The inelastic scattering process gives rise to neutrons of low energy. A small fraction will have energies of a few hundred volts or less; of these an even smaller fraction will have energies within the cadmium resonance band. Owing to the extremely high efficiencies with which these two classes of neutrons can be detected, the production of both classes of neutrons has been observed.

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The Compton Effect with Gamma-Rays

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This paper summarizes the results of recent studies of the Compton effect using Geiger-Müller counters. The results show that no time lag as great as 10^{-4} second can exist in the Compton scattering process and that the angular relationship given by the photon theory is verified to within $\pm 20°$.

THE results of several recent experiments[1] have seemed to indicate that in the Compton scattering of x-rays or gamma-rays the recoil electron appears at the same instant that the quantum is scattered. The angular relationship predicted by the theory of Compton-Debye[2] between the directions of the recoil electron and scattered photon pairs is, however, more difficult to establish. The original cloud chamber experiments of Compton and Simon[3] made with hard x-rays scattered by air indicated an agreement with theory. More recently, the counter experiments of Bothe and Maier-Leibnitz[1] and the cloud chamber experiments of Crane, Gaerttner and Turin[4] have seemed consistent with the angular relationship predicted by theory. Some experiments by the present writer[5] which attempted to fix the angular relationship sharply, yielded results that did not support the theoretical predictions. The publication of these findings aroused an active interest in the subject which resulted in several new experiments[6] and theoretical discussions[7] that have added greatly to the knowledge of these phenomena.

An experiment to check the angular relationship between the paths of the recoil electrons and scattered photons has been made with the apparatus shown schematically in Fig. 1. The beam of gamma-rays is directed against the scatterer at S, which in this experiment is an aluminum foil of thickness 0.00165 cm. The source is a radon tube giving off the gamma-rays of Ra C which are filtered through 0.32 cm of lead. The collimating system consists of a series of lead shields surrouncing a brass tube 0.8 cm in diameter and 28 cm long. The end of this system as shown in Fig. 1 is designed to prevent most of the gamma-rays scattered from the brass tube from going

[1] W. Bothe and H. Maier-Leibnitz, Zeits. f. Physik **102**, 143 (1936), Gött. Nachr. **2**, 127 (1936), Phys. Rev. **50**, 187 (1936); J. C. Jacobsen, Nature **138**, 25 (1936); W. E. Burcham and W. B. Lewis, Proc. Camb. Phil. Soc. **32**, 637 (1936); R. S. Shankland, Phys. Rev. **50**, 571 (1936), Phys. Rev. **51**, 1024 (1937).

[2] A. H. Compton, Phys. Rev. **21**, 483 (1923); P. Debye, Physik. Zeits. **24**, 161 (1923).

[3] A. H. Compton and A. W. Simon, Phys. Rev. **26**, 289 (1925).

[4] H. R. Crane, E. R. Gaerttner and J. J. Turin, Phys. Rev. **50**, 302 (1936).

[5] R. S. Shankland, Phys. Rev. **49**, 8 (1936).

[6] Ibid. also A. Piccard and E. Stahel, J. de phys. **7**, 326 (1936); E. J. Williams and E. Pickup, Nature **138**, 461 (1936).

[7] P. A. M. Dirac, Nature **137**, 298 (1936); N. Bohr, Nature **138**, 26 (1936); F. Cernuschi, Comptes rendus **203**, 777 (1936); M. Taketani, Kagaku, 6 (1936); B. Hoffmann, A. G. Shenstone and L. A. Turner, Phys. Rev. **50**, 1092 (1936); E. J. Williams, Nature **137**, 614 (1936); R. Peierls, Nature **137**, 904 (1936).

directly to the counters. The gamma-ray counter G is set at an azimuth $\phi=90°$.

As the gamma-rays from Ra C consist of a line spectrum, it is necessary to compute the angle θ at which the recoil electron is ejected by each component of the incident radiation, when the scattering angle ϕ is 90°. The calculation is made by the Compton-Debye formula:

$$\cot\theta=(1+\alpha)\tan\tfrac{1}{2}\phi,$$

where $\alpha\equiv h\nu/mc^2$. A weighted mean of the values of θ computed from these equations is 22.5°. The electron counter R is mounted on an arm pivoted directly under the scatterer so that it can be set in the azimuth $\theta=22.5°$, and also be moved to the setting indicated by $-\theta$ in Fig. 1. The conservation of momentum in the scattering process will make it impossible for a recoil electron to be ejected to the counter R in the $-\theta$ position by a quantum scattered to the gamma-ray counter at G. The value of $-\theta$ chosen was $-22.5°$ since in this position the background discharge rate in the counter will be the same as in the $+\theta$ position. This fact was verified by an auxiliary experiment.

During the experiments the electron counter R was alternated from one position to the other every half-hour and the coincident discharges in the counters G and R were recorded. Table I gives the observed coincidence rates in the counters after they have been corrected for the decay of the radon tube during the course of the observations. The average strength of the source was 93 millicuries. The first and third columns give the coincidence rates observed with the electron counter R set in the position $+\theta$, demanded

TABLE I. *Observed rates of coincident discharges in counters G and R corrected for decay of gamma-ray source.*

R at $+\theta$ (min.$^{-1}$)	R at $-\theta$ (min.$^{-1}$)	R at $+\theta$ (min.$^{-1}$)	R at $-\theta$ (min.$^{-1}$)
1.24	0.78	1.08	0.98
1.22	1.07	1.09	0.85
1.46	0.88	1.19	0.82
1.30	0.79	1.78	0.71
1.50	1.11	0.99	0.50
1.50	0.89	1.22	0.62
1.19	0.91	1.51	0.61
		1.21	0.92

Mean Rates	1.30 min.$^{-1}$	0.83 min.$^{-1}$
Natural Uncertainty of Mean	± 0.09	± 0.07
Mean Error of Mean	± 0.05	± 0.05
Total Coincidences	195	141

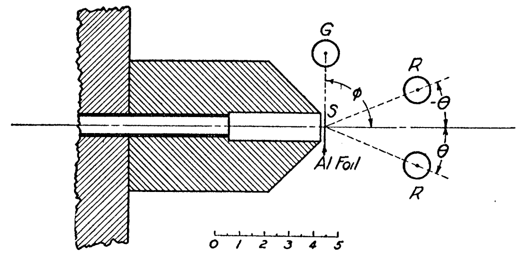

FIG. 1. Schematic diagram of apparatus.

by the photon theory of scattering. The second and fourth columns give the values observed with R set in the symmetrically opposite position $-\theta$. The mean coincidence rates are given, together with the natural uncertainty and the mean error of the mean.

The difference between the mean coincidence rates observed in the two positions shows a surplus of 0.47 min.$^{-1}$ in the position predicted by the theory. The natural uncertainty in this quantity is 0.12 min.$^{-1}$ while the mean error of the mean is 0.07 min.$^{-1}$, as computed by the Bessel formula. The criterion usually given in the theory of errors for an effect to be significant is that its magnitude should be at least three times the mean error of the mean. Here the effect is nearly seven times this quantity.

The data of Table I are plotted in Fig. 2 in the form of histograms. The abscissae give the observed coincidence rates, and the ordinates show the number of groups that were observed in the various ranges. A statistical analysis of these data has been made[8] which gives the following results. The data taken with the electron counter set in the $-\theta$ position show that the average $\bar{n}=0.835$ min.$^{-1}$; the standard deviation $\sigma=0.166$; and the asymmetry factor $\beta_1^{\frac{1}{2}}=0.11$. These quantities suggest that the observations approximate the normal distribution law: $[(2\pi)^{\frac{1}{2}}\sigma]^{-1}\times\exp[-(n-\bar{n})^2/2\sigma^2]$ where n is the observed coincidence rate. A smooth curve of this form has been drawn in the upper graph of Fig. 2. It is not unexpected that the residual coincidences should be distributed according to the normal law since they are due principally to the chance juxtaposition in time of the discharges of the separate counters.

[8] T. C. Fry, *Probability and Its Engineering Uses*, pp. 299–305; 470–471.

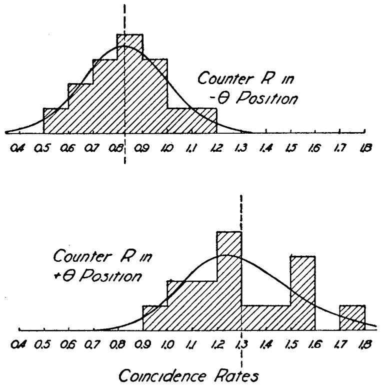

FIG. 2. Histograms of coincidence rates for different counter positions.

The data when the counter R is in the $+\theta$ position yield the average $\bar{n}=1.30$ min.$^{-1}$; the standard deviation $\sigma=0.214$; and the asymmetry factor $\beta_1^{\frac{1}{2}}=0.67$. The large asymmetry exhibited by these data suggests that they may be distributed according to the Poisson law: $\epsilon^n e^{-\epsilon}/n!$ Here ϵ is the expectation, and n is the coincidence rate measured from the background rate. Since the effect producing the shift of the second set of data with respect to the first is superimposed on the background coincidence rate, the origin for the Poisson function will be at 0.835 min.$^{-1}$. The smooth curve in the lower graph of Fig. 2 is of the Poisson type. The true coincidences due to the Compton effect should follow the Poisson law[9] and it is interesting that the data themselves suggest such a distribution. The greater value of σ found in the $+\theta$ position is due to the fact that the background and the true coincidences are here superimposed. Although the data are hardly extensive enough to base any further conclusions on the shapes of these curves, yet it is interesting to note that when they are graduated by the usual methods,[10] smooth curves result which are very similar to those drawn in Fig. 2.

The mean coincidence rates given in Table I are the sum of true coincidences and background coincidences. The latter are due principally to the finite resolving time of the electrical adding circuit used to record the coincidences. Cosmic-ray showers and other factors may also contribute a few coincidences in both the $+\theta$ and $-\theta$ positions. The residual coincidence rate was determined experimentally in two ways. First, the scattering foil was removed and the coincidence rate measured with the Ra C source in place and the electron counter set in the azimuth $-\theta$. The observed coincidence rate was 0.68 ± 0.10 min.$^{-1}$. Then, with the Ra C source removed, the counters were stimulated with old radon tubes so that the individual discharge rates in the counters were nearly the same as in the principal experiment, and the coincidence rate was again determined. The result of this comparison experiment gave a rate of 0.57 ± 0.09 min.$^{-1}$. The average of these determinations is used in the calculations below. The time constant of the adding circuit has not been further reduced since it is possible that the real coincidences may be missed in so doing; while theory indicates that there should be no lag in the scattering process itself of sufficient magnitude to be determined experimentally, yet the possibility exists that small time lags will be introduced by the amplifying circuits themselves which would vitiate the results. Other factors contributing to the background rate are the increased single counting rate in the electron counter due to scattered gamma-rays and to secondary beta particles ejected from the walls of the collimating system by the primary gamma-ray beam. Some of these will reach the electron counter and tend to raise the background coincidence rate. However, this will not obscure the principal effect looked for if the electron counter is operating near its maximum efficiency. That this was the case was shown by placing a radioactive source near the electron counter while the main experiment was in progress; the very large increase observed in both the single counting rate and the coincidence rate proved that the apparatus was functioning properly.

With the background coincidence rate determined above and the data of Table I, the following values for the rates of true coincidences observed in the two positions of the electron counter R are given: In the $+\theta$ position the observations give $1.30-0.62=0.68$ min.$^{-1}$; and in the $-\theta$ position the rate is $0.83-0.62=0.21$ min.$^{-1}$. The true coincidences observed in the $-\theta$ position can hardly be ascribed to chance.

[9] T. C. Fry, *Probability and Its Engineering Uses*, pp. 214–240.

[10] E. T. Whittaker and G. Robinson, *The Calculus of Observations*, pp. 285–316.

They are probably due to recoil electrons that have been deflected through an angle of about $2\theta°$ in leaving the foil. If the angle of deflection suffered by a recoil electron from its direction of ejection from an atom is given by Δ, and $P(\Delta)$ is the probability of a deflection of this magnitude, then the relation between Δ and $P(\Delta)$ may be given by the normal distribution function:

$$P(\Delta)=\frac{0.4769}{\pi^{\frac{1}{2}}\Delta_m}\exp\left[\frac{-(0.4769)^2\Delta^2}{\Delta_m{}^2}\right],$$

where Δ_n is the angle of half-scattering. For the position $+\theta$ the value of Δ is zero and $P(0)$ has a value proportional to 0.68 min.$^{-1}$. For the $-\theta$ position, Δ equals 45° and $P(45°)$ is here proportional to 0.21 min.$^{-1}$. A substitution of these values in the equation gives $\Delta_m=\pm 20°$ as the experimental value of the "angle of half-scattering" of the distribution function. This indicates that half of the recoil electrons ejected in the initial direction $+\theta$ are deflected by less than 20° in leaving the scatterer.

The value of Δ_m determined from this experiment can be compared with the value computed with the formula given by Bothe[11] for the multiple scattering of electrons passing through a thin foil: $\Delta_m=2.6e^2E^{-1}Z(\pi nt)^{\frac{1}{2}}$. Here t is the thickness of the foil, n is the number of atoms per cubic centimeter, Z is the atomic number, and E is the energy of the electron. This equation, however, applies to the case where the electron passes through the entire foil. In the present experiment the recoil electrons are liberated within the foil itself. This means that the equation would be correct for electrons passing through the entire foil, while those liberated on the side of the foil near the counter would suffer a negligible deflection. For sufficiently thin foils, the formula should be correct if t is used as half the foil thickness. For the foil as actually used, it works out that $\Delta_m=\pm 11.4°$.

Several factors are present which would tend to make the observed value of Δ_m greater than that computed by the Bothe formula. These are the nonhomogeneity of the gamma-rays used, the scattering by the gas in the chamber, and the finite angles subtended by the counters. The net effect of these factors would be to diminish the number of coincidences observed in the predicted direction, although the number observed in the other position would not be appreciably increased by this cause. It has been suggested[7] that the true coincidences observed in the $-\theta$ position might be related to a double Compton effect or to other mechanisms that would make the usual application of the laws of conservation of energy and momentum inadequate, but the essential features of the present experiments require no such interpretation. The wave mechanics suggests that, as in the case of the ejection of photoelectrons, the spread of the electrons about the most probable direction of ejection must be greater than that predicted by the simple corpuscular picture of the process. The experiments of Kirchner[12] support this view. In any event there seems no reason to doubt that the present experiment supports the angular relationship given by the theory of Compton-Debye to within $\pm 20°$.

Four other experiments have been performed by the writer which have shown a directional effect in the emission of recoil electrons. These experiments have been summarized in Table II. The first column gives the material of the scatterer; the second column gives the angle Δ between the predicted position of the electron counter and the comparison position; while the third column gives the angle θ of the Compton-Debye theory. The fourth and fifth columns give the thickness of the scatterer in centimeters and the initial energies of the recoil electrons expressed in million electron volts, respectively. The sixth column gives a quantity which should determine the relative number of recoil electrons deflected to the electron counter when it is in the comparison position where theory predicts no

TABLE II. *Results of six experiments which show a directional effect in the emission of recoil electrons.*

SCATTERER	Δ	θ	t	F	$\frac{Z}{E\sin(\Delta/2)}\left(\frac{t'\rho}{A}\right)^{\frac{1}{2}}$	$\frac{R_+-R_0}{R_--R_0}$
Aluminum	45°	22.5°	0.00165	0.35	1.30	3.24
Aluminum	45°	22.5°	0.0038	0.35	1.96	2.52
Air	60°	45°	1.5	0.18	1.07	5.5
Cellophane	12°	30°	0.0028	1.1	0.95	6.22
Paraffin	60°	45°	0.05	0.18	3.44	1.74
Beryllium	60°	45°	0.04	0.18	4.80	1.38

11 W. Bothe, Zeits. f. Physik **13**, 368 (1923).

12 F. Kirchner, Physik. Zeits. **27**, 385, 799 (1926); Ann. d. Physik **81**, 1113 (1926).

coincidences should be observed. The last column gives the ratio between the true coincidences observed in the predicted position and in the comparison position of the electron counter.

The first line of the table summarizes the experiment with aluminum as described above in some detail. The second line gives a similar experiment with a somewhat thicker aluminum foil. The average strength of the radon tube used in this experiment was 82 millicuries. The fourth line summarizes the experiment of Bothe and Maier-Leibnitz[1] with the more homogeneous gamma-rays from Th C″. The experiments in the third, fifth, and sixth lines were performed with the counters in a somewhat different arrangement than shown in Fig. 1. Here they were placed parallel to the axis of the gamma-ray beam. This allows beta-particles with a wider range of θ values to enter the electron counter, thus minimizing the effect of the nonhomogeneity of the gamma-ray source. The predicted position for the electron counter was therefore in the plane containing the quantum counters and the incident beam; while in the comparison position the electron counter was rotated 90° out of this plane about the incident beam as an axis. In the latter position, the conservation of momentum would not be satisfied and no coincidences should be observed except those due to the disturbing factors discussed above. In these experiments four gold-walled counters were used to record the scattered photons; these counters have a higher efficiency for gamma-rays than those with walls made of elements of lower atomic number. The average strength of the radon tube was 120 millicuries. Since the several experiments were performed under such different conditions, the only practicable way of attempting to correlate them is by means of the ratios given in the final column of Table II.

Figure 3 shows the relationship between this ratio for the several experiments and the electron diffusion factor given in column five of the table.

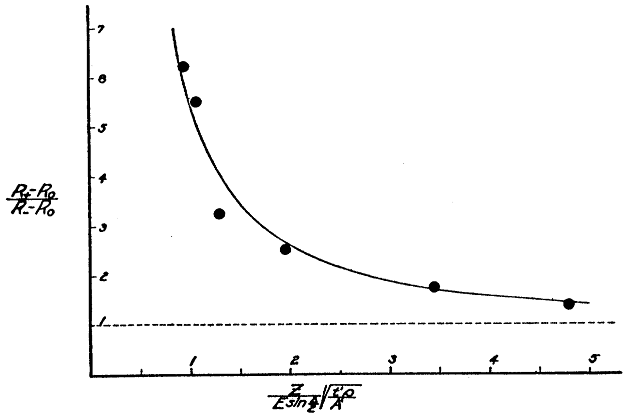

FIG. 3. Correlation of the data in Table II.

The circles represent the six experiments and a smooth curve of the form $(y-1)=cx^{-b}$ has been drawn to emphasize the general tendency. The two points that fall below this simple smooth curve represent the experiments in which the nonhomogeneity of the source would have the greatest effect in diminishing the angular effect. It is interesting to note that a more definite positive effect is observed with the paraffin scatterer than with a thinner scatterer of beryllium under the same conditions. This is of course due to the lower average atomic number of the paraffin. The fact that six distinct experiments may be correlated in this fashion gives strong additional evidence that the surplus of coincidences observed in the predicted position is due to the Compton effect and not to any spurious cause.

Taken as a group, these experiments lead to the conclusion that no time lag as great as 10^{-4} second can exist in the Compton scattering process, and that the angular relationship given by the theory of Compton-Debye is verified to within $\pm 20°$. The limit in the time factor is set by the nature of the electrical circuits used, while the angular resolution is determined very largely by the diffusion of the recoil electrons.