequency.

Apparatus and Standards

The apparatus for the present work was largely new and was built subsequent to the work of Part I. Particular care was taken on the numerous details of the equipment, much of which cannot be mentioned here. Some general precautions taken included: continuous thermostating of the whole room as well as the more precise thermostating of the standard cells and the Helmholtz coils; complete radiofrequency shielding of all electrical apparatus; a high speed pumping system (oil diffusion pumps because of the gold coated measuring chamber) which gave a pressure under working conditions of 1 to 2×10^{-6} mm/Hg; the keeping of liquid air on

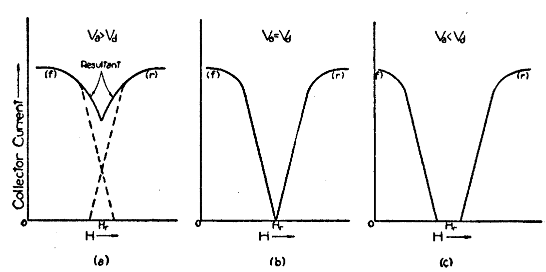

FIG. 4. Effect of contact potentials in gaps. V_a = potential between slits (1) and (2). V_d = potential between slits (5) and (6).

the trap between pumps and chamber at *all* times when the chamber was sealed and evacuated to prevent, as far as possible, the contamination of the gold surfaces with oil vapor; the careful reduction of stray magnetic fields from various parts of the equipment and from elsewhere in the building until the total of such d.c. fields was about 1.5×10^{-4} oersteds (effect on e/m negligible as the axial component of this is measured as a part of the earth's field) and of the a.c. fields to about one-quarter of this; the placing of all apparatus containing iron as far away as possible with a minimum distance of 1.5 meters from the chamber.

(A) Measuring chamber

A new chamber was constructed having many refinements in mechanical precision and accuracy. Some idea of the care put into its construction can be gained from the fact that ten months time of a skilled mechanic was required for its construction. (Auxiliary equipment made by the machine shops, which included the angle and radius measuring device, the form for the standard solenoid, etc., required another five months.)

A schematic diagram of the whole chamber has been given in Fig. 1. A detail of the region of the electron gun and collector is shown in Fig. 5 (same lettering of parts in both). A heavy brass can A with a removable cover has a heavy tube U passing through its center. A spider V consisting of a cylinder with four arms carrying slits (2), (3), (4), and (5) is accurately centered, aligned and *entirely* supported by a cone on U. The purpose of this design is to avoid any motion of the slit system due to compression of the chamber when evacuated. All four of these slits have conical joints, as at J, by means of which the edges of the slits could be adjusted to parallelism with the axis of U. All slits were adjustable also in a radial direction through the rotation of eccentric pins (not shown) and were held tightly in their adjusted positions by concealed leaf springs. The axial length of opening of all slits was stepped down to 2 mm by thimbles as at T. The spacing of slits (2) and (5) relative to (1) and (6) respectively (i.e., the distance d) could be varied by sliding the cylinders N in the cylinders O. Keys prevented rotation.

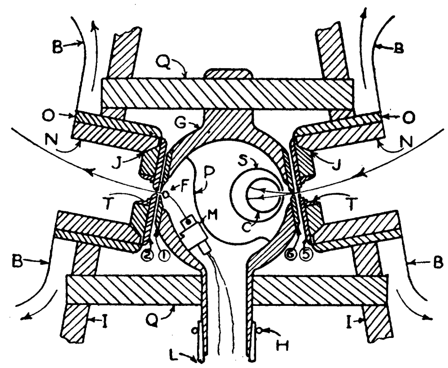

FIG. 5. Cross section of chamber in vicinity of electron gun G and collector C.

The electron gun mounting G (a cylinder perpendicular to the plane of figure) was supported on two ground quartz rods Q, the lead-in L to it being in part flexible braided copper tubing. A water cooling coil at H (supplied through flexible leads along L) kept the gun at the same temperature as the rest of the chamber. The gun subtended an angle of about 20° on the axis. The unipotential oxide coated filament at F was 5 mm long and indirectly heated by a hairpin heater which produced a negligible external magnetic field. The geometry connected with this filament and slit (1) is most important. The primary requirement is that, as the slit width Δr is reduced towards zero, the extent of the radiofrequency field approach in the limit the accurately measurable outer face of the slit (1). Other requirements are that sufficient emission be attained through the penetration of the radiofrequency field and that the distance from the face of F to the outer face of slit (1) be a minimum to avoid curling inward of the electron paths by the magnetic field before the electrons attain appreciable speeds. These requirements were satisfied by reducing the thickness of the slits (1) to 0.005 inch at their edges and by placing the filament by means of a microscope 0.005 ± 0.001 inch back of them. The filament was supported at M and enclosed in a "box" by a partition P with end caps above and below. Leads to it came through L and *through* the inductance plugged into the end of it.

The radiofrequency was fed to the electron gun through L and to the chamber through the concentric copper tube E (the chamber being grounded). Brushes K along the outer edge, top and bottom of the arms I provided a direct path from E to the slits (2) and (5). Pumping around the gun was accomplished through a large number of holes in the arms I and thence around the baffles B. The holes were covered with screen to further improve the radiofrequency shielding.

The collector at C was shielded by a shield S extending up from the bottom of the chamber. The outer edges of S and C were on the same radius as the outer slits, but the inner edges were on successively shorter radii to allow for curling of the electrons at lower velocities. The spacing between S and the back of slit (6) was about 1.5 mm.

To reduce the effect of surface charges on insulating films formed under electrom bombardment[5, 6] all of the surfaces of regions (1), (2) and (3), Fig. 1, were coated with gold. This was applied by evaporation from helical filaments after thorough cleaning, followed by the running of a discharge under a partial vacuum. In regions (2) and (3) the filaments were introduced temporarily through holes in the cover. In region (1) two filaments surrounded by gold grids were permanently mounted at R. This region frequently required recoating since the electron current bombarded a large fraction of the walls over a central strip about 5 mm wide. An electrode for running a discharge could be lowered (under vacuum) from the cover, after which it was withdrawn leaving no insulating surfaces visible to the beam.

The Helmholtz coils which produced the magnetic field were accurately aligned and spaced by mounting them on extensions of tube U above and below the vacuum chamber. To avoid the long time (3 or 4 hours) required for temperature, and hence for current equilibrium, the coils were equipped with water jackets supplied with thermostated water by a constant gravity flow system. The whole chamber was mounted on three legs and its axis kept parallel with the earth's field by use of the amplifier arrangement described in Part I, page 409.

(B) Radius and angle measurement

The value of e/m found is directly dependent on the angle θ given by Eq. (11). The electron radius r must be known only approximately for calculating the mass ratio, Eq. (18), but enters more precisely through the magnetic field H_r since the field constant of the Helmholtz coils is a function of the radius of the electron orbit.

A device to measure both angles and radius was designed as follows: a long internal cone was turned in the upper end of the tube U, Fig. 1. A male cone inserted in this carried a heavy plate in a plane perpendicular to the axis. The plate had a (silver) angle scale ruled along its outer periphery for some 35° and also carried a microscope with micrometer eyepiece located with axis parallel to the main axis and passing through the electron orbit. A second microscope with micrometer eyepiece was mounted on the outer (turned) face of the chamber C, Fig. 1.

If cross hairs in the rotating microscope are turned radially and made successively coincident with faces of slits (2), (1), (6) and (5) and the readings on the angle scale noted through the fixed microscope, the angles θ_f and θ_d are readily obtained. Since a complete circle was not ruled, errors of eccentricity could not be directly checked. Through the cone construction used in centering and pivoting, through the avoidance of dust particles on the cone, and through the most careful centering of the plate on the divided circle when the scale was ruled, it is believed the maximum possible eccentricity was 0.0003 inch, which would cause an error in θ of only 3 parts in 10^6. Through the courtesy of the Mount Wilson Observatory the scale was ruled on its large (approximately 4 foot) divided circle. The two microscope system was used in ruling, and lines were ruled every 5′ of arc. Measurements were always made using several pairs of lines on the angle scale but the results never differed by more than a few seconds of arc. To insure accuracy of setting, both micrometer eyepieces had cross hairs consisting of a *pair* of closely ruled lines, the object sighted on being centered between them. In spite of this there is a subjective error possible when sighting on the face (or end) of a slit due to the light and dark contrast. To check this the contrast was reversed in direction by sighting on the edges of the polished steel spacers used in setting the gap d. Agreement to one second of arc (probably this small only by chance) was obtained from the average of several sets of readings. Furthermore, the measured angle θ_d checked within 2″ arc that computed from the length d and radius r.

The observational probable error in measuring an angle, based on the external consistency of five determinations taken over a period of a year, was $\pm 3''$ arc. In spite of the apparent sum in Eq. (11), analysis shows that the observational error in θ is the same as for one angle. The probable error in θ was then estimated in seconds of arc as follows: observational $\pm 3''$; instrumental $\pm 3''$; indefiniteness of a slit edge due to rounding, etc. $\pm 4''$; total probable error $= \pm 6''$. An additive allowance of 4″ for constant errors brings the final p.e. in the angle θ to $\pm 10''$ arc in 340° or ± 8 parts in 10^6.

For the radial setting of the slits, the micrometer eyepiece of the rotating microscope was provided with both a fixed and a moving pair of cross hairs. One slit was then set to bisect the inner, the other to bisect the outer cross hair. This was done successively at all six slits and resulted in an accurately constant slit width Δr and slit radius r (mean). The actual values of r and Δr were obtained from a specially ruled glass scale. This scale (which had a large hole through its center) was placed on a table with a vertical column and cone (duplicate of that in U, Fig. 1), and centered on the axis of the cone. By placing the arm with microscope in this cone the radius of both the fixed and movable cross hairs could be determined from the closely spaced lines on the scale. The table was adjustable in height (for focus) and *both* the table and arm could be rotated to detect errors of eccentricity. The table moved about cylindrical surfaces so carefully ground that no error from play could be detected. Errors from illumination of the lines at different angles were eliminated by a device through which the light source was centered on the optical axis. The scale was calibrated in terms of a newly calibrated (N.B.S.) decimeter scale.

The probable error in the radius measurement was estimated as follows: since a motion of a slit of 2×10^{-4} cm was easily apparent, the p.e. in setting is taken as $\pm 1\times 10^{-4}$ cm; error from rounded edges, $\pm 2\times 10^{-4}$ cm; lack of exact parallelism of edges with axis of U (Fig. 1) $\pm 0.5\times 10^{-4}$ cm; calibration of scale $\pm 2\times 10^{-4}$ cm; total p.e. $= \pm 3\times 10^{-4}$ cm. An additive allowance of 2×10^{-4} cm was made for constant errors, giving a final p.e. in the radius of $\pm 5\times 10^{-4}$ cm, or about ± 5 parts in 10^5. The resulting error in the magnetic field H_r happens to be one seventh of this or $\pm 7\times 10^{-6}$ parts. Temperature corrections were made to the standard temperature used, namely 23.6°C.

The validity of taking the effective radius as the average of the inner and outer slit radii must be considered. It is well known, particularly in connection with 180° velocity filters,[8] that the divergence of the beam from the source causes a spread of the focal point into a short line extending radially inward. In the present case slits (5) and (6) are somewhat in front of the second focus (at which point the error would be zero). This, combined with the relatively small slit width used ($\Delta r/r \doteq 2.5\times 10^{-3}$) makes the effect small. An order of magnitude computation indicates that the shift of the center of gravity of the beam was probably not more than one part in a million of the radius, and hence is negligible.

(C) Collector current measurement

The electron current to the collector C, Fig. 5, was measured by an FP 54 electrometer tube used in a DuBridge and Brown circuit.[9] The maximum sensitivity with a leak of 8×10^{10} ohms was 1.4×10^{-16} amp./mm. Originally a large zero shift was observed with changing magnetic field due to a magnetron effect of the magnetic field (order of 15 oersteds) on the FP 54. This was eliminated by so placing and orienting the FP 54 that this shift was substantially zero over the range of magnetic fields used.

Since the collector was situated near the center of a source of radiofrequency radiation of the order of one-third of a kilowatt, shielding would not prevent blocking of the tube. The trouble was avoided by the insertion of a resistance of 10^8 ohms between the control grid and the collector side of the leak. At the high frequencies used (of the order of 5×10^7 cycles) this resistor removes most of the radiofrequency voltage from the grid to itself. Even at maximum sensitivity the switching on of full radiofrequency power

[8] W. A. Wooster, Proc. Roy. Soc. **A114**, 729 (1927).
[9] DuBridge and Brown, Phys. Rev. **4**, 532 (1933).

did not produce a detectable deflection of the galvanometer.

The collector and shield S (Fig. 5) were mounted eccentrically on a ground joint so that a small rotation of the system produced in effect a radial motion of the collector slit. The proper location of the collector slit (1 mm wide) was therefore easily found. Quantitative measurements showed that over a large "central" region no shift in the position of the current minimum could be detected.

No retarding potential was put on the collector to draw back secondaries since it was found that such a potential, for reasons not evident, caused a slight irregularity and asymmetry in the current minimum.[10]

(D) Frequency source and measurement

The general requirements in the radiofrequency source were the following: (1) it must furnish a continuous range of frequencies between 40 and 60 megacycles (7.5 to 5 meters); and (2) peak voltages up to 4000 volts; (3) the frequency must be constant and known to at least one part in 10^5.

The continuous range of frequency with good constancy was sufficiently well satisfied by using a low power electron coupled[11] master oscillator. The output of the oscillator was tuned to the second harmonic. This was followed by four stages of class C power amplification to yield the power of about one-half kilowatt necessary to produce the 4000 volt peak. This is shown schematically in the lower part of Fig. 6, the lower numbers in each block indicating the number and type of tubes used. The last amplifier fed through a concentric tube transmission line T_3 into a shielded box (adjacent to the e/m chamber) and was coupled to the inductance L. The capacity of this tank circuit consisted primarily of that of the electron gun to the adjacent slits. A *small* trimmer was added at C in order to be able to tune this circuit over a small frequency range. The use of two tuned tank circuits connected by a matched impedance transmission line undoubtedly produced a much more nearly sinusoidal voltage on the electron gun than the arrangement in Part I, and also greatly reduced in the e/m chamber *dc* magnetic fields from the plate current flowing through the inductances. To aid in this latter the r.f. chokes in the last two stages were made in the form of space wound toroids.

Leads to the electron gun filament, as mentioned earlier, pass *through* the tubing of which L is made and are taken out in a shielded cable at the point F (r.f. ground). At V is indicated a wire acting as an antenna which picked up a constant fraction of the electron gun voltage and impressed it on the grid of a tube voltmeter immediately adjacent. The latter was used to adjust the peak voltage to the desired value, as a monitor to check constancy, and as an aid in exact tuning of the amplifier. The voltmeter was calibrated by finding the maximum value of the magnetic field for a given r.f. voltage at which electrons can be sent around the e/m chamber (the inner slit radius was used for the voltage computation). The voltage (in volts) is given by

$$V=\frac{c^2}{(e/m_0)\times 10^8}\left(\frac{m}{m_0}-1\right).$$

Using Eq. (18) the voltage is obtained as a function of the frequency ν:

$$V=\frac{c^2}{(e/m_0)\times 10^8}\left[\left\{1-\left(\frac{r\theta\nu}{c}\right)^2\right\}^{-\frac{1}{2}}-1\right]. \quad (19)$$

Frequencies were obtained in terms of a government standard, using the carrier wave of one of the larger broadcast stations as an intermediary. Through arrangements made with the Federal Radio Commission, the San Pedro Monitoring Station made checks on the carrier frequency at times during which measurements were being made. In addition the management of KFI (the station whose carrier was used) made available the record of their checks taken at half-hour intervals in terms of their own standard. The latter was calibrated weekly by the crystal-clock system of RCA Communications Laboratory at Point Reyes. These two checks were in substantial agreement and indicated that the carrier frequency seldom deviated from its nominal value (640 kc) by more than one part in 10^5 and was usually within 4 cycles or 6 parts in 10^6.

No attempt was made to demodulate the carrier and to produce harmonics directly be-

[10] The use of a retarding potential did not cause any change in the location of the minimum.

[11] J. B. Dow, Proc. I. R. E. **19**, 2095 (1931) and Q. S. T. **16**, No. 1, 23 (1932).

cause it was felt that enough frequency modulation would be left to be troublesome. Instead a local oscillator (electron coupled) was set to within a fraction of a cycle of KFI's frequency through use of a broadcast receiver and a zero beat indicator (upper part Fig. 6). High harmonics of this (11th to 87th) were selected by a tuned output circuit and fed into a detector with tuned output. A small fraction of the output of the master oscillator was also fed into the detector by the transmission line T_1. Thus the nth harmonic of KFI's fundamental could be heard from the speaker beating with the mth harmonic of the master oscillator. Since the frequency of the master oscillator was doubled, the output frequency is given by

$$\nu = 2(m/m)\nu_0. \tag{20}$$

In some of the earlier results the multiplying factor was 6 instead of 2.

The probable error in the frequency ν was estimated as follows: variation of carrier from nominal value ± 4 parts in 10^6; operational, in setting and maintaining the zero beats ± 2 parts in 10^6; total p.e. $= \pm 3.5$ parts in 10^6. An additive allowance of 2.5 parts in 10^6 was made for constant errors giving a final p.e. in the frequency of ± 6 parts in 10^6.

The question was raised during the course of the work as to the exactness of the integral relationship of harmonics at these frequencies (order of 50 megacycles) where the size of a tank circuit is beginning to become an appreciable (though small) fraction of a quarter wave-length. A test was made to see if results were consistent when different harmonic combinations giving little change in frequency were used. The combinations[12] used were 6(54/5) and 6(11/1) with a frequency difference of only 1.8 percent. Measurements, made with a minimum possible lapse of time between to eliminate other possible variations, gave results differing in e/m_0 by only 1 part in 10^6, which is less than the observational probable error.

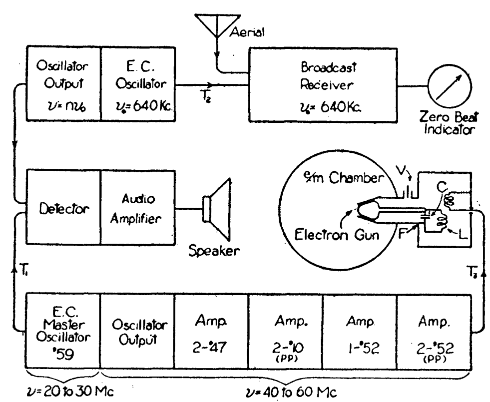

FIG. 6. Source of radiofrequency and scheme for frequency measurement.

(E) Magnetic field determination

The magnetic field is made up of two parts: that produced by the current i_H in the Helmholtz coils and that due to the earth's field (about 4 percent of the former). The first involves knowledge of the Helmholtz constant k_R on the central plane at a radius R equal to that of the electron orbit; the latter requires the value of the Helmholtz constant k_0 on the axis at the central plane, since the earth's field was measured by reversing the Helmholtz current and determining the current i_e necessary to balance the Helmholtz field on the axis against the earth's field. The expression for the magnetic field at the electron radius R is then:

$$H_R = k_R i_H + k_0 i_e. \tag{21}$$

Instead of using a flip coil and ballistic galvanometer to determine the balance with the earth's field, a much more sensitive and rapid arrangement was used consisting of a continuously rotating coil (40 r.p.s.) connected to a high gain amplifier (up to 10^6 voltage gain). The output was indicated by a sensitive detector circuit with a long time constant to smooth out transients. Ten readings spread out over ten minutes (to give a better time average) gave a result with an observational probable error of only ± 2.5 parts in 10^5, which effects H_R by ± 1 part in 10^6. The earth's field was measured before and after each run, small differences being linearly interpolated with time.

Both i_H and i_e were measured by the usual potentiometer method. A careful calibration of the Wolf potentiometer used was made and the small ohmic corrections applied to each reading. The currents could be read directly to one microampere (about 1 part in 10^6 of i_H). The standard of voltage consisted of three standard

[12] See Eq. (20).

cells kept in a thermostat with a temperature constancy better than 0.01°C. Two of the cells had new N.B.S. certificates shortly after the beginning of the present work (1933). The absolute value of the voltage standard was reestablished late in 1936 by a new cell with an N.B.S. calibration. Spurious errors of more than a few microvolts were eliminated by careful construction of the circuit and switching plugs.

(1) *Helmholtz coils.*—The same coils were used as in the work of Part I. Each consisted of 399 turns (19 layers, 21 turns/layer) wound in the channel of a cast aluminum wheel with stiff fiber insulation between layers. The mean radius was about 20 cm. The coils were hermetically sealed with beeswax to prevent moisture absorption.

The magnetic field constants were calculated directly from the geometry in Part I. Although the absolute values obtained were not used in the present work it was necessary in the process of comparison with a standard to know the manner of variation of the field over a region in the vicinity of the electron orbit. The method used to determine this variation took into account the cross section of the coils and was based on a direct adaptation to the present problem of Lyle's method[13] for the computation of mutual inductance between coaxial coils. Since his method is based on finding an approximately equivalent single filament which will give the same magnetic potential at all points, it is equally applicable to magnetic field computations. By computing the field constant for a series of radii centered about the electron orbit of radius $R(\doteq 9.9$ cm) and spaced at equal intervals, an expression of the following form (valid over interval $r=9.5$ to 10.3 cm) was obtained by central differences:

$$k_r = k_R[1-B(r-R)-C(r-R)^2 - D(r-R)^3 - E(r-R)^4]. \quad (22)$$

(2) *Standard solenoid.*—The absolute values of the Helmholtz field constants were determined by comparison with a carefully constructed solenoid. In choosing the general dimensions of the latter the choice had to be made between the usual long solenoid involving in this case certain difficult mechanical features, and a short solenoid without the mechanical difficulties and crowding. The latter was chosen. Although it is now felt the former would have been better, the only essential difference has been the increased work required to measure the short solenoid geometry.

The standard solenoid consisted of 100 turns of about No. 21 B & S gauge copper wire wound with a spacing of 36 turns/inch and having a radius of about 5.6 cm. In precise work the form used has generally been of marble but more recently[14] of quartz, glass or porcelain. The surfacing of these materials requires considerable technique. To avoid the troubles associated with such materials a form was evolved after some experimentation which consisted of a heavy copper cylinder threaded with rather flat grooves (120°). On this were thoroughly baked two coats of Bakelite varnish. Grooves were turned in the Bakelite matching those in the copper and leaving a uniform film of Bakelite 0.010 cm in thickness. The wire wound in these grooves was made especially for the purpose[15] (commercial wire not being sufficiently round) and deviated from uniformity in diameter by only 0.00005 inch (maximum to minimum). The advantages of this design are: the Bakelite provides an easily machinable surface which has sufficient insulation and which, due to its negligible thickness, cannot appreciably change the coil dimensions by moisture absorption; the coefficients of expansion of wire and form are the same; and (of the greatest importance) the thermal conductivity between wire and form is so high that no appreciable temperature difference can exist with a reasonable current (only 0.12°C with one ampere). A disadvantage is that a period of six months or more after winding must elapse before the diameter is measured in order to allow for flow of the Bakelite under the wire tension. With an initial wire tension of 1.1 kg, the observed diameter decrease was 0.002 cm or 1.8 parts in 10^4. The form was turned with an internal cone to fit over that on the tube U, Fig. 1, so as to make the solenoid coaxial and centered relative to the Helmholtz coils.

The wire was specially treated to remove kinks

[13] See page 38, Sc. Papers of N.B.S., No. 169.

[14] See for example "An Absolute Determination of the Ohm," Curtis, Moon and Sparks, N.B.S. J. Research **16**, 5 (1936).

[15] Acknowledgment is made to the Cleveland Wire Works of the General Electric Company for making and donating the special wire used.

and wound on the solenoid under constant tension. During the winding, the wire diameter was measured in a radial direction every half turn using a micrometer reading directly to 0.00001 inch so that errors from the small wire ellipticity were eliminated. The calibration of this micrometer was later checked on the comparator.

The over-all solenoid diameter was measured at 45° intervals on every turn (i.e., four measurements per turn). This was accomplished by a special apparatus built to fit on the carriage of a Pratt-Whitney precision lathe. The apparatus consisted of two anvils sliding in grooves on a vertical base plate and each carrying a small piece of glass with a diamond ruled line. The anvil ends, lapped flat, and wide enough to cover only one turn of wire were situated on opposite ends of a solenoid diameter. The carriage was attached to the screw so that the anvils followed the turns as the coil was rotated. The pressure against the wire (37 grams or 1.3 oz.) was the minimum required to avoid sticking. Doubling the pressure gave no appreciable difference. By the use of a microscope and a calibrated micrometer eyepiece, small displacements of the anvils could be measured relative to two fixed lines on a sheet of plate glass mounted on the back of the base plate and in the same focal plane as the moving lines. The distance between those fixed lines was determined by the National Bureau of Standards. The distance between the anvil lines when their faces were in contact was measured in terms of a decimeter also calibrated at the N.B.S.

Several readings on a given diameter usually had a spread less than 2×10^{-5} cm. The average eccentricity (maximum less minimum diameter) was 2.5×10^{-4} cm and the diameter increased fairly uniformly from one end to the other by 9.1×10^{-4} cm. To determine the effect of this diameter change, the field at the solenoid center was computed by dividing the solenoid into ten equal sections and obtaining the contribution from each. The analysis showed that the field was greater than the value obtained from the average diameter by 3.6 parts in 10^6.

The observational probable error in the mean diameter of the whole coil was obtained from the four diameter measurements by separating them into two groups each consisting of a 90° pair. This avoids eccentricity errors in the comparison. The two groups differed by 1.7×10^{-5} cm, which indicates a probable error of $\pm0.6\times10^{-5}$ cm, but to be conservative the observational p.e. will be taken as $\pm1\times10^{-5}$ cm. This and the other estimated probable errors r entering into the diameter are listed in Table I. Five lengths were required to convert micrometer eyepiece readings to diameters, and the five corresponding probable errors are listed in addition to the above observational p.e. All but the first of these involves the accuracy of the decimeter standard and the calibration of the Gaertner comparator, which are also listed. In addition, an additive allowance of two microns is made for constant errors, giving a final p.e. in radius equivalent to ±2.7 parts in 10^5.

The measurement of the pitch of the solenoid was made by placing the coil directly on the carriage of the comparator. Here again four transits were made the length of the coil but spaced at 90° intervals. To determine the effect of variations in pitch on the magnetic field, data were taken for a ten section analysis. The mean pitch of a ten turn section was taken as the mean spacing of five turns at the beginning and five at the end; for example the pitch of the third section (from the 20th to the 30th turns) was taken as one-tenth the distance from the mean coordinate of turns 18, 19, 20, 21 and 22 to that of turns 28, 29, 30, 31 and 32. The coordinate for a single turn was taken as the mean of the readings on each side. This eliminates errors from varying wire diameter. The ten section analysis showed that the effect of the pitch variation (caused by an unbalanced rotating member of the lathe carriage) was such as to lower the magnetic field at the center by 1.55 parts in 10^5. A twenty section analysis would probably not change this figure more than 1 or 2 parts in 10^6.

A list of probable errors r is given in Table II. The observational probable error was computed from the consistency of the four transits. The maximum deviation from the mean was about 1×10^{-6} cm (in about 0.07 cm) and the p.e. $=\pm2.6\times10^{-7}$ cm. The probable errors in the decimeter standard and in the comparator calibration were the same as in the diameter measurements (see Table I) for a length equal to that of the coil. Hence for *one* pitch the values are 0.01 of these. An additive allowance for constant errors was again made, giving a final p.e. in pitch equivalent to ± 1.4 parts in 10^5.

The mean radius and pitch of the winding on the solenoid can then be used to compute the magnetic field at the center, or as was actually done the mutual inductance with the flip coil. However, the result should be corrected by ± 3.6 parts in 10^6 for the radius variation, by -15.5 parts in 10^6 for the pitch variation, and by ± 4.2 parts in 10^6 due to the effective current radius being less than the radius of the geometric center of the wire, or a net correction of -7.7 parts in 10^6. The result so found had a probable error R given by the equation

$$R = [r_a^2(\partial H/\partial a)^2 + r_p^2(\partial H/\partial p)^2]^{\frac{1}{2}}, \qquad (23)$$

where r_a and r_p are the probable errors in the mean radius and pitch, found above to be, respectively, $\pm 15\times 10^{-5}$ cm and $\pm 1.0\times 10^{-6}$ cm; and $(\partial H/\partial a)$ and $(\partial H/\partial p)$ are the rates of change of field with radius and pitch, equal respectively to 1.211 and 38.11 oersteds/amp. R, the p.e. in the field of the standard solenoid, was found from these to be $\pm 1.86\times 10^{-4}$ oersteds/amp., or $+2.0$ parts in 10^5.

(3) *Calibration of Helmholtz field on the axis.*—The Helmholtz field at the axis was directly determined in terms of the standard solenoid by the usual method of opposing the two and measuring the balance point with a flip coil and ballistic galvanometer.[16] The axes of the coils were made accurately parallel to the earth's field, and the earth's field linkages had to be added to those of one field, namely the Helmholtz.

This comparison method makes the flux linkages of the flip coil with the standard solenoid equal to the sum of those from the Helmholtz field and the earth's field, that is

$$M_s i_s = M_H i_H + M_H i_e,$$

where M is a mutual inductance and i is a coil current. By straightforward reasoning it can be shown that the Helmholtz field constant k_0 on the axis is given by

$$k_0 = (M_s'/m_H')(i_s/i_H + i_e), \qquad (24)$$

where i_s and i_H are the standard solenoid and Helmholtz currents at balance, i_e is the Helmholtz current sufficient to produce a field equal to that of the earth, M_s' is the *calculated* value of the mutual inductance between flip coil and solenoid, M_H' is the same with the Helmholtz coils, and m_H' is defined by $M_H' = m_H' k_0'$, in which k_0' is the *calculated* Helmholtz field constant. Thus m_H' is the flux linkage with the flip coil from the Helmholtz field when the latter is equal to unity on the axis. It depends only on the manner of variation of the Helmholtz field in the flip coil region and not at all on the absolute value of the field.

The value of M_s', the mutual inductance of the solenoid with the flip coil, was calculated by the formula of Searle and Airey,[17] and as a check by the formula of Roiti,[17] the results agreeing to 1 part in 10^7 and giving the value 519,989.9 e.m.u. after the necessary correction of -7.7 parts in 10^6 (see preceding section). Since this is about $\frac{1}{3}$ percent less than would exist with a uniform solenoid field, it was necessary to calculate the uncertainty introduced into the result by uncertainties in the flip coil geometry. This was done in detail, together with some supplementary experimental measurements, and it was found that the probable error due to the flip coil geometry was not greater than ± 7 parts in 10^6. In passing it is to be noted that if *uniform* fields are being compared, this error is zero due to the ratio M_s'/m_H' in Eq. (24).

The value of m_H', the flux linkages from the Helmholtz coils per unit central field, was also calculated by the formula of Searle and Airey,[17] and as a check by the assumption of a uniform Helmholtz field, the latter approximation giving a value only 1.0 parts in 10^6 less. The result was 5507.131 e.m.u.

The value of i_s, the solenoid current at balance, was obtained by least squares from a plot of the galvanometer throws against i_s for values near balance, i_H being held constant. i_e was similarly obtained. Determinations were made on four days between 1 and 5 A.M. so as to avoid errors from the much larger earth's field (or local field) fluctuations in day time. Complete temperature corrections were computed but were negligibly small, due primarily to the design of the standard solenoid which kept the rise in temperature of the

[16] The continuously rotating coil and amplifier arrangement is not suitable for the comparison of fields, one or both of which vary appreciably in the volume occupied by the flip coil.

[17] Sc. Papers of the N.B.S., No. 169, pp. 61 and 57.

winding above that of the form to 0.12°C (with $i_s'=1$ amp.), and due to the thermostating of the room.

In the derivation of Eq. (24) it has been tacitly assumed that all surrounding space has a permeability equal to unity. The obvious deviation from this assumption requires investigation. During construction of the measuring chamber samples were prepared of every piece of brass used. Samples were also cut from one of the Helmholtz coil forms and from the standard solenoid. The susceptibility of each was measured and the results, expressed as the deviation from unity of the permeability, varied from -1×10^{-5} to $+1\times10^{-5}$. Since these materials occupied only a part of the space in the immediate vicinity, it is felt that the probable error from them is not more than ±5 parts in 10^6. As to material somewhat more removed: the placing of apparatus containing iron and the fields from all apparatus including circuits has been discussed earlier. There remains the effect of the iron reinforcing in the concrete floor at a distance of about 1.25 meters. A good estimate can be obtained from work[18] done at the National Bureau of Standards in the absolute determination of the ohm. Their glass form standard of inductance has approximately the same ratio of length to diameter as both the present standard solenoid and Helmholtz coils. Making allowances for the relative sizes, the iron in the floor probably increased the fields, and therefore the mutual inductance, of the Helmholtz coils by 5 parts in 10^6, and of the solenoid by less than 1 part in 10^6. The first increase would appear in the *measured* value of k_0 and the second is negligible. Hence no uncertainty results.

The value of k_0, the Helmholtz constant on the axis, was found by Eq. (24) to be

$$k_0=17.92116 \text{ oersteds/amp.},$$

a value 2.68 parts in 10^4 less than that calculated directly from the Helmholtz geometry.

A list of probable errors r is given in Table III.[19] The observational error given is the internal p.e. based on the p.e. (from least squares) of each of the four determinations. The external p.e. based on consistency of the four results was 1.1 parts in 10^5, a value slightly smaller than the internal p.e. An additive allowance for constant error brings the total p.e. in the Helmholtz field constant on the axis to ±4.0 parts in 10^5.

(4) *Ratio of Helmholtz field on axis to that on electron orbit.*—This ratio was determined through the use of two small flat coils placed in the plane of the electron orbit, one centered on the orbit (radius R) and one on the axis. These ratio coils were connected in series-opposing to a ballistic galvanometer so that the resulting deflection D_1 when the Helmholtz field was suddenly reversed was proportional to the *difference* of the two

TABLE I. *Probable errors in measurement of standard solenoid diameter (wire centers).*

Length	Origin of Error	Probable Error $r\times10^5$ cm
	Obs. p.e. in direct diameter measurement	1
1	P.e. in glass scale (taken as $\frac{1}{2}$ limit of error given by N.B.S.)	5
2	Meas. of anvil distance	3
3	Calibration of micrometer eyepiece (one side)	1
4	Calibration of micrometer eyepiece (other side)	1
5	Measurement of wire diameter	5
2–3–4–5	Decimeter standard (taken as $\frac{1}{2}$ limit of error given by N.B.S.)	5
2–3–4–5	Calibration of comparator in terms of decimeter	3

For diameter $(\Sigma r^2)^{\frac{1}{2}}$ $= 10\times10^{-5}$ cm
Allowance for constant errors $= 20\times10^{-5}$ cm
Final p.e. in diameter $= \pm30\times10^{-5}$ cm ($=\pm2.7$ parts in 10^5)
Final p.e. in radius $r_a=\pm15\times10^{-5}$ cm ($=\pm2.7$ parts in 10^5)

TABLE II. *Probable errors in measurement of standard solenoid pitch.*

Source of Error	Probable Error $\times10^7$ cm
Observational p.e.	2.6
Decimeter standard (taken as 0.5×0.01 limit of error given by N.B.S.)	5
Calibration of comparator ($=0.01$ p.e. in a 7 cm length)	3

$(\Sigma r^2)^{\frac{1}{2}}$ $= 6.4\times10^{-7}$ cm
Allowance for constant errors $= 3.6\times10^{-7}$ cm
Total p.e.: $r_p=\pm1.0\times10^{-6}$ cm ($=\pm1.4$ parts in 10^5)

[18] Curtis, Moon and Sparks, N.B.S. J. Research **16**, 32 (1936).

[19] The solenoid, Helmholtz coils and flip coil were so accurately centered and aligned that no appreciable errors could occur from this cause.

mutual inductances and to the Helmholtz current i_1, i.e.:

$$D_1 = c(M_0 - M_R)i_1.$$

The constant c can be eliminated if only one coil is used, for instance the one on the axis. The other is left in the circuit but removed to some distance. In this case the deflection is:

$$D_2 = cM_0 i_2.$$

By straightforward reasoning it can then be shown that the ratio of the Helmholtz field constant on the axis k_0 to that on the electron radius k_R is

$$k_0/k_R = [1-(D_1 i_2/D_2 i_1)]^{-1}\cdot(m_R/m_0). \quad (25)$$

If the single coil used is the one on the electron orbit, the deflection then being D_3 and the Helmholtz current i_3, the ratio becomes

$$k_0/k_R = [1+(D_1 i_3/D_3 i_1)](m_R/m_0), \quad (26)$$

in which m_R is defined by $M_R = m_R k_R$ and m_0 by $M_0 = m_0 k_0$. Here M_R and M_0 are the mutual inductances with the ratio coils on the electron orbit and axis respectively. m_R and m_0 are therefore the flux linkages per unit field at the center of the coil and depend on the manner of variation of the Helmholtz field (not on its absolute value) and on the geometry of the ratio coils.

In the above it has been assumed that the ratio coils are identical. The error resulting from their dissimilarity can be reduced to a negligible amount by interchanging them and taking the average of the two resulting ratios. The presence of a large amount of brass in the field causes a delay in the reversal because of the eddy currents induced, but the final change in flux linkages is unaltered. It is required only that the ballistic galvanometer period be long enough and that the rates of change of flux through the ratio coils be substantially the same. This last was accomplished by adjusting the relative amounts of brass around the two coils.

The ratio coils were very carefully made and each consisted of 594 turns, layer wound. The coil length was about 0.5 cm and the outer diameter about 2.6 cm. The diameter of each layer was measured at six points. Although such care is of some value for the ratio problem, it was taken primarily so that these coils might be used to check the geometry of the flip coil employed in the axis determination of the preceding section.

The value of m_0, the flux linkage with a ratio coil on the axis per unit axial field, is rigorously given by the formula

$$m_0 = \pi \sum_{n=1}^{n=N_L} a_n^2 N_T, \quad (27)$$

where a_n = radius of the nth layer, $N_T = 18$ = number of turns per layer, $N_L = 33$, the number of layers. This was computed for both coils, and the coils were found to differ by 2.13 parts in 10^3, although they had exactly equal inner and outer diameters.

The value of m_R, the flux linkage with the ratio coil on the electron orbit per unit field at radius R, can be most conveniently calculated from a development (exact on the central plane) of the following form:

$$m_R = m_0 - 2N_T \sum_{n=1}^{n=N_L} \int_{R-a_n}^{R+a_n} [B(r-R)+C(r-R)^2+D(r-R)^3+E(r-R)^4] r \cos^{-1}\left(\frac{r^2+R^2-a^2}{2rR}\right) dr, \quad (28)$$

where a is simply the radius a_n treated as a continuous variable in the integrand, and the bracket term is the fractional variation of the field from the value at R as given in Eq. (22). The integral was evaluated graphically. Sufficient precision is easily attainable since the whole negative term is less than 0.1 percent of m_0. The summation was evaluated for both ratio coils. These results were later checked by expressing the integrand in an approximate form which could be integrated, the result differing only 8 parts in a million from the graphical value (average) of m_R. These results from Eq. (28) were increased by 6.72 parts in 10^5 to allow for the small variation of the Helmholtz field off the central plane (i.e., along the 0.25 cm half-length of the ratio coils). This correction also was graphically determined. For comparison, m_R was of the order of 8 parts in 10^4 less than m_0.

In the taking of the data for Eqs. (25) and (26), two further precautions were observed: (1) all deflections were made closely the same in order to avoid any ballistic galvanometer error from nonlinearity; (2) besides taking data with the coils interchanged, additional data were taken for D_1 with the outer ratio coil rotated through 180° in order to average out eccentricity errors. The results: Eqs. (25) and (26) give ratios the average of which at the electron radius was $k_0/k_R = 1.033152$. From this and the value of k_0 given in the preceding section, the magnetic field constant at the electron radius (9.89810 cm) is

$$k_R = 17.34610 \text{ oersteds/amp.}$$

which is 10.02 parts in 10^4 less than that calculated directly from Helmholtz geometry.

A list of the probable errors r entering into k_R is given in Table IV. The observational probable error was calculated from the consistency of the ratios found by Eqs. (25) and (26), using completely independent sets of data and was ± 1.3 parts in 10^5. From the check on m_R mentioned it is believed that the p.e. in the calculation of m_R/m_0 is certainly not more than ± 0.3 part in 10^5. The effect of uncertainties in the geometry and location of the ratio coils was studied. The only dimension of importance was found to be the radius R of the center of the outer ratio coil. The fractional change in k_R is one-seventh that in R. Due to the care exercised in the precise mechanical construction of coils and holder, and in the measurement and checks of R, it is believed that the p.e. in R can be taken as ± 0.001 cm. This makes the fractional p.e. in $k_R = \pm 1.4$ parts in 10^5. Small temperature corrections were made in R and in all other contributing factors. The uncertainty in k_0 must be included in k_R. An allowance for constant errors (in addition to that in k_0) brings the total p.e. in the Helmholtz field constant on the electron orbit to ± 6.0 parts in 10^5.

(5) *Discussion of calibration results for the Helmholtz coils.*—The foregoing results of the calibration in terms of the standard solenoid gave measured values lower than those calculated directly from the Helmholtz geometry by 2.68 parts in 10^4 on the axis and 10.02 parts in 10^4 on the electron orbit. The reasonableness of this result should be examined. Since with the Helmholtz coils used, the rate of change of magnetic field on the electron orbit with changing Helmholtz radius was practically zero (a most

TABLE III. *Probable errors in the Helmholtz field constant k_0 on the axis.*

Source of Error	Probable Error $r\times 10^5$ Parts
Observational p.e. (internal)	1.4
Standard solenoid	2.0
Flip coil geometry	0.7
Permeability of surroundings not unity	0.5
$(\Sigma r^2)^{\frac{1}{2}}$	2.6
Allowance for constant errors	1.4
Total p.e.	± 4.0

useful property), there is one and only one solution to the necessary changes in the Helmholtz radius and spacing which will account for the results. It is that the radius is 0.0042 cm less and the total spacing 0.012 cm greater than the measured amounts. The first change is quite likely as a result of compression of the fiber interlayer insulation, as the radius was measured four years previous to calibration. Since the spacing between the Helmholtz forms was measured shortly before calibration, the mean centers of the windings must deviate this much from the geometrical centers of the forms. This deviation is quite possible. It is therefore concluded that the measured magnetic field constants are in reasonable agreement with the earlier calculated values.

(6) *Earth's magnetic field nonuniformity and its compensation.*—The uniformity of the earth's field, or more precisely the net field in the room, was measured by placing the rotating coil used in the measurement of the earth's field on an arm at the electron orbit radius. This could be rotated to any point on the orbit. The field was found to vary (maximum to minimum) around the orbit by practically one percent. The cause was undoubtedly a group of vertical iron pipes in a wall about 5 meters away. Since this amounts to 4 parts in 10^4 of the total field, a compensation device was necessary.

The arrangement used was suggested by Professor Barnett and consisted of two compact coils placed on the axis of the chamber, one a half meter above, the other a half meter below the plane of the electron orbit. The two coils were oriented with axes parallel to the electron plane and in the direction of the field maximum-minimum. With opposite polarities there results on the electron plane only an axial component (i.e., no radial component) which varies as cos ϕ, where ϕ is the azimuth angle. Thus correction is made for linear variation (in the maximum-minimum direction) of the room field. Since the room field did not vary quite linearly, it was possible to reduce the variation only from one percent to 0.25 percent. This, however, means that the deviation from the mean was only 5 parts in 10^5 of the total field, and since the direction of maximum-minimum happened to be symmetrically oriented with respect to the slit system, no appreciable error should result.

The mean value of the field around the orbit is higher than that at the center, and this requires a small correction of -6 parts in 10^6 in the value of e/m_0.

Results

In obtaining the data for the computation of e/m_0 by Eq. (16), the general procedure was to vary the magnetic field (i.e., vary the Helmholtz current i_H) and observe its value at the resonance minimum, with frequency held constant. Other experimental conditions held constant through all observations included: (1) The emission current to slit (1) was held at 10 μa; (2) the ratio of the peak of the radiofrequency voltage to that at which the electrons were accelerated was kept at the arbitrary value of 1.24; (3) the room was thermostated to a temperature at, or as near as possible to, 23.6°C; (4) the temperature of the Helmholtz coils also was maintained at about 23.6°C by adjustment of the temperature of the thermostated cooling water.

Helmholtz currents were recorded to a precision of 1 μa (they were of the order of one ampere) and all known corrections including temperature and ohmic errors in the potentiometer were applied to the same precision. These temperature corrections included standard resistance (kept in an oil bath), electron radius and Helmholtz spacing (the Helmholtz radius correction was negligible). These corrections were applied to the observed i_H so as to give the value which would have been obtained at 23.6°C (the temperature of all standardization). To facilitate comparison of runs at the same frequency the total magnetic field current i_T was defined as

$$i_T = i_H + (k_0/k_R)i_e, \tag{29}$$

so that the total magnetic field (see Eq. (21)) is then simply

$$H_R = k_R i_T = 17.34610 i_T. \tag{30}$$

H_R is now used with the added significance of the field *at resonance* and is therefore identical with H_r of Eq. (1).

(A) Shape of resonance minimum

A typical plot of current to the collector C, Fig. 1, as a function of the Helmholtz current is given in Fig. 7 for the region of resonance and on to the cut-off point. An enlarged detail of the lower quarter of the minimum is given in Fig. 8 in which a very narrow appendix is seen at the bottom. This tip of the minimum is further magnified in Fig. 9.

The shape of the resonance minimum is important since disturbing factors affecting it could cause the apparent minimum to differ in location from the true minimum. A small anti-clockwise tilting is evident in Fig. 7. However, if the large sloping background is subtracted most, though not quite all, of this tilting is removed. The deep, narrow tip of Fig. 8 is due to the fact that electrons in the vicinity of slit (6), when slowed down below a certain minimum velocity, suffer such an inward change of direction as to entirely miss the slit in the collector shield. The slowing down occurs, of course, *equally* on both sides of resonance. The correctness of this explanation was demonstrated by doubling the distance from slit (6) to the collector shield. The tip was then observed to triple in width. (In making this test a sleeve was inserted inside the gun to keep constant the capacity from shield to gun.) Part of the width of the tip was at times due to contact potentials in the gaps (see Fig. 4c). In two-thirds of the final essential data (upper two curves of Fig. 15) the tip was only 40 to 50 percent as wide as in Fig. 9.

In studying Fig. 8, it is noticed that the point P determined by extrapolation of the straight sides is to the left of the center of the tip by about 2.5 parts in 10^4. One reason the value of the field at the center of the tip has been taken as representing resonance and not P is that the location of P is not constant over a period of weeks and further can be shifted at will (over a small range) *without moving the tip* by applying, for example, a small retarding potential between collector and shield (collector positive). Another most important property of the tip of the minimum is that along the lower three-quarters its center, as shown in Fig. 9 by the line of mean abscissas, is absolutely vertical to 1 or 2 parts in 10^6.

(B) Slit correction data

As explained in the section on the "Effective Angle," observations on the current minimum must be taken for a series of slit widths Δr and the extrapolated value of the magnetic field used in Eq. (16) to get e/m_0. In addition, it was desired to check experimentally the validity of Eq. (11) by varying the slit spacing d and observing the constancy of the results. The data for these objectives are given in the first quarter of Table V and are plotted in Fig. 10. Curves drawn through the points must have zero slope at $\Delta r=0$ because the slit penetration by the field approached zero asymptotically as $\Delta r/d\rightarrow 0$. First, the solid curves were drawn graphically. Later, dotted curves were obtained by least squares *assuming* a function of the form $y=a+bx^2$. With separate plots it is apparent that the graphical fit is better for the upper curve and poorer for the lower. Because of this and because of the arbitrariness of the assumed function (limited by the small number of points) an average of graphical and least squares results is probably as reliable a value as can be obtained.

In all of the work that follows the slit width was held at 0.250 mm and the larger d used. The slit correction to the observed magnetic field current is then (see Fig. 10) $i_s=-0.000049$ ampere when the frequency is 4.14720×10^7 cycles. Similar data at a higher frequency of 5.18400×10^7 cycles gave $i_s=-0.000052$. The magnetic field current in the remainder of Table V has been corrected in proportion to these values.

The values of e/m from the two slit spacings differed by only 1.5 parts in 10^5, which is within the sum of the probable errors of extrapolation and is therefore considered a good check on the validity of Eq. (11).

(C) Observed e/m_0 as a function of electron energy

The most important check in any free electron determination is the constancy of results with varying electron energy. The initial data for this are found in the middle of Table V,[20] points 1 to 9, and are plotted as the lower curve in Fig. 11. An increase of about 2 parts in 10^4 in e/m_0 was found with a 57 percent energy increase (of the same order and sign as in the preliminary results, Part I).

A possible cause of this increase might be a phase difference in the radiofrequency voltage between the accelerating and decelerating gaps as a result of the greater capacity on the decelerating side of the electron gun due to the collector shield S (Fig. 5). To check this, an added variable capacity was provided for the accelerating gap in the form of a disk placed parallel to the face of slit (2) and held by a sleeve in contact all around with the cylinder θ, Fig. 5. A strip the width and length of the slits (2) was cut out of the middle of the disk so as not to change the slit spacing d. Quantitative analysis seemed impossible but rough estimates indicated 0.5 $\mu\mu$f should provide an effective balance.

Observations were made with an added capacity of 2.1 $\mu\mu$f (points 10 to 14 of Table V and dotted curve, Fig. 11). The results were essentially the same below 2100 volts, though the resonance minimum was shallow. But towards the higher energies the minimum became so very shallow and broad that its apparent location was appreciably shifted by the sloping background,

TABLE IV. *Probable errors in the Helmholtz field constant k_R at the electron radius.*

Source of Error	Probable Error $r\times 10^5$ Parts
Observational p.e.	1.3
Ratio of m_R/m_0	0.3
Effect on k_R of uncertainty in radius R	1.4
Field constant on axis, k_0	4.0
$(\Sigma r^2)^{\frac{1}{2}}$	4.4
Allowance for constant errors additional to that in k_0	1.6
Total p.e.	± 6.0

[20] The calculated values in this table were computed by formula as follows: electron voltage by Eq. (19); the external probable error R_e by least squares from the values for the associated runs; the mass ratio by Eq. (18); the apparent e/m_0 by Eq. (16) with θ (calculated by Eq. (11)) equal to 5.922561 radians for $d=1.308$ mm, and $\theta=5.920636$ radians for $d=1.594$ mm, and with H_R as obtained from Eq. (30).

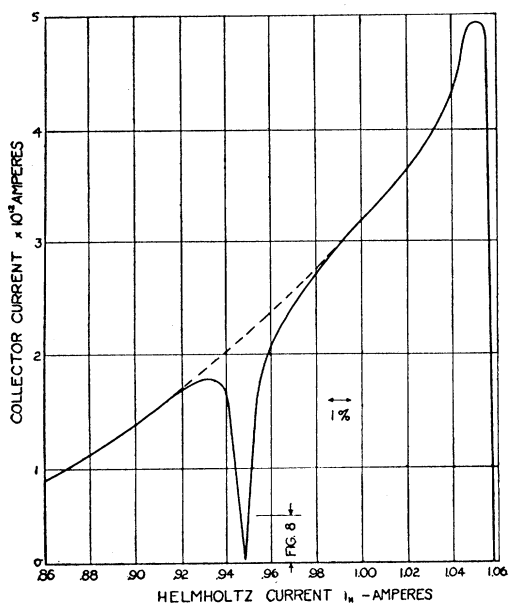

FIG. 7. Collector current in general region of resonance minimum. Frequency $\nu=4.9920\times10^7$ cycles, slit width $\Delta r=0.250$ mm, slit spacing $d=1.594$ mm.

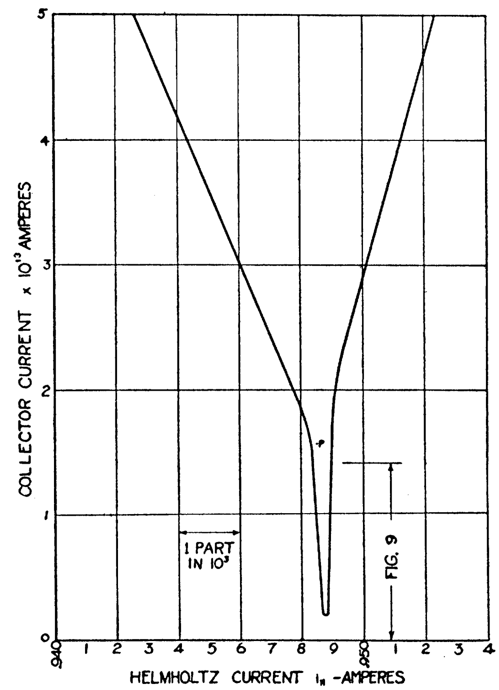

FIG. 8. A magnified section of Fig. 7 showing the bottom of the resonance minimum.

and rapidly increasing values of e/m_0 were found. The effect of unbalance due to considerable *over* correction having thus been shown, the correct estimated value of 0.5 $\mu\mu$f was tried (points 15 to 17). The minimum was now found to have returned to its normal depth but it was narrower by a factor of two than that shown in Fig. 9. This narrowing was probably not due to the change in capacity from 0 to 0.5 $\mu\mu$f but to a change in some other factor such as contact potentials (see Fig. 4c). No significant change from the $\Delta C=0$ results was found using 0.5 $\mu\mu$f. The apparent e/m_0 averaged 7 parts in 10^5 higher but the curve was tilted slightly to the horizontal (see Fig. 11) so as to suggest an approach to the same value at high energies.

It was now apparent that the condition of the gold surface in the deflecting chamber was influencing the results. To obtain quantitative data, a complete new coat of gold was evaporated on the deflecting chamber and new data immediately obtained with the same capacity of 0.5 $\mu\mu$f (points 18 to 21). The apparent e/m_0 shifted about 1 part in 10^4 higher and the curve was tilted still more to the horizontal. Another example of this shift is point (10) taken with $\Delta C=2.1$ $\mu\mu$f and all new gold. Due to experimental troubles a week's time and perhaps 15 hours of electron bombardment elapsed before

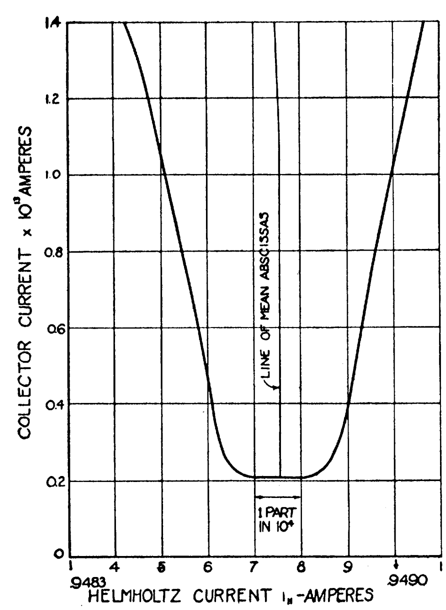

FIG. 9. A section of Figs. (7) and (8) with still greater magnification showing the bottom of the resonance minimum. During the taking of two-thirds of the final data the width was 40 to 50 percent of that shown here.

the taking of point (11), which was 1.4 parts in 10^4 lower. Still another example is the average value of the results, taken with new gold, of Fig. 10, namely $(\Delta e/m_0)/(e/m_0) = -5.45$ parts in 10^4 (on Fig. 11, would be near point (18) just below the curve), which is 2.42 parts in 10^4 higher than the average of points 1, 2, 3 and 4 taken with an old surface. From those examples and a study of earlier data it is evident that *if the gold surfaces of the deflecting chamber were a few days old and had suffered electron bombardment for perhaps fifteen hours, a reduction occurs in the apparent e/m_0 results of the order of 1 or 2 parts in 10^4, the curves becoming steeper the greater the reduction.*

(D) Surface potential study

It is believed that in spite of good vacuum conditions, small electron currents and the use of gold surfaces, the foregoing effect was due to the formation of an insulating film as described by Shaw[6] and Stewart.[5] Most of the electron bombardment occurs in compartment (1), Fig. 1. It therefore seemed of value to check this explanation roughly by inserting an insulated metal strip in compartment (1) (see dotted line, Fig. 1). This strip approximately covered the area bombarded by electrons and hence a positive potential applied to it should cancel the assumed negative charge on the insulating film (the latter of course would now be considered to have reformed on the strip). Qualitatively the results,

TABLE V. *Main data: first 7 points for slit correction (see Fig. 10); last 21 points show variation in e/m_0 with electron energy (see Fig. 11), taken with various capacities added to one side of electron gun to test possibility of phase error. A "run" consists of 10 consecutive observations, a "point" of two or more runs taken at different times.*

Curve	Point	Number of Runs	Slit Width Δr (mm)	Frequency ($\times 10^7$ Cycles)	Electron Voltage V (Volts)	Total Magnetic Field Current i_T (Amperes)	Probable Error, R_e ($\times 10^{-6}$ e.m.u.)	Mass Ratio m/m_0	Apparent e/m_0 ($\times 10^7$ e.m.u.)	$\frac{1.760-e/m_0}{1.760}$ ($\times 10^{-4}$ Parts)
Slit correction:	1	4	0.250	4.14720	1689	0.807 724	6	1.003 303		
	2	4	.300	"	"	.807 757	9	"		
$d=1.308$ mm	3	4	.400	"	"	.807 832	4	"	1.759 029	−5.52
	4	3	.550	"	"	.807 979	5	"		
	5	3	.250	"	"	.807 423	4	"		
$d=1.594$ mm	6	5	.400	"	"	.807 491	2	"	1.759 055	−5.37
	7	2	.550	"	"	.807 590	4	"		
	1	4	.250	"	"	.807 585	5	"	1.758 593	−7.99
	2	3	"	"	"	.807 579	2	"	1.758 606	−7.92
Variation with	3	4	"	"	"	.807 591	3	"	1.758 580	−8.07
energy:	4	4	"	4.16000	1700	.810 067	4	1.003 324	1.758 653	−7.65
	5	2	"	4.53120	2018	.882 809	3	1.003 947	1.758 829	−6.65
$C=0$ $\mu\mu$f	6	2	"	4.54400	2030	.885 344	1	1.003 969	1.758 786	−6.89
	7	2	"	4.60800	2087	.897 905	5	1.004 082	1.758 806	−6.78
	8	6	"	4.99200	2452	.973 386	2	1.004 796	1.758 871	−6.41
	9	6	"	5.18400	2646	1.011 149	3	1.005 175	1.758 968	−5.86
	10	8	"	4.14720	1689	.807 453	4	1.003 303	1.758 880	−6.36
	11	2	"	4.16000	1700	.810 073	5	1.003 324	1.758 640	−7.73
	12	2	"	4.60800	2087	.897 917	3	1.004 082	1.758 782	−6.92
$C=2.1$ $\mu\mu$f	13	2	"	4.99200	2452	.973 328	2	1.004 796	1.758 975	−5.82
	14	2	"	5.18400	2646	1.011 014	5	1.005 175	1.759 203	−4.53
	15	2	"	4.16000	1700	.810 013	1	1.003 324	1.758 771	−6.98
$C=0.5$ $\mu\mu$f	16	2	"	4.60800	2087	.897 846	2	1.004 082	1.758 921	−6.13
(Old gold)	17	3	"	5.18400	2646	1.011 087	4	1.005 175	1.759 076	−5.25
	18	4	"	4.16000	1700	.809 876	5	1.003 324	1.759 068	−5.30
$C=0.5$ $\mu\mu$f	19	3	"	4.60800	2087	.897 749	3	1.004 082	1.759 111	−5.05
(New gold)	20	2	"	5.18400	2646	1.010 997	0	1.005 175	1.759 233	−4.36
	21	2	"	5.56800	3056	1.086 727	3	1.005 977	1.759 274	−4.13

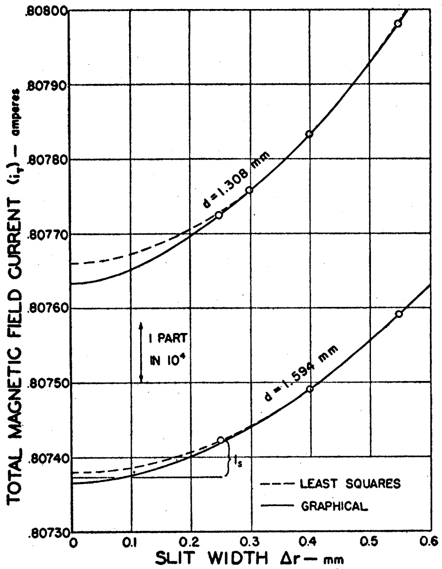

FIG. 10. Effect of slit width on location of resonance minimum. $\nu=4.14720\times10^7$ cycles.

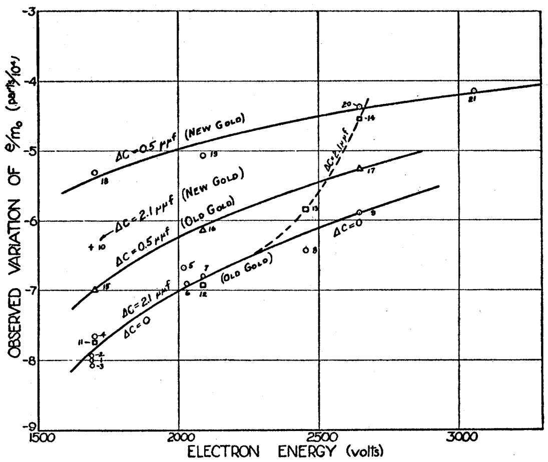

FIG. 11. Variation of observed e/m_0 with electron energy. Slit width $\Delta r=0.250$ mm, slit spacing $d=1.594$ mm. At zero variation $e/m_0=1.7600\times10^7$ e.m.u. ΔC is the extra capacity added at the accelerating gap. The curves were determined by least squares assuming the form $y=a-b/x$.

Fig. 12, check the surface charge explanation: the observed values of e/m_0 are *raised* and by amounts greater for lower electron voltages than higher, thus giving a more nearly constant value of e/m_0 with varying electron energy.

Quantitatively the points (18) and (20), on which the variation is based, require a *difference* in the two curves of Fig. 12 of 0.94 part in 10^4 to make the results independent of energy. This (by extrapolation) requires a strip potential of roughly +9.4 volts to neutralize the surface potential on a new gold surface. Similarly for an old gold surface the $\Delta C=0$ curve indicates a surface potential of approximately 19 volts. The existence of surface potentials of this magnitude on metals such as gold, or even brass, has apparently not been previously reported. Even with positive ions under conditions of high current density and poor vacuum conditions, one volt is about the maximum reported.[6a] Shaw[6] has given some data for a transient phenomenon he terms "surface polarization" (electron beams moving parallel to metal surface). For gold under the present conditions, his work would indicate potentials of the order of 0.01 to 0.25 volt. He gives no data for the permanent films formed by electron impact with which we are mainly concerned here. Stewart[5] states that these films have minimum resistances of 10^6 ohms (over an area of perhaps (?) 10 mm^2) and rupturing voltages of 4–14 volts. Nevertheless the positive experimental evidence that a reduction in the observed value of e/m_0 does occur with aging of the gold surfaces, the initial value being again found with a new surface, seems to indicate definitely such large surface potentials[21] as the cause of the variation of the observed e/m_0 with energy.

(E) Study of effect of space charge in front of filament

Two other factors have been found which change the observed e/m_0 values. Both of these are associated with the space charge in front of the filament. Fig. 13 is an instantaneous plot of the potential from the face of the filament to the deflecting chamber, when the slit width Δr is not zero. The effective accelerating distance is then not d but AB, and the location of A is a function of the space charge existing at that instant. A change in A[22] alters the observed e/m_0 through the change of the effective angle θ.

(1) *Effect of varying the filament emission.*—Increase of electron emission through raising the

[21] Since the strip test is an approximation to actual conditions the potentials found are also to be taken as approximate, but should be correct within a factor of two.

[22] The location of point A would also be changed if the filament were not replaced at the same distance from slit (1) after recoating. In practice it was found that results never differed after recoating by more than 1 part in 10^6 and averaged much closer than this.

filament temperature always caused a decrease in the observed e/m_0, as is shown in Fig. 14. The point A would be pushed outward from the filament by increased space charge. This would decrease θ and hence increase the observed e/m_0 since it is computed with constant θ. Since the observed shift is negative it is thus evident that increased negative surface potentials due to the increased density of current incident on the films is a more important factor resulting in a reduction of the observed e/m_0.

(2) *Variation of the peak radiofrequency voltage.*—If the frequency and emission were held constant and the peak radiofrequency voltage V_m increased from a value near the resonant electron voltage V_0, the observed e/m_0 was found to increase slightly: 0.6 part in 10^5 at $V_m/V_e=1.24$, 1.2 parts in 10^5 at 1.35, 2.3 parts in 10^5 at 1.47. The space charge and field in front of the filament are not static but continually changing in step with the radiofrequency. During the half of each cycle in which slit (2) is negative no current flows and the space charge grows, probably spreading out even in front of slit (1). Then as the potential reverses, current again flows and the space charge is gradually reduced. But the time elapsing between zero potential and a potential of V_e volts is less the greater the peak V_m, and hence with more of the space charge left, the point A is farther from the filament and the observed e/m_0 is greater since it is computed with constant rather than with decreasing angle.

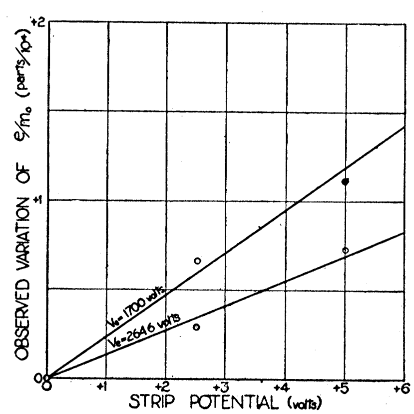

FIG. 12. Effect on e/m_0 of potential on an auxiliary strip placed to simulate the negative surface charges but used with a positive potential so as to cancel them. Data taken at two electron energies.

This applies to the electrons accelerated on the rising part of the cycle (see Part I, page 407). Those accelerated on the falling part of the cycle might experience no change in angle or possibly a smaller increase. Thus, increasingly higher peak voltages should also cause increasingly divergent effective angles for the two groups of electrons, the sense of the changes being such as to make the minimum more shallow. This effect was observed though it was small.

The essential point in consideration of both these space charge effects is that no net error results in e/m_0 with the procedure adopted: i.e., all results taken with the same emission current (10 μa to the first slit), the same ratio of peak to electron voltage ($V_m/V_e=1.24$) and with the slit correction determined at various frequencies. In fact, it makes no difference where the point A occurs as long as it is fixed for all observations at a given frequency, since the method of extrapolating results to zero slit width Δr always moves it in the limit (through the i_s correction) to the face of slit (1).

The effect of surface changes remains after the slit correction. If, however, the surface charge is constant (constant emission current and small lapse of time) and the results are extrapolated to infinite electron energy, this error also should be removed.

(F) The value of e/m_0

The experimental situation is then briefly this: no indication of an error due to a phase difference between accelerating and decelerating slits was

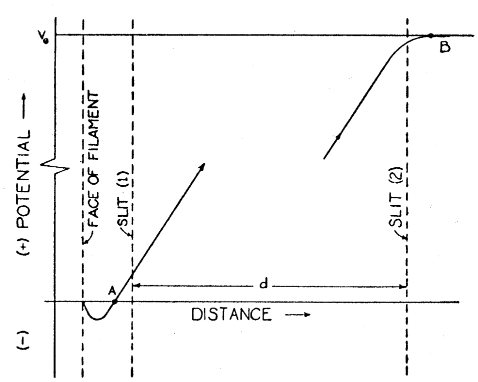

FIG. 13. Qualitative picture of potential in region of filament and accelerating gap at the time the electrons dealt with are accelerated, showing effect of space charge on accelerating distance $A-B$.

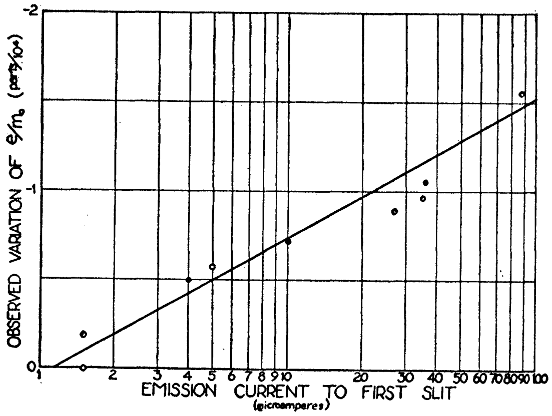

FIG. 14. Effect of varying filament emission on e/m_0. Taken at an electron energy of 2087 volts by varying the heater current of the oxide coated cathode. All other data in this paper were taken at about 10 μa.

found (quantitative data presently); error from the space charge in front of the filament changing the effective angle θ was eliminated by constant experimental conditions and by the method of extrapolating results to zero slit width; error from surface charges on the walls of the deflecting chamber remains but can be eliminated by extrapolating the e/m_0 observations to infinite electron energy.

To make this extrapolation with the limited number of points available it is necessary to assume a function to represent the curves of Fig. 11. (The $\Delta c=2.1$ $\mu\mu$f curve was taken under obvious out-of-phase conditions and hence has been disregarded.) The function assumed was $y=a+b/x$. To check this, the data have been replotted in Fig. 15 against the reciprocal of the electron energy, and it is seen that, within the accuracy of the data, a linear variation occurs. The straight lines of Fig. 15 and the curves of Fig. 11 (equivalent) were determined by least squares.[23] The resulting values of the zero intercepts together with their observational probable errors, also determined by least squares,[24] are given in Table VI. The observational probable error in the average is indeed small, that determined by external consistency being $R_i=\pm 9.0$ parts in 10^6 and by internal consistency $R_i=\pm 9.4$ parts in 10^6. The close agreement between R_e and R_i indicates[25] not only the consistency of the data in the value of e/m_0 found under varied experimental conditions (with new and old surfaces and with appreciable variation of the capacity on the acceleration side of the electron gun) but is also a rather good check on the validity of the functional relation to which the points have been fitted. This last is particularly true in view of the variation in the slope of the three curves by a factor of two.

The mean value of e/m_0 given in Table VI must be changed by several small corrections as follows (expressed as parts in 10^5): $+7.2$ from international to absolute amperes;[26] -0.6 from the nonuniformity of the room magnetic field resulting in the mean value around the electron orbit averaging slightly higher than that measured at the center; -1.3 from a small error found in the Helmholtz magnetic field ratio calculations; total correction $+5.3$ parts in 10^5. This gives the final value[27] of e/m_0 as

$$e/m_0=1.7597.$$

The experimental data of this section constitute a determination of the total magnetic field current i_T of Eqs. (29) and (30). The estimated probable errors in this determination are listed in Table VII. The observational error is that of the mean zero intercept as given in Table VI. The uncertainty in i_T involves the p.e. in the slit correction i_s and in the ratio k_0/k_R (see Eq. (29)). The p.e. in the latter is that part of the determination of k_R which is not common to that of k_0 (to obtain, omit p.e. in k_0 from

[23] In making this computation each point was given unit weight, rather than being weighted according to the external probable errors R_e given in Table V, because it was felt that the *variation* in the probable errors so computed from the consistency of two or more runs did not really represent the relative accuracy of the points. In fact, frequently the p.e. (internal) of a single run was *less* than the external p.e. of the mean of two or more runs simply because the precision of a single observation was so high and because there were uncontrollable systematic variations between runs (such as aging of the gold surface). The actual p.e. of most of the points in Table V can probably be taken as 4 or 5×10^{-6} amp. (estimated by taking a large number of runs for a single point over a period of several days with gold surface and filament renewed when necessary).

[24] R. T. Birge, Phys. Rev. **40**, 224–227 (1932).

[25] R. T. Birge, Phys. Rev. **40**, 213–223 (1932), particularly page 219.

[26] H. L. Curtis and R. W. Curtis, N.B.S. J. Research **12**, 665 (1934).

[27] The preliminary result (reference 1) differs from this due primarily to the inaccuracy of the formula there used for the effective angle (Eq. (4)) and also probably to the difference between the actual magnetic field and that calculated from the Helmholtz geometry. Due to the impossibility of calculating either of these errors the preliminary result cannot be recomputed and compared with the final result.

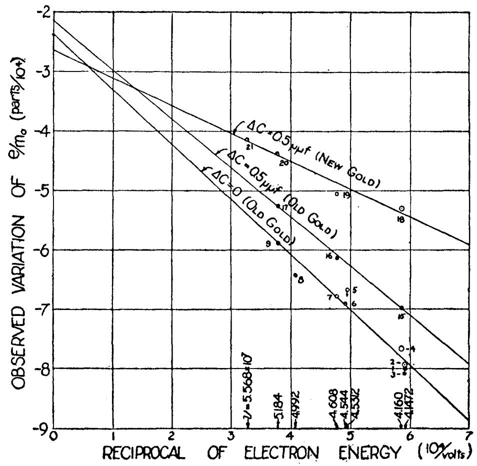

FIG. 15. Variation of observed e/m_0 with electron energy. Same data as Fig. 11 replotted against the reciprocal of electron energy. The radiofrequencies at which the observations were taken are also indicated along the abscissa scale. At zero variation $e/m_0=1.7600\times10^7$ e.m.u. The straight lines were determined by least squares, the zero intercepts giving the indicated values of e/m_0.

Table IV). The uncertainty introduced into i_T from this uncertainty in the ratio k_0/k_R (see Eq. (29)) is only about 4 percent of the latter because i_e is only about 4 percent of i_T. These probable errors, together with the standard voltage resistance and potentiometer errors, give a total p.e. of ±5.8 parts in 10^5. In addition, a generous allowance has been made for constant errors of 14.2 parts in 10^5 (almost one and a half squares in Fig. 15). This covers possible errors in extrapolation (including the assumed function), errors in phase of the radiofrequency voltage, and errors from penetration of the radiofrequency field into the deflecting chamber (felt to be negligible because of shielding precautions). The final p.e. in the total magnetic field current i_T is then ±20 parts in 10^5.

Estimated probable error

The probable error in each factor entering into Eq. (16) for the computation of e/m_0 has been evaluated in detail at the point in the article where the factor has been discussed. It is to be recalled that each of these components has already had added to it an allowance for constant errors, these allowances amounting on the average to more than a 100 percent increase. It should also be mentioned that the probable errors used for N.B.S. calibrated standards have been taken as one-half of the limit of error given by the Bureau. This is equivalent to saying that there is about one chance in five of their limit of error being exceeded.

In Table VIII the probable errors for each of the main factors have been listed together. In addition to the uncertainty in the Helmholtz constant k_R at any given radius there is a small additional probable error due to uncertainty in the actual radius used (i.e., in the radius of the electron orbit) which is one-seventh as great as the latter. The final probable error based on these components is ±2.09 parts in 10^4 or $(\pm0.00037)\times10^7$ e.m.u. The latter figure will be rounded off so that there results

adopted probable error in

$$e/m_0=(\pm0.0004)\times10^7\ e.m.u.$$

This is equivalent to ±2.3 parts in 10^4 or 1 part in 4400.

It is evident that the probable error is made up almost entirely of the allowance for constant errors in i_T since the magnetic field calibration enters but slightly and the other factors are negligible. In fact, if the error is calculated without the allowance for constant errors a value only one-fourth as large is obtained, namely ±0.0001.

DISCUSSION

The value of e/m_0 found in the present work is

$$e/m_0=(1.7597\pm0.0004)\times10^7\ \text{e.m.u.}$$

This is higher by 1.2 parts in 10^3 than the recent value of $(1.7576\pm0.0003)\times10^7$ as given by Birge,[28] a value determined primarily by the two recent spectroscopic determinations (fine structure of H^1-H^2) of Shane and Spedding[3]) and of Gibbs and Williams[4]) because the estimated

TABLE VI. *Results (from Fig. 15) of three determinations of e/m_0, the intercepts representing the fractional variation from an arbitrary value of $e/m_0=1.7600$. All data determined by least squares.*

Curve	Zero Intercept $\times10^4$ Parts	Obs. Probable Error $\times10^5$ Parts	Weighted Mean and Observational Probable Error
$C=0$ (old gold)	-2.39	±2.87	-2.26 parts in 10^4 or
$C=0.5$ " "	-2.15	±1.11	$e/m_0=1.759602$ ±0.000016
$C=0.5$ (new gold)	-2.63	±2.20	$R_e/R_i=0.96$

[28] R. T. Birge, Nature **137**, 187 (1936).

probable errors are appreciably smaller than those of all other determinations made previously. The other recent spectroscopic determination of Kinsler and Houston,[2] using the Zeeman effect, has a consistent but slightly lower value. The weighted mean of these three is $(1.75774 + 0.00023) \times 10^7$ e.m.u. and this may be taken as the most recent "spectroscopic" value. The author's value is higher than this by 1.1 parts in 10^3, or to put it differently, the values differ by 3.0 times the sum of the probable errors. Computing the weighted mean of the spectroscopic and author's values, the ratio of the external to the internal probable errors[25] R_e/R_i is found to be large compared with unity, namely 2.87. The chance of this occurring by purely statistical fluctuation[29] is about 1 in 120. The author's result then apparently reintroduces the question of the disagreement between spectroscopic and free electron values of e/m_0. By this there is implied not any real difference in e/m_0 but rather an error in the experimental work of either method or in the theory involved in the spectroscopic method.

There is, however, some new spectroscopic work which, when it has been carried beyond the present preliminary stage, may help to clear up the difficulty. Houston[30] has recently developed a new method of obtaining the fine structure separation from the microphotometer patterns. The method is essentially a Fourier analysis and should yield values much less subject to error. The value of e/m_0 he obtains in his fine structure study of H^1-H^2 is $(1.7601 \pm 0.0015) \times 10^7$ (the probable error was given to the author by Houston), a value very close to the author's result. No conclusions can be drawn until this probabler error is reduced, but the fact that Houston's result is considerably higher than the other two fine structure determinations at least raises the question as to what changes would result if the fine structure data of the others had been analyzed by this method. In addition there is the possibility that the fine structure theory itself is in need of revision, particularly in view of the fact that the observed separations of the two main components of Hα and of Dα are 2 percent less than the values predicted by the theory. If upward revision of the fine structure e/m_0 should result from either of these two possibilities (method of analysis or theory) there would still remain a discrepancy with the careful Zeeman effect work.[2]

During the writing of this discussion word has been received[31] of the final results of a deflection determination using crossed fields by A. E. Shaw of Chicago. The value found is 1.7571 ± 0.0013. This result is 1.5 parts in 10^3 lower than the author's, the two differing by 1.5 times the sum of their probable errors. In this case, however, Shaw's probable error is large enough to give a ratio of R_e/R_i much nearer unity, namely $R_e/R_i = 1.19$. The deviation of this from unity is less than half the probable error in the ratio, so that no real discrepancy is indicated. That is to say, the probable errors of the two measurements are sufficiently large that the difference in

TABLE VII. *Probable errors in the determination of the total magnetic field current i_T at resonance.*

Source	Probable Error $r \times 10^5$ Parts
Observational error in determination of i_T	0.9
Uncertainty in the slit width correction i_s	1.2
Effect on i_T of p.e. of 3.5×10^{-5} parts in (k_0/k_R)	0.1
Standard of voltage (taken as $\frac{1}{2}$ of N.B.S. limit of error)	5.0
Standard resistance (taken as $\frac{1}{2}$ of N.B.S. limit of error)	2.5
Potentiometer (ohmic errors and thermal e.m.f.'s)	0.5
$(\Sigma r^2)^{\frac{1}{2}}$	5.8
Allowance for constant errors including extrapolation, etc.	14.2
Total p.e.	20.0

TABLE VIII. *Probable error in e/m_0 estimated from the various main components entering into its determination.*

Source	Probable Error $r \times 10^5$ Parts
Angle θ	0.8
Frequency ν	0.6
Helmholtz constant k_R	6.0
Additional uncertainty in k_R from p.e. of 5 parts in 10^5 in electron radius	0.7
Total magnetic field current i_T	20.0
Final p.e. in e/m_0	20.9

[29] Throughout this discussion the chance of a discrepancy occurring by statistical fluctuations has been based on the formula for the proportional p.e. in the estimated p.e. as given by Deming and Birge (Rev. Mod. Phys. **6**, 149 (1934), *viz.*: $u = 0.4769/(n-1)^{\frac{1}{2}}$ where n is the number of observations.

[30] W. V. Houston, Phys. Rev. **51**, 446 (1937).

[31] Result communicated to the author in a letter from Dr. Shaw.

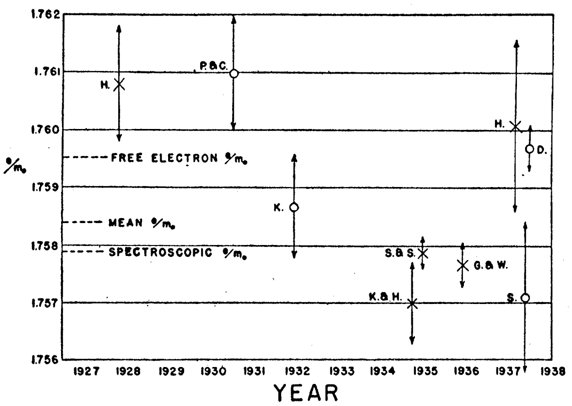

FIG. 16. Chronological plot of e/m_0 values used in estimating the present probable value of e/m_0. ($\times$) are spectroscopic values, $\odot$ are free electron values. Arrows show probable errors.

the e/m_0 values is well within the statistical fluctuation to be expected.

The preceding comparisons have been limited entirely to quite recent work (1934 and later). There is, however, some earlier work which should be included in making comparisons and estimating the present probable value of e/m_0. The results which are felt to be significant are listed in Table IX and are chronologically plotted in Fig. 16.[32] Houston's work[33] on H^1-He is included and though made with older methods of analysis is of interest since it is quite possible that an error in the fine structure theory would effect the results less with helium than with deuterium. Perry and Chaffee's[34] and Kirchner's[35] linear acceleration values have also been included. In all cases only the final results of an individual or group at a given institution have been retained.

If the data are treated as a *whole* to obtain the present probable value of e/m there results the value: $e/m_0 = 1.75842 \pm 0.00026$. For this the probable error ratio R_e/R_i is 1.42. While the chance of this occurring by statistical fluctuation is only 1 in 10, one cannot attach much weight to this alone as proving a discrepancy.[36]

If, however, the data are divided into the two groups indicated in Table IX, namely spectroscopic and free electron, the discrepancy is again evident since the inclusion of these additional e/m_0 results produces only small shifts in the weighted means as follows: in the spectroscopic value a shift of $+0.00020$, giving 1.75794 ± 0.00026; in the free electron value a shift of -0.00016 from the author's result giving 1.75954 ± 0.00033. The weighted mean of these has a ratio $R_e/R_i = 2.55$, again differing considerably

[32] Kretschman's work (Phys. Rev. **43**, 417 (1933)) has not been included since it yields a value of $(e/m)(e/h)$ if the x-ray wave-lengths are assumed correct. It is of interest to note that, as shown by Birge (Nature **133**, 648 (1934)), if the x-ray value of $e = 4.803 \times 10^{-10}$ e.s.u. is assumed and also the validity of the Rydberg formula, Kretschman's results give a value of $e/m_0 = 1.760$. Robinson's work (Phil. Mag. **18**, 1086 (1934)) has been omitted for the same reason.

[33] W. V. Houston, Phys. Rev. **30**, 608 (1927). Corrected by Birge, Rev. Mod. Phys. **1**, 1 (1929), but probable error used as given by Houston to author.

[34] C. T. Perry and E. L. Chaffee, Phys. Rev. **36**, 904 (1930).

[35] F. Kirchner, Ann. d. Physik **8**, 975 (1931) and **12**, 503 (1932).

[36] It is only when the chance becomes quite small, say of the order of 1 in 50 or 1 in 100, that a discrepancy can definitely be assumed. See reference 25, in particular page 223.

TABLE IX. *Summary of e/m_0 determinations used in estimating the present probable value of e/m_0 and in showing the discrepancy which has again appeared between the spectroscopic and free electron determinations.*

EXPERIMENTERS	DATE	METHOD	e/m_0 $\times 10^{-7}$ e.m.u.	PROBABLE ERROR $r \times 10^4$	WEIGHT $=225/r^2$	e/m_0 $\times 10^{-7}$ e.m.u.	e/m_0 $\times 10^{-7}$ e.m.u.
Houston[33]	1927	Fine structure H^1-He	1.7608	10	2.3	$1.7579 \pm 3-$ ($R_e/R_i = 1.19$)	$1.7584 \pm 3-$ (From the whole: $R_e/R_i = 1.42$ From the two groups: $R_e/R_i = 2.55$)
Kinsler and Houston[2]	1934	Zeeman effect	1.7570	7	4.6		
Shane and Spedding[3]	1935	Fine structure H^1-H^2	1.7579	3	25.0		
Gibbs and Williams[4]	1935	"	1.7577	4	14.1		
Houston[30]	1937	"	1.7601	15	1.0		
Perry and Chaffee[34]	1930	Linear acceleration	1.761	10	2.3	$1.7595 \pm 3+$ ($R/R = 1.01$)	
Kirchner[35]	1932	" "	1.7587	9	2.8		
Shaw[39]	1937	Deflection	1.7571	13	1.3		
Dunnington	1937	"	1.7597	4	14.1		

[39] A. E. Shaw, Phys. Rev. **51** (L), 887 (1937).

from unity. The chance of this occurring by purely statistical fluctuation is about 1 in 35. Although not as great a discrepancy is indicated with the inclusion of these additional data, the discrepancy is still very apparent, and amounts to 1 part in 1100 in the value of e/m_0.

The situation then is briefly this: both the author's value and the free electron value (mean of four determinations) are in definite disagreement with the spectroscopic value (mean of five determinations). The possibility that this disagreement is not real as far as the fine structure results are concerned but is due to the method of analysis is suggested by Houston's recent work.[30] This work, along with other earlier work,[37] also suggests the need of a refinement of the fine structure theory. That the trouble may be in the author's work is certainly also to be considered.

It should be mentioned that the increase in e/m_0 indicated by the author's results does not remove the present puzzling discrepancy in the fundamental physical constants,[28, 38] for this increase amounts to only about one-fifth of that necessary to bring agreement.

In conclusion it is desired to emphasize two points in connection with the author's value: (1) the probable error adopted is not simply an observation probable error. Rather, the precision of all the measurements was refined until the observational probable errors were negligible, leaving only basic uncertainties in the method and in the interpretation of the physical phenomena observed. (2) All of the final measurements were completed and the apparatus dismanteled before any computations were made of the value obtained. *During the taking of these measurements there was absolutely no knowledge whatever as to whether the results were coming out "high" or "low."* The author feels that this is a most essential precaution in precision measurement since, no matter how earnestly the experimenter endeavors to avoid bias, it seems impossible for him to make the many judgments necessary without being influenced to some extent by a knowledge of whether the results are coming out high or low.

It is again a pleasure to thank Professor E. O. Lawrence for suggesting the method used and for his encouragement. My appreciation is expressed to Professors W. R. Smythe, W. V. Houston and I. S. Bowen for many helpful discussions, to Professor R. A. Millikan for his interest in the work and his efforts in making possible its completion, and to Professor R. T. Birge for his interest in the problem and his criticism of the manuscript. Acknowledgment is made to the physics department of the University of California for their kindness and cooperation in making a loan of a considerable amount of special equipment built there for the preliminary work (Part I). My sincere thanks are given to G. H. Worrall whose skill as an instrument maker and understanding of the problem made possible the unusual degree of precision attained in the measuring chamber, and to B. E. Merkel who made most of the other machined parts. Many students assisted in the work. In particular the work of Herbert Ribner and Charles Perrine, Jr. should be mentioned. Grateful acknowledgment is made to the American Philosophical Society and to the Carnegie Corporation for grants which made possible the latter part of the work.

Note added after manuscript was finished: As stated near the beginning of *Section E* in *Apparatus and Standards* the standard voltage was reestablished late in 1936 with a new cell. Since a standard established with a single cell is always subject to question, a further check in terms of two additional new cells has just been completed. The standard determined by the mean of the three new cells is 0.000009 volt lower than that from the first cell alone. This increases the value of e/m_0 by 9 parts in 10^6 (i.e., by less than two units in the fifth decimal), a quite negligible change. The author wishes to express his appreciation to Dr. B. H. Sage for the loan of the two new standard cells.

By an oversight, correction was not made to the magnetic field constant k_R of Eqs. (29) and (30) to allow for its variation off the central plane. The integrated average over the extent of a slit length (2 mm) is 1.0 part in 10^5 greater than the value on the central plane. The value of e/m_0 should then be reduced by essentially this same amount.

These two corrections (standard cell and magnetic field) practically cancel each other leaving no net change in the value of e/m.

[37] See references given in Houston's paper, reference 30.
[38] J. DuMond and V. Bollman, Phys. Rev. **51**, 400 (1937) (in particular, Part IV).