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Scattering of Fast Electrons in Hydrogen

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Electrons of 2000-volt energy scattered by hydrogen molecules are found to be divided into two distinct groups, those scattered elastically and those scattered inelastically. The former have been scattered by the nuclei, while the latter have been scattered by the atomic electrons. The theoretical ratio of the number inelastically scattered to the number elastically scattered differs according to whether one adopts the classical theory of scattering or the wave mechanical. The difference comes in through the fact that when particles are scattered by similar particles, such as electrons by electrons, the two theories lead to different expressions. The experimental results are in much closer agreement with the predictions of the wave mechanical theory than with those of the classical theory. A slight difference between the results obtained with helium and those obtained with hydrogen may be attributed to diffraction effects which are present when scattering is due to diatomic molecules.

IN a former paper[1] the authors investigated the ratio of the number of inelastically scattered electrons to the number of elastically scattered electrons, when a beam of electrons is directed into helium, as a function of the angle of scattering in order to test whether the classical or the wave-mechanical description of the scattering of like particles by like particles is supported by experiment. The principle of the method may be summarized as follows. In order that a 2000-volt electron may be deflected by a helium nucleus or by an atomic electron through an appreciable angle (say 20° or more), classical orbit theory shows that it must pass by the deflecting center within a distance which is considerably smaller than the average distance between the atomic electrons and the nucleus in the helium atom. Consequently if we find fast electrons scattered through an appreciable angle by atoms of helium, it is legitimate to assume that the electron has been scattered either by a nucleus or by one of the atomic electrons, and not by the cooperation of two or more scattering centers. This allows us to regard helium as providing a mixture of electrons and nuclei acting independently of each other. Among the electrons scattered at a definite angle by such a gas, there will be two groups, those scattered by the nuclei and those scattered by the atomic electrons. They will be clearly distinguished from each other by the fact that those scattered (elastically) by the nuclei lose no energy, while those scattered (inelastically) by the atomic electrons lose a finite amount of energy which is greater the larger the angle of scattering.

The probability that an electron of velocity v and mass m will be scattered into unit solid angle at an angle ϕ with the original direction by another electron, initially at rest, is, according to the classical theory, as given by Darwin[2]

$$\alpha_1 = (e^4/m^2v^4)4\cos\phi\,(\operatorname{cosec}^4\phi + \sec^4\phi). \quad (1)$$

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[1] A. L. Hughes and S. S. West, Phys. Rev. **50**, 320 (1936).

[2] C. G. Darwin, Phil. Mag. **27**, 499 (1914).

A different formula

$$\alpha_2=(e^4/m^2v^4)4\cos\phi\,(\operatorname{cosec}^4\phi+\sec^4\phi-\Phi\operatorname{cosec}^2\phi\sec^4\phi)\quad(2)$$

is given by wave mechanics.[3] (For angles over 20° and electron energies of 2000 volts or more, Φ is very close to 1.) When an electron is scattered by a nucleus (i.e., by an unlike particle) both the classical method and the wave-mechanical method lead to the same formula

$$\alpha=(Z^2e^4/4m^2v^4)\operatorname{cosec}^4(\phi/2),\qquad(3)$$

where Ze is the charge on the nucleus.

Experimentally we measure α'/α, the number of inelastically scattered electrons divided by the number of elastically scattered electrons, and compare this with $\alpha_1'/\alpha(=Z\alpha_1/\alpha)$ or with $\alpha_2'/\alpha=(Z\alpha_2/\alpha)$, to decide whether the classical or the wave-mechanical description is supported experimentally.

In the paper cited, it was shown that the results obtained with helium were decisively in favor of the wave-mechanical theory. This paper gives an account of a repetition of the experiment using hydrogen in place of helium. Since the apparatus and mode of obtaining results are precisely the same as before, all description of the apparatus and methods of measurement are here omitted and the reader is referred to the previous paper.

The results for scattering of electrons by hydrogen are summarized in Table I. These are plotted in Fig. 1, which shows the classical theoretical curve, the wave-mechanical theoretical curve, and the experimental points. The estimated uncertainties in the experimental values are given numerically in Table I, and are shown by vertical lines on Fig. 1. It may be concluded definitely that the results for hydrogen are much closer to the values predicted by the wave-mechanical theory than to those predicted by the classical theory. We may conclude, therefore, that the collision of one electron with another can be described satisfactorily by the wave-mechanical theory but not by the classical theory. The experimental values for 24.5° and 29.5° show a slight, but apparently definite, deviation from the wave-mechanical curve by amounts somewhat greater than the estimated experimental uncertainty. On the other hand, the experimental values for the larger angles (on which we are inclined to put less reliance because the scattered currents are smaller and more difficult to measure) show no definite deviation from the curve.[4]

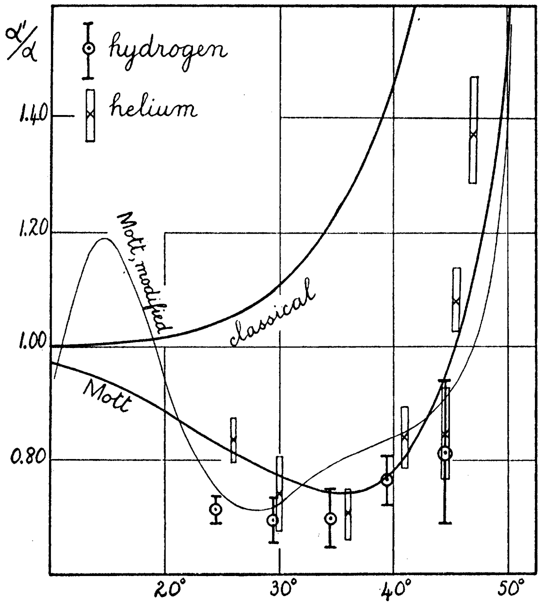

FIG. 1. Ratio of inelastically scattered to elastically scattered electrons.

TABLE I. *Ratio of inelastic scattering to elastic scattering in hydrogen, for 2000-v electrons.*

Angle	Experimental α'/α	Theoretical α_1'/α	Theoretical α_2'/α
24.5°	0.715±0.022	1.043	0.833
29.5°	0.695±0.040	1.098	0.773
34.5°	0.698±0.049	1.214	0.745
39.5°	0.765±0.042	1.432	0.768
44.5°	0.812±0.126	1.870	0.943

For purposes of comparison the values obtained previously for the scattering of 2000-v electrons in helium are plotted in the diagram. (These values have each been multiplied by two,

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[3] N. F. Mott and H. S. W. Massey, *The Theory of Atomic Collisions* (Oxford University Press, 1933), p. 76.

[4] A careful independent determination of the experimental ratios at 24.5° and 29.5° was made by Mr. R. F. Moody. He obtained the values 0.711 and 0.697 which check very well with the values given in Table I, and so give us considerable confidence in the results at these two angles. Unfortunately circumstances did not permit a check at other angles. It should be stated that the readings as taken by an observer give no hint whatever of the final ratio which is obtained only after a considerable calculation involving many readings. The method is outlined in the previous paper.

since the ordinates for the theoretical curve for hydrogen are just twice those for helium.)

A possible explanation of the slight difference between the fit of the hydrogen and the helium points and the theoretical curve at 24.5° and 29.5° may be found in the fact that in hydrogen the scattering is done by diatomic molecules while that in helium is due to monatomic molecules. We know from experiments on the *elastic* scattering of fast electrons (5000 to 50,000 volts) by polyatomic molecules, that well marked diffraction effects occur, which for the case of diatomic molecules can be described by $\alpha(1+\sin x/x)$, where α is the elastic scattering due to a single atom and $x=2\pi d \sin(\phi/2)/\lambda$, in which ϕ is the angle of scattering, λ the electron wave-length and d the distance apart of the atoms in the diatomic molecule.[5] Following the usual procedure we shall assume that the diffraction factor $(1+\sin x/x)$ applies only to the coherent scattering (i.e., the elastic scattering), while the incoherent, or inelastic, scattering is not modified by diffraction effects. Consequently this consideration leads to replacing α_2'/α by $\alpha_2'/\alpha(1+\sin x/x)$. In computing x we assume $d=0.74$A for the interatomic distance in the hydrogen molecule.[6] The modified curve, $\alpha_2'/\alpha(1+\sin x/x)$, is shown as a light line in Fig. 1. It is clear that the two experimental points on which we place most reliance are closer to the modified curve than to the unmodified curve. On the other hand the other points are possibly closer to the unmodified curve, but the accuracy with which they were determined does not permit a clear cut decision as to which of the two alternative curves they fit better. It is perhaps well to mention that the *maximum* scattered electron currents were of the order of 10^{-15} amp. and that Fig. 1 is drawn on a very open scale.

Objection may be raised, with some justification, to the procedure of superposing the diffraction concept which implies that the two nuclei in each molecule cooperate, on a viewpoint in which the essential feature is that in deriving the (inelastic/elastic) ratios we postulate that the scattering centers are wholly independent of each other and act individually. Closer analysis indicates that the diffraction concept is added only to the elastic scattering whose function in this investigation is merely to afford a standard with which to compare the inelastic scattering. There is no attempt to apply a diffraction "correction" to the inelastic scattering; our picture of *this* loses nothing of its original simplicity and directness. We may infer that our conclusion that the scattering of electrons by electrons is best described by a wave-mechanical theory still holds; experiments with hydrogen instead of helium introduce an additional problem of just how to handle the diffraction effects in the elastic scattering which may be present in the diatomic molecule.

Inasmuch as various circumstances have compelled us to discontinue this investigation for the present, it was deemed advisable to publish the results as they stand, even though they have raised, but left unanswered, the intriguing question whether or not the diffraction factor $(1+\sin x/x)$ must be included in a complete theory. Should the answer be in the affirmative, these measurements may give us, in principle at any rate, a new method of determining the interatomic distance in the hydrogen molecule. We wish to acknowledge the assistance given to us by Mr. R. F. Moody.

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[5] Mott and Massey give a more exact formula for hydrogen, but since, in it, the dominating term is $(1+\sin x/x)$, it will be sufficient for our purpose to consider it alone.

[6] R. S. Mulliken, Rev. Mod. Phys. **4**, 1 (1932).