Precise Magnetic Torque Measurements on Single Crystals of Iron*

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A rugged balance of sufficiently high sensitivity and a rotating magnet capable of producing a field of over 3000 oersteds are used to make magnetic torque curves. The equations for the torque curve of a single crystal disk of any orientation whatsoever are derived on the assumption that the magnetic anisotropy is adequately described by a single constant. Furthermore, a method is developed for predicting the orientation from a knowledge of the angles of zero torque alone, and both the experimental and theoretical limitations of this method are discussed. It is shown experimentally that the use of cylindrical disks introduces errors that may be avoided by using ellipsoids. An experimental torque curve for a single crystal ellipsoid of 2.8 percent Si iron gave excellent agreement with the curve calculated on the basis of just the first anisotropy constant, $a_F = 1.70\times10^5$ ergs/cc. Corrections for magnetization not being parallel to applied field are briefly discussed, and it is shown that in practical cases, the magnetization cannot deviate appreciably from the plane of the specimen. The possibility is mentioned of computing a fictitious value of the second anisotropy constant because of error in interpreting experimental data. It is proposed that the anisotropy constants be obtained directly from accurate torque measurements, in order to furnish data for the computation of magnetization curves to be checked against experiment.

INTRODUCTION

THE purpose of this and succeeding papers is to develop magnetic measurements as a tool in studying the grain orientation of ferromagnetic materials so that, in the first place, the physical condition of any given sample may be better and more conveniently defined than is at present possible and, in the second place, so that the physical effect of the various mechanical and thermal treatments to which metals are subjected may be studied in greater detail in the hope of throwing some much needed light on the processes of plastic deformation and recrystallization. The magnetic attack on the problem seems to offer promise because the magnetic properties depend on grain orientation to a marked degree, and because the theory is now sufficiently developed to permit a fairly accurate mathematical description of many phenomena.[1]

In order to develop an interpretation of the magnetic properties of polycrystalline aggregates, especially when directional characteristics are involved, it is essential that we be absolutely certain of the corresponding treatment of single crystals. The following paper discusses this point in some detail.

ENERGY OF SATURATED CRYSTALS

In discussing ferromagnetism it is customary to assume that small regions are spontaneously magnetized to saturation in different directions, and that the observed magnetization of a macroscopic sample is the resultant effect of these regions. In general it is not possible to say how the directions of magnetization of these regions are distributed. The only simple case is the one in which the entire sample is magnetized to saturation in some one direction, so that all the small regions are magnetized in this same direction. In this paper, we shall confine our attention to this case.

The fundamental quantity determining the magnetic behavior of saturated crystals is the energy E_θ which we shall express in terms of ergs/cc. It can be shown[2] that for cubic ferromagnetic crystals in the absence of external forces,

$$E_\theta = a_F\Sigma'\alpha_i^2\alpha_j^2 + a_F'\alpha_1^2\alpha_2^2\alpha_3^2 + \cdots, \tag{1}$$

where the α's are the direction cosines of the saturation magnetization I_S with respect to the cubic axes, the summation being taken for $i=1, 2, 3$, $i\neq j$. In terms of a notation commonly used,[3] $a_F = \frac{1}{2}K_1$, $a_F' = K_2$. The number of constants a_F,

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[1] The reader can find references in E. C. Stoner's *Magnetism and Matter* (Methuen, 1934), p. 434.

[2] For a complete discussion which takes into account the effects of magnetostrictive distortion and which explains more fully the basis of our notation, see *Introduction to Ferromagnetism*, by F. Bitter (McGraw-Hill, 1937).

[3] R. M. Bozorth, Phys. Rev. **50**, 1076 (1936).

a_F', $\cdots$ required to describe E_θ must be determined experimentally for each material, since addition of alloying elements to pure iron or nickel may greatly change the values of these constants.

Let us assume for the present that E_θ can be accurately represented by the first term alone, which we have found experimentally to be true for silicon iron, as will be discussed later in the paper. Then Eq. (1) reads

$$E_\theta = a_F \Sigma' \alpha_i^2 \alpha_j^2. \tag{2}$$

The surface defined by this equation for $a_F > 0$ is shown in Fig. 1 where the length of a line from the center of the model to the surface represents the energy of a crystal magnetized to saturation in the corresponding direction. (A constant term has been added to the expression for E_θ to make it easier to construct a physical model.) The minima occur in the [100] directions, the maxima in [111]. If a_F were negative, as for nickel, the maxima and minima would change places. In general, E_θ is a function of the distortion of the crystal and of the temperature, but we shall consider only crystals kept at one temperature and undistorted except for magnetostriction, the effects of which are taken into consideration in the derivation of Eq. (1).

In order to magnetize a single crystal to saturation in a single direction, it is necessary to apply an effective field H of the order of 10^3 oersteds. The experimentally observed energy is no longer E_θ but $E_{\theta,H}$ where

$$E_{\theta,H} = E_\theta - HI_S \cos \psi, \tag{3}$$

ψ being the angle between H and I_S. Except for certain directions, ψ is not zero, although it tends to approach zero as H becomes indefinitely large and we shall show later how to correct experimental results to give us E_θ. If Eq. (2) is correct, then an experimental determination of E_θ for a single crystal would at once fix its orientation.

If, however, the crystal to be examined has the form of a disk, or more properly of an ellipsoid, it is convenient to confine the observations to the plane of the disk, and the interpretation of the results is somewhat more complicated. Essentially the observation that can be made is the shape of the curve which is formed by the intersection of the model of Fig. 1 with a plane passing through its center, and the problem is to deduce the orientation of the plane from the shape of the resulting curve. This problem has been solved by the methods outlined below.

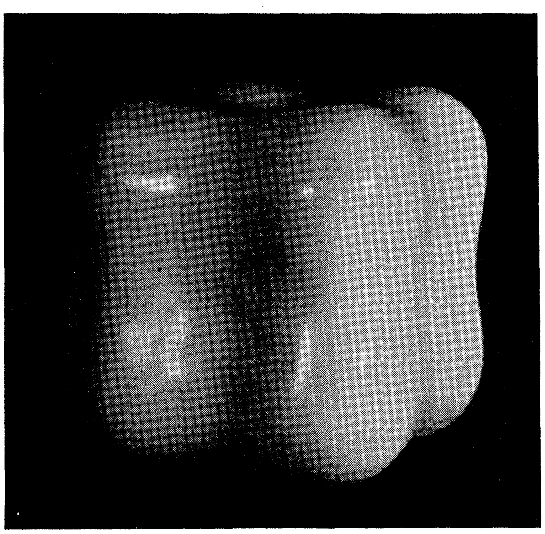

FIG. 1. Model of E_θ for iron, $a_F > 0$.

TORQUE-MEASURING APPARATUS

The experimental observations may most conveniently be made by measuring the torque T acting about the axis of the disk[4] when it is magnetized to saturation along any diameter. If θ measures the direction of magnetization in the plane of the disk, we have $T = -\partial E_\theta / \partial \theta$, and E_θ may be found by integrating the experimentally found torque curve, after it has been corrected for the deviation of I_S from H. We shall see, however, that it is more convenient to work with the torque curve itself rather than to bother with integrating it.

Apparatus for measuring such torque curves has been in use for more than twenty years. That which we have used was designed to give somewhat greater accuracy than has generally been needed, and to produce sufficiently intense fields so that the sample is practically saturated since under such conditions the observed torque becomes independent of the magnitude of the applied field. A photograph of the apparatus is

[4] As we shall see later, the incomplete saturation of the edges of a disk makes it necessary to substitute an ellipsoid for a disk so as to secure best results; hence, in all that follows, "disk" should read "ellipsoid" if strict accuracy is wanted.

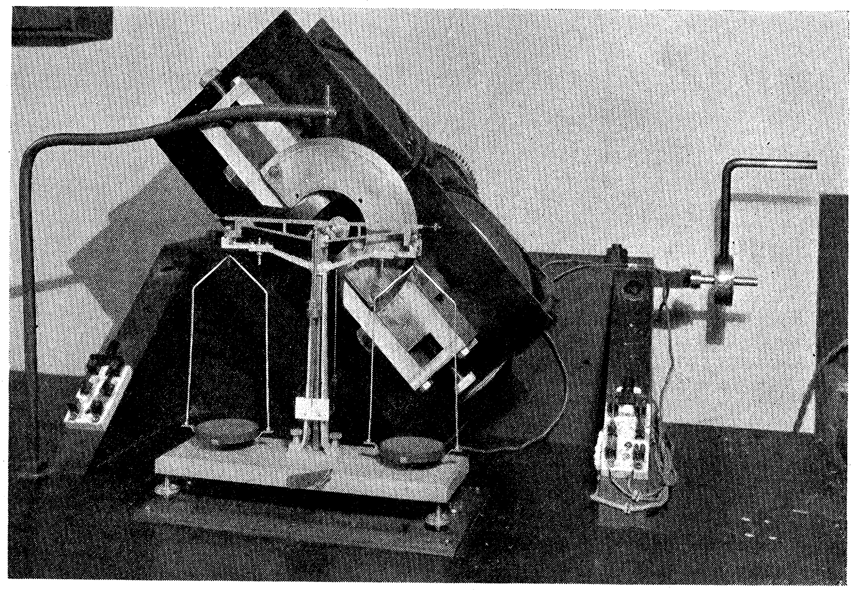

FIG. 2. Experimental set-up showing torque balance and rotating pole pieces.

shown in Fig. 2. The sample, a one-inch disk, is mounted in a light aluminum chuck with its axis accurately in line with the knife-edges of a small balance from which all ferromagnetic parts were carefully removed. Slight traces of ferromagnetism can be corrected for by a test run without a specimen. When the balance is in position, the sample is between the poles of an electromagnet which can be turned so as to produce magnetization along any diameter of the disk. Specially shaped poles are used to produce as uniform a field as possible.[5] The sample is in unstable equilibrium halfway between the poles, and any error in the alignment will cause the sample to jump to one pole face or the other. The labor in making this adjustment may be considerably lessened without sacrificing needed sensitivity by using heavy lead-weight pans. The sensitivity of the balance was in fact purposely reduced by attaching a brass weight to the pointer of the balance so that readings could be made for unstable orientations of the disk in the field. Even with these alterations, a change of 5 mg in the balancing weight could be easily detected, corresponding to a precision of better than 0.2 percent of the maximum weight which was about 3 g for several different disks. Other forms of torque-measuring apparatus may be found described elsewhere.[6]

THEORETICAL TREATMENT OF TORQUE CURVES

Since the theory on which are based the studies of preferred orientation of polycrystalline materials is developed from E_θ as given by Eq. (2), it is important to find the effect of neglecting both the higher order terms and the distortion of the curve arising from lack of parallelism between I_S and H.

As the first step let us investigate the possibility of exactly determining the orientation of a single crystal disk on the assumption that Eq. (2) holds exactly. An obvious limitation on the uniqueness of the solution is the fact that the two sides of the disk are magnetically indistinguishable, so that for any given orientation of the cubic lattice in the disk there is always a twin orientation with the plane of the disk serving as the reflection plane, the two orientations giving identical magnetic results. Since $T=-\partial E_\theta/\partial\theta$ and $E_\theta=a_F\Sigma'\alpha_i^2\alpha_j^2$, it is easily shown that

$$T/4a_F=\Sigma\alpha_i^3\partial\alpha_i/\partial\theta. \quad (4)$$

[5] We wish to thank Professor H. B. Dwight for his assistance in designing these pole pieces.

[6] E. Dahl and F. Pfaffenberger, Zeits. f. Physik **71**, 93 (1931); N. Akulov and N. Brüchatov, Ann. d. Physik **15**, 741 (1932); K. J. Sixtus, Physics **6**, 105 (1935); H. J. Williams, R. S. I. **8**, 56 (1937).

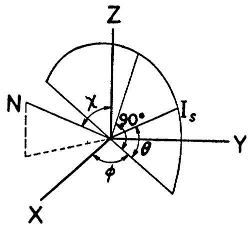

FIG. 3.

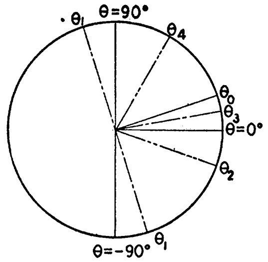

FIG. 4.

FIG. 3. Angles defining orientation of disk. X, Y, Z are the cubic axes of the crystal; N is the normal to the disk; I_S is the direction of the magnetization in the plane of the disk; θ is the angle measured in the plane of the disk.
FIG. 4. Positions of zero torque in plane of the disk.

In terms of the angles defined in Fig. 3,

$$\begin{aligned} \alpha_1 &= \cos\theta\cos\phi - \sin\theta\sin\phi\cos\chi, \\ \alpha_2 &= \cos\theta\sin\phi + \sin\theta\cos\phi\cos\chi, \\ \alpha_3 &= \sin\theta\sin\chi. \end{aligned} \tag{5}$$

The angle θ is measured experimentally from an arbitrary diameter of the disk but in the derivation of the theory it is measured from the intersection of the disk with the XY plane. The X, Y, Z axes are of the form $<100>$.

From Eqs. (4) and (5) it can be shown that[7]

$$T/a_F = A_1 \sin 2\theta + A_2 \sin 4\theta + B_1 \cos 2\theta + B_2 \cos 4\theta,$$

where

$$\begin{aligned} A_1 &= \tfrac{1}{4}\sin^2\chi[(1-7\cos^2\chi) - (1+\cos^2\chi)\cos 4\phi], \\ A_2 &= -(7/8)\sin^4\chi - [(1/8)\sin^4\chi + \cos^2\chi]\cos 4\phi, \\ B_1 &= -\tfrac{1}{2}\sin^2\chi\cos\chi\sin 4\phi, \\ B_2 &= -\tfrac{1}{2}(1+\cos^2\chi)\cos\chi\sin 4\phi. \end{aligned} \tag{6}$$

The general behavior of a curve fulfilling the conditions of Eq. (6) is illustrated in Fig. 7. The quantities which can be determined with the greatest experimental accuracy are the four angles of zero torque occurring within an angle of 180°. Two of these positions correspond to energy minima and give stable zeros, while the other two give unstable zeros. There is no difficulty in distinguishing between the two types of zeros experimentally.

[7] Similar equations were obtained independently by S. L. Burgwin (private communication) but were not developed further. R. M. Bozorth, Phys. Rev. **50**, 1076 (1936), has obtained analogous equations for the torque but with the constants expressed in terms of the Miller indices of simple crystallographic planes and directions.

USE OF TORQUE ZEROS IN ESTABLISHING ORIENTATION

The four distinct values of θ for which $T=0$ can be found by successive approximations for any pair of values of χ and ϕ. Because of symmetry, it is necessary to investigate only those values of χ and ϕ which lie within the spherical triangle whose corners are [100], [110] and [111]. This was done for 5° intervals both of χ and ϕ, with some smaller intervals where needed. Since actual measurements of the angles of zero torque have to be made with respect to an arbitrary reference direction, it is necessary to consider not the angles, but rather the differences between them.

Let us take the angles as in Fig. 4, where $\theta=0°$ gives the reference line on the disk; $\theta=+90°$ and $-90°$ establish the sense of rotation; at θ_1 and θ_3 are found the stable zeros, at θ_2 and θ_4 the unstable zeros; and at θ_0 is the line of intersection of the disk with the XY plane (see Fig. 3). We assume from now on that $a_F>0$ as for iron; if $a_F<0$, as for nickel, then the stable and unstable zeros exchange places, but otherwise the argument is the same. If for convenience we let θ_{ij} be the absolute value of the *acute* angle between θ_i and θ_j, the experimental determination of the angles of zero torque gives θ_{13} and θ_{24} in the new notation. These two angles can be found immediately from the calculated values of θ_1, θ_2, θ_3 and θ_4. The simplest way of expressing the relation between orientation of the axes in the disk and the θ's was found to be the plotting of two families of contours, $\theta_{13}=$constant and $\theta_{24}=$constant, on a projection of the spherical triangle mentioned above. This map is shown in Fig. 5. The projection of the spherical triangle is not the commonly used stereographic one, but one in which the meridians are distorted into parallel lines in order that ordinary graph paper can be used.

A study of the map shows that appreciable changes of orientation near the [100] and [110] directions will not noticeably affect the values of θ_{13} and θ_{24}, as could be expected from the shape of the energy surface shown in Fig. 1. In about a third of the triangle the orientation is not defined uniquely on account of the double intersection of many of the θ_{13} curves with some of the

θ_{24} curves. This occurs only for $\theta_{24}<76°$. To distinguish between two such possible orientations it is necessary to calculate A_1, A_2, B_1 and B_2 for each, substitute in Eq. (6), and by using the known value of a_F for the material in question, to compare with the experimental torques. The closer the orientation approaches to the [111] corner, the less will be the magnitude of the torques, the torques finally vanishing for all angles when the disk normal becomes the [111] direction.

It is clear from Figs. 3 and 4 that the orientation of the disk with respect to the cube axes can be specified completely (except for the twinning position) only if θ_0 can be located. Exactly as was done for θ_{13}, a map was constructed for θ_{01}, as shown in Fig. 6. In order to locate θ_0 it is necessary to have some means of distinguishing between θ_1 and θ_3, and also between θ_2 and θ_4. Inspection of the calculated θ's of zero torque showed that θ_{41} is always the largest of the angles between any two consecutive zeros, except for a narrow range of orientations in which $\theta_{41}=\theta_{12}$. When there is a largest angle, as is usually the case, θ_4 and θ_1 are immediately identified, and this also fixes θ_3 and θ_2. θ_0 always lies between θ_4 and θ_3, so that it is uniquely located. In case $\theta_{41}=\theta_{12}$ it is unnecessary to distinguish between θ_2 and θ_4 since θ_{01} is found to be 90° to within a small fraction of a degree, making it immaterial in what direction from θ_1 it is measured.

To summarize here, we have obtained two families of curves relating the orientation to the angles of zero torque on the assumption that the energy of magnetization is given by Eq. (2). If

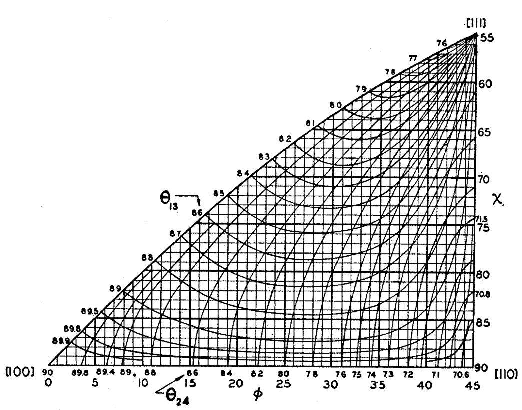

FIG. 5. θ_{13} and θ_{24} as functions of orientation.

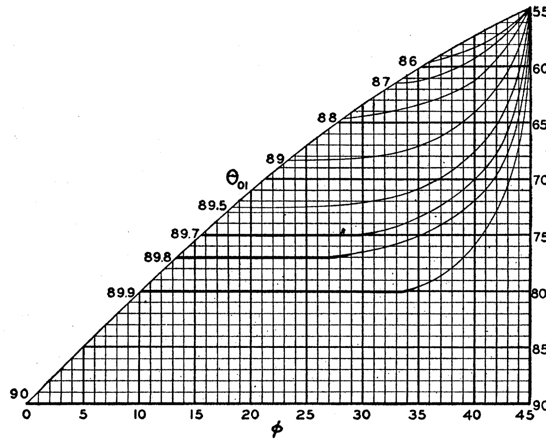

FIG. 6. θ_{01} as function of orientation.

we measure experimentally θ_{13} and θ_{24}, then χ and ϕ can be read off from Fig. 5, and θ_{01} can be found for this value of χ and ϕ from Fig. 6. An uncertainty may be introduced for a certain region of orientations on account of the double intersection of the curves θ_{13} with the curves θ_{24}, but the proper choice of χ and ϕ can be made if a_F is known. Unfortunately, a given χ and ϕ determine not one but two orientations on account of the inherent magnetic similarity of the opposite faces of the disk.

Although the method outlined above has at most a limited application as a practical means of determining the orientation of a single crystal disk, it should prove to be quite useful in studying the form of the energy anisotropy, because the knowledge of θ_0 gained by this method makes it possible to measure precisely the harmonic coefficients of the experimental torque curve. If these are compared directly with the coefficients calculated for the known x-ray orientation according to Eq. (6), we can determine a precise value of a_F in case the agreement is good. On the other hand, if we find any discrepancies to exist in the 2θ and 4θ terms or if higher harmonics, i.e. terms in 6θ, 8θ, etc. are present, we can more easily proceed to evaluate the higher order anisotropy constants such as a_F'. We shall return to the subject of more accurate forms of E_θ later. It should be pointed out here that the 2θ and 4θ terms arising from the presence of higher order anisotropy constants will make the orientation wrong to the extent that these terms interfere with the ones calculated according to Eq. (6).

Correction to Measured Torque Curves

The direction of I_S is found by setting $\partial E_{\theta,H}/\partial\theta=0$ since in this case the torque $T=-\partial E_\theta/\partial\theta$ is balanced by the component of I_S perpendicular to H. Letting $\psi=\theta_H-\theta$ where θ_H determines the direction of H, we can find from Eq. (3) that

$$\sin\psi=-T/I_SH. \qquad (7)$$

For fields of the order of 2500 oersteds, the maximum value of the correction will be only 2° or 3° so that the angular correction is proportional to the torque. If we are determining the orientation of a crystal by means of the torque zeros, then the correction obviously disappears.

We have assumed in the derivation of Eq. (7) that I_S always lies in the plane of the disk even though I_S and H, are not parallel and we shall now show that in all practical cases I_S does not deviate by more than a fraction of a degree from the plane of the disk.

Let us place the disk in the XY plane and have the external field H in the Y direction. The saturation intensity I_S will, in general, make a small angle ω with the XY plane, and the projection of I_S on XY will make a small angle ψ with Y. The deviation of I_S from XY is determined by minimizing $E_{\theta,H}$ so we have $\partial E_\theta/\partial\omega-\partial(\mathbf{H}\cdot\mathbf{I})/\partial\omega=0$. Letting, L, M, N be the demagnetizing factors along X, Y, Z, the components of the field are of the form H_X-LI_X, where $H_X=H_Z=0$, $H_Y=H$, $I_X=I_S\cos\omega\sin\psi$, $I_Y=I_S\times\cos\omega\cos\psi$, $I_Z=I_S\sin\omega$, and $L=M$. Then $\mathbf{H}\cdot\mathbf{I}=HI_S\cos\psi\cos\omega-I_S^2(M\cos^2\omega+N\sin^2\omega)$ and

$$\partial E_\theta/\partial\omega=I_S\sin\omega[2I_S(M-N)\cos\omega-H\cos\psi]. \qquad (8)$$

In the most unfavorable case ω is given by the largest value of $\partial E_\theta/\partial\omega$ which is $\pm 1.12a_F$; this occurs when I_S is in the direction of maximum torque in the (110) plane. Setting $\cos\omega=1$ and $\cos\psi=1$,

$$\sin\omega=\frac{-1.12a_F}{I_S[2I_S(M-N)-H]}. \qquad (9)$$

For the ellipsoid used in our experiments, $a_F=1.70\times10^5$, $I_S=1625$, $M-N=-12.03$ (the ratio of diameter to thickness was 54.4). For these constants, Eq. (9) becomes

$$\sin\omega=117/(39{,}100+H).$$

For $H=0$, $\omega=10'$ and for $H=5000$, $\omega=9'$. If we had used an ellipsoid whose diameter to thickness ratio was ony ten, we would find $\omega=13'$ and $11'$ for the largest possible deviations in the two cases.

We have thus shown quantitatively that the demagnetizing field normal to the plane of the disk (ellipsoid, to be exact) is so strong as to make appreciable deviations from magnetization in the plane of the disk impossible.

Experiments on Single Crystals

Preliminary experiments were carried out with seven single crystals of silicon iron,[8] which were all in the form of disks about 2 cm in diameter and 0.05 cm thick. An external field of about 2700 oersteds was used, making the effective field about 2400 oersteds.

The values of χ and ϕ as deduced from the angles of zero torque, did not check at all satisfactorily with the x-ray orientations (obtained by the usual back-reflection method), which were known to an accuracy of 1°. The discrepancies ranged from $1\frac{1}{2}°$ to $5\frac{1}{2}°$ and corresponded to errors in θ_{13} and in θ_{24} of 1° to 2°. As the latter could be measured to an accuracy of about 0.1°, torque curves of two specimens were analyzed on a Coradi harmonic analyzer, and terms of small but not at all negligible amplitude were found in addition to the expected terms in 2θ and 4θ.

The maximum torque had been observed to increase as much as 2 percent on changing the external field from 2700 to 3100 oersteds, and as this was not a shift of the curve in the manner of Fig. 7 but a real increase in the maximum, we filed one of the disks into an ellipsoid of revolution in order to avoid the complicated effects of nonsaturation in a disk. The proper shape was found by projecting the magnified image of the disk which was fixed on a rotating shaft onto a screen containing a drawing of an ellipse of the proper size. After the filing operation, at least

[8] These disks were kindly lent by Dr. K. J. Sixtus of the General Electric Co. Research Laboratories. The chemical analysis of the disks was: 2.8 percent Si; 0.02 percent C; 0.10 percent Mn; 0.021 percent S; P, nil.

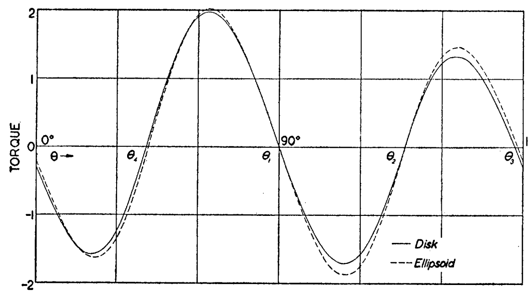

FIG. 7. Experimental torque curves for single crystal in disk and in ellipsoid form.

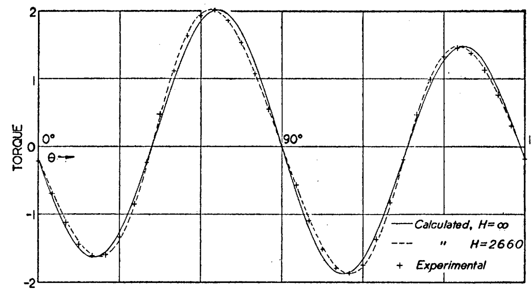

FIG. 8. Torque curves calculated for infinite field and for $H=2660$ oersteds, together with experimental points for ellipsoid.

some of the distorted surface layer was removed by applying a piece of waste soaked in $6N$ nitric acid to the rotating sample. Although any remaining distorted material might affect the results, it seems reasonable to believe that this material was randomly oriented and gave rise to no net torque; thus the only result would be to decrease the effective mass of the sample.

Torque measurements made on the ellipsoid for positions of torque maxima showed no detectable change when the field was increased as previously, so that the crystal could be assumed to be completely saturated. Torque curves of this single crystal both in the form of a disk and of an ellipsoid are shown in Fig. 7. The x-ray orientation of the sample is given by $\chi=84°$, $\phi=17\frac{1}{2}°$. Not only do the curves differ in their peaks (the curves are for the same mass of sample and the torques are in arbitrary units) but the zeros of torque do not coincide. The torque zeros of the ellipsoid curve given an orientation of $\chi=82.8°$, $\phi=18.0°$, or $1\frac{1}{2}°$ away from the x-ray orientation, while for the same sample in the form of a disk $\chi=80.2°$ and $\phi=20.3°$, a discrepancy of about $4\frac{1}{2}°$. Since the assumed x-ray orientation may itself be in error by 1°, the discrepancy of $1\frac{1}{2}°$ in the case of the ellipsoid is reasonably small and we can say that the agreement is very satisfactory.

In Fig. 8 two torque curves have been drawn, one calculated for $H=\infty$ and the other for $H=2660$ oersteds, the external field in the experimental work on the ellipsoid. Both curves were calculated for the orientation of the ellipsoid as given by x-rays, and the ordinate scale was calculated for the experimental value of a_F. The experimental points are shown as crosses, and it is seen that they fall fairly accurately on the dashed curve, the one for $H=2660$. Close inspection reveals that in some places the correction for the field is a little too large, but this slight disagreement very likely arises from the use of an inexact x-ray orientation in calculating the curve for $H=\infty$.

RESULTS OF TORQUE MEASUREMENTS

In order to show the changes that occur in the torque curves, it may be worth while to give the results of the harmonic analyses for the disk, for the ellipsoid and also for the ellipsoid when corrected according to Eq. (7). In Table I, one unit corresponds to 200 mg as weighed on the balance, so that the last decimal place is not to be regarded too seriously. The harmonic analyses could be checked within 0.01 or 0.02 unit. The torque curve is of the form $\Sigma A_n \sin 2n\theta + B_n \cos 2n\theta$. We see from this table that for this orientation at least the higher harmonics present

TABLE I. *Values of the coefficients in the harmonic analysis for the disk, for the ellipsoid, and for the ellipsoid corrected for H.*

	DISK	ELLIPSOID	ELLIPSOID, CORRECTED FOR H
A_1	1.52	1.48	1.44
A_2	−8.22	−8.87	−8.87
A_3	−0.16	−0.11	0.04
A_4	0.41	0.54	−0.06
A_5	0.04	0.00	—
A_6	—	0.03	—
B_1	−0.82	−0.52	−0.53
B_2	−0.77	−0.54	−0.62
B_3	0.04	0.03	−0.02
B_4	0.07	0.07	0.01
B_5	0.02	0.01	—
B_6	—	−0.01	—

in the disk and ellipsoid curves disappear when the field correction is applied. On the other hand, this correction does not greatly affect the value of the 2θ and 4θ terms, from which a_F is obtained; but the shape of the sample does affect the value of a_F.

The value of a_F derived for the ellipsoid is 1.70×10^5 ergs/cc and the estimated uncertainty is 0.01×10^5 ergs/cc, provided the randomly distorted surface is negligible in thickness. For the disk, a_F was calculated to be 1.60×10^5 ergs/cc, which is 6 percent too low, as would be expected because the disk near its edges is not properly magnetized to contribute its share of the torque.

In view of the possible experimental errors, the agreement between the experimental and calculated torque curves for the ellipsoid is highly satisfactory and it would be unwarranted to seek further information about the form of the anistropy from the minute discrepancies that may be left.[9] As for the disk, its shape affects not only the value of a_F but may in some cases give rise to a fictitious second anisotropy constant to take care of the 6θ and 8θ terms which occur because of the shape.

To summarize, we found that $a_F=1.70\times10^5$ ergs/cc for 2.8 percent silicon iron and that our results are adequately described by this single constant.

Results of Other Investigators

In discussing Webster's[10] torque measurements on a single crystal of iron cut as a fairly thick disk parallel to the (100) plane, Bitter[11] found an apparent increase of a_F with the field, even at very high fields of several thousand oersteds. This can now be explained satisfactorily by the incomplete saturation of a disk-shaped sample.

Gans[12] used the results of Honda and Kaya[13] to calculate a value of a_F' for pure iron. In the curves given by Honda and Kaya $\theta_{13}=87°$ instead of 90°, as should be the case for the ellipsoid used, which had [110] for its axis of revolution; this leads to an uncertainty in the angular measure. Also an effective field of only 507 oersteds may be insufficient to eliminate small low field deviations from the curve which would be obtained at several thousand oersteds. Bozorth[14] shows in Fig. 1 of his paper the slight effect on the torque curve of a value of a_F' similar to the value found by Gans. The uncertainties in the original data seem to be large enough to make them unsuitable for extracting information about the magnitude of a_F'.

Williams[15] has made torque measurements on a disk cut parallel to the (110) plane. The discrepancy between his calculated and experimental curves can be explained by reference to Figs. 7 and 8.

The existence of a second anisotropy constant has been postulated to explain the existence of ferromagnetic alloys with [110] as a direction of easy magnetization. In order that we can have more than a qualitative check of the magnetization curves, such as has been obtained by Bozorth,[14] it is necessary to determine accurately the form of the anisotropy and this is possible only if the pitfalls discussed above are avoided. With a really accurate knowledge of the anisotropy constants, as obtained directly from torque measurements on single crystal ellipsoids cut in the proper directions, it should be possible to construct quite accurate magnetization curves. In such a case, any discrepancy between theory and experiment would call for a re-examination of the basic magnetization theory rather than cast doubt upon the correctness of the anisotropy constants which are involved.

[9] We found that by letting $a_F'=0.5\times10^5$ erg/cc (see Eq. (1)), $A_3=0.04$ and $B_3=-0.025$, which would agree nicely indeed with the values in the last column of Table I. Yet if we introduce a_F' to account for these very small coefficients, we should introduce an energy term of the form $a_F''\times$(eighth order terms in α) in order to account for A_4, which is of the same size as A_3. The introduction of these higher order terms would account not only for harmonics that may be of an entirely accidental nature, but would give rise to appreciable terms in 2θ and 4θ, which in turn would affect the calculated value of a_F. Thus the addition to E_θ of a correction term in a_F' alone does not seem justifiable, at least in the present case.

[10] W. L. Webster, Proc. Roy. Soc. **107**, 496 (1925).

[11] F. Bitter, Phys. Rev. **39**, 371 (1932).

[12] R. Gans, Physik. Zeits. **33**, 924 (1932).

[13] K. Honda and S. Kaya, Tôhoku Univ. Sci. Reports **15**, 721 (1926).

[14] R. M. Bozorth, reference 7.

[15] H. J. Williams, reference 6.

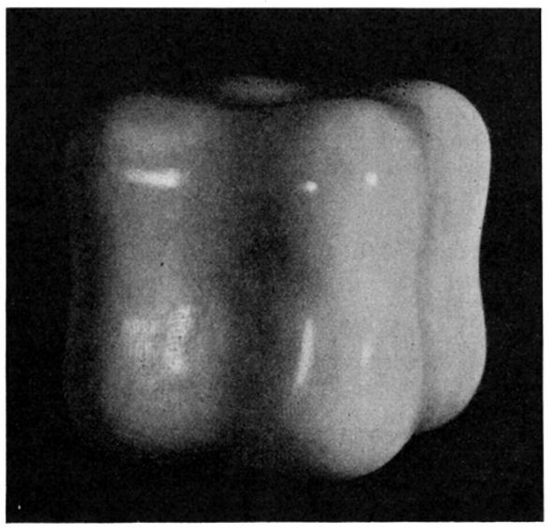

FIG. 1. Model of E_θ for iron, $a_F > 0$.

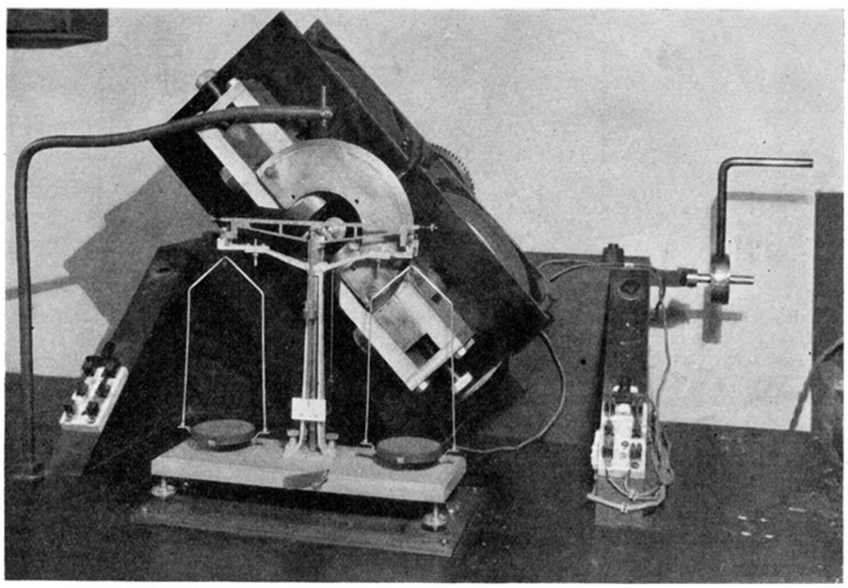

FIG. 2. Experimental set-up showing torque balance and rotating pole pieces.